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Analysis of Planar Graded-Index Optical Waveguides with Strong Asymmetry of a Refractive Index Profile by the Beam Propagation Method

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In the paper application of the beam propagation method (BPM) to investigation of wave processes in regular and nonregular graded-index waveguide structures with strongly asymmetric refractive index profile is analyzed. On purpose to match the BPM requirement to smoothness of the refractive index profile, the profile with strong asymmetry, at the problem solution, is approximately exchanged by a symmetrical one. For exponential and Gaussian profiles of the refractive index the BPM calculations are compared to the strict and WKB solution respectively. Calculation of a wave pattern of a mode field coupling into the substrate at the tapered edge of a waveguide with strongly asymmetric profile is demonstrated.

Key words and phrases: graded-index optical waveguide, beam propagation method.

1. Introduction

A substantial number of works has been devoted to the electrodynamic analysis of planar graded-index optical waveguides. The basic research technique is observed, for example, in books [1, 2]. Among them are: i) strict analysis based on a modal field description of a waveguide, applicable to the limited number of refraction index profiles $n(x)$ of the waveguide layer for which the strict solution of an electrodynamic problem is known; ii) approximate ray methods, in particular, the WKB method valid in cases of a smooth variation of the refractive index, if the index change at a wavelength distance is much less than $n(x)$; iii) numerical methods, including the one based on an approximate description of the continuous refractive index distribution by a step-function with a subsequent matching of fields at the layer boundaries [3]. These and similar methods provide a possibility to obtain fields and mode propagation constants of regular waveguides. However, problems of waveguide optics are far not settled by propagation of light in regular waveguides. Any waveguide components and circuits (transitions, bends, splitters, couplers, etc.) in which an energy exchange occurs are constructed on the base of nonregular waveguides with parameters being a function of the longitudinal coordinate z. Application of the above methods to analysis of nonregular waveguide structures leads to remarkable difficulties, and to gain the solution is possible, as a rule, in the elementary cases [4]. Along with it, throughout last years, an approximate numerical beam propagation method (BPM) first offered for study of laser light propagation in the atmosphere [5] and then in graded-index optical fibers [6] is being widely used. Absence of necessity to determine eigenfunctions of waveguides, application of the Fast Fourier Transform routine ensure the BMP accuracy and high computing efficiency.

The key requirement which narrows down the field of applicability of the BPM is the mentioned above smoothness of the refractive-index function of the graded-index layer. However, in practice there are many regular and nonregular waveguide structures, for example, slab waveguides on a substrate in integrated optics [7] with a graded-index layer matching the smoothness requirement, but having a step of refractive index on the boundary between the cover (usually air, $n_1 = 1$) and substrate ($n_2 \approx 1.5 \div 3.6$). Such profiles of index of refraction are usually referred as strongly asymmetric [2] or profiles with strong asymmetry [7]. Immediate application of the

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BPM for examining light propagation in such structures is incorrect. Nevertheless, the problem can be solved in an approximate approach if we combine the BPM and a known method of approximate analysis of fields in waveguides with strong asymmetry of the refractive index profile [7].

2. Solution Technique and the Purpose of the Work

The way of solution is that on purpose to match the requirement of the BPM to smoothness of the refractive index profile, a profile with strong asymmetry at the problem solution is approximately substituted by a symmetric one. In doing so, the field of a waveguide mode in the substrate of the asymmetric waveguide is approximately presented by half of the field distribution of an odd mode of the respective symmetrical waveguide, and the field in the cover is assumed be equal to zero. This approach is based on the fact that in a waveguide with strong asymmetry of refractive index profile the electromagnetic field penetrates into the cover only on a negligible distance, that smaller, than greater is the step of refractive index at the boundary cover — substrate. This condition provides a possibility of neglecting the field in the cover and considering its intensity at the boundary cover — substrate $x = 0$ to be zero (we assume that x is a transverse coordinate and the graded-index profile varies in this direction, z — the direction of propagation, and y — the second transverse coordinate on which the problem does not depend). As the BPM does not take into account polarization of waves, one can choose between tangent to the boundary components of electric field: $E_y(x)$ for the TE-modes, or $E_z(x)$ for the TM-modes. Differently, we choose the modes of the symmetric waveguide with the field that matches boundary conditions at the equivalent electric wall in the plane $x = 0$. We will restrict further consideration to TE-modes and observe an electric component $E_y(x)$ which, in compliance with the said above, should be an odd function of the coordinate.

Using the BPM, first of all one should set an external field $E_{y,0}(x)$ at the input $z = 0$ of the waveguide. This field whenever possible should be close to the expected distribution of the waveguide mode. Further, there follows a consecutive scaling to step Δz of the wave process propagating down the longitudinal axis. For the problem solution, i.e. the field of a waveguide mode, we will take a steady-state field distribution at a long distance from the waveguide input. Strictly speaking, this approach is fully correct if only one (fundamental) mode propagates in the waveguide; however, just this one is as a rule of practical interest.

The purpose of this work is to estimate the possibility of applying the described technique to analysis of light propagation in slab graded-index waveguide structures with strong asymmetry of a refractive index profile, with the accent on regular waveguides, as in this case it is possible to compare the results obtained by the BPM with strict electrodynamic solutions.

3. Technique and Results of the Numerical Analysis

In this work we used a version of the BPM based on the Fast Fourier Transform routine [6]. The calculations are realized by means of a computer program. All calculations were carried out on a grid of 512×200 points in the x- and z-directions respectively. The grid step Δx was of 0.25 μm at a wavelength of 0.63 μm, and the length of the step Δz varied according to the refractive index profile and was of an order of some microns. To avoid the inaccuracy due to reflection at the lateral borders of a computing grid, absorption smoothly increasing at approach to the grid ends was entered into the refractive index distribution. The waveguide with a symmetric profile was placed in the middle of a computing grid $x = 0$. An external field distribution at the waveguide input $z = 0$ was set in the form of an odd function

$$E_{y,0}(x) = x \exp\left(-\frac{x}{w}\right), \tag{1}$$

where w is the parameter that defines matching of the external field and the field of the principal mode of the waveguide. An arbitrary amplitude factor is omitted in this expression.

Fig. 1 shows the results of the BPM calculation of a wave process in a single-mode waveguide with an exponential refractive index profile [1]

$$n(x) = \begin{cases} n_1, & x < 0, \\ n_2 + \Delta n \exp\left(-\frac{x}{d}\right), & x \geqslant 0, \end{cases} \tag{2}$$

where Δn is the refractive index change on the surface of the substrate $x = 0$, and d — the thickness of the graded-index layer. The profile parameters are as follows: $n_1 = 1$, $n_2 = 2.47$, $\Delta n = 0.061$, $d = 0.4\,\mu$m. We see in Fig. 1a, that a wave process launched at the input of the waveguide $z = 0$ promptly gains a steady-state character, and it is possible to take for the waveguide mode the field distribution $E_y(x)$ in the left part of the pattern. Fig. 1b shows the process launched at another value of the parameter w of the input beam. We observe considerable oscillations of the field in the initial section of the waveguide attended by an intensive radiation into the substrate, which indicates a nonoptimal choice of the value w in the second case.

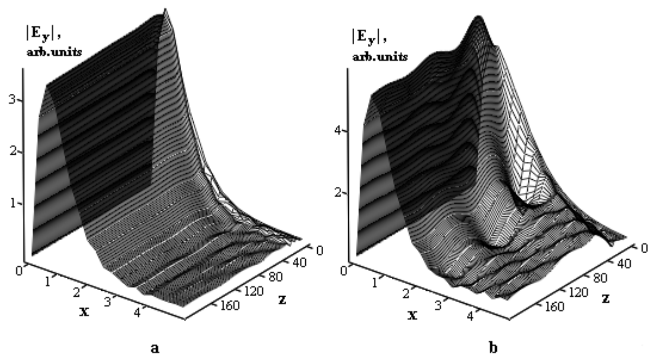

Figure 1. Field pattern in the substrate of a single-mode waveguide with exponential refractive index profile (2) computed by the BPM. The input beam width $w = 0.47\,\mu$m (a), $0.65\,\mu$m (b). Distances along x- and z-axes are given in micrometers

Fig. 2 shows the calculated electric field of the fundamental mode of a waveguide with an exponential profile (2). Curve 1 is the strict solution of a problem taking into account the field in the cover [1]. Curve 2 is approximation of curve 1 by half of the field distribution of an odd mode of a waveguide with symmetric exponential profile; this field is also obtained on the basis of the strict solution [1]. Curve 3 is computed using the WKB method [2,8] which, as is known and clearly visible in the figure, yields physically invalid result at the vicinity of the turning point, however, far from it the WKB solution approaches the strict solution. Nevertheless, the WKB approximation is often applied in cases when the strict solution is not known. At last, curve 4 was calculated by the BPM approach discussed here. A smooth dependence was obtained on discrete data by the third order spline interpolation. It is seen in Fig. 2 that curves 2 and 4 are in close agreement and, therefore, an evaluation inaccuracy while using the BPM is related in the core to replacement of the problem for a waveguide with strong asymmetry of the refractive index function by a problem about a symmetric waveguide, that shows up in loss of the reflectivity phase at the boundary $x = 0$.

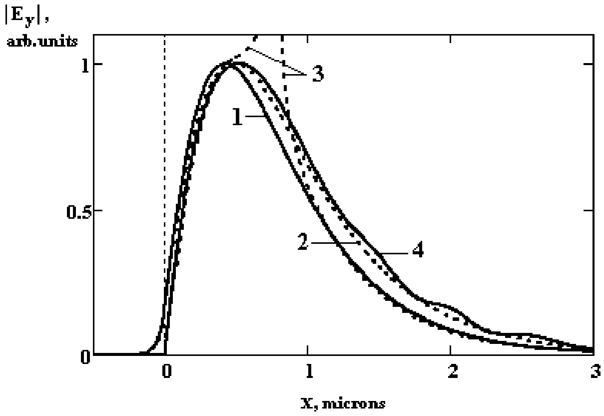

Figure 2. Electric field amplitude of the TE-mode in a single-mode waveguide with refractive index profile (2) **calculated by various methods**

Fig. 3 displays the results of calculation of electric field in a single-mode waveguide with Gaussian refractive index profile

$$n(x) = \begin{cases} n_1, & x < 0, \\ n_2 + \Delta n \exp\left(-\frac{x^2}{d^2}\right), & x \geqslant 0, \end{cases} \tag{3}$$

and typical parameters: $n_1 = 1$, $n_2 = 1.5$, $\Delta n = 0.1$, $d = 0.8\,\mu$m [8].

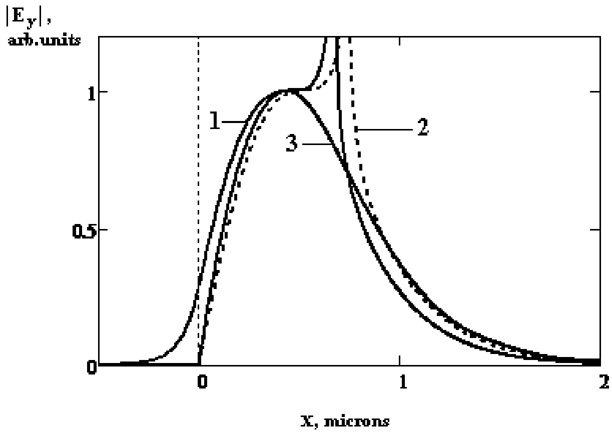

Figure 3. Electric field amplitude of the TE-mode in a single-mode waveguide with Gaussian refractive index profile (3) **calculated by various methods**

The strict electrodynamic solution for function (3) is not known, so for comparison we used the WKB approximation. Curve 1 presents the field of the mode computed by the WKB method taking into account the field in the cover. Curve 2 is the result of WKB calculation of the field of the respective odd mode in the symmetric waveguide, which approximates the field of the waveguide with strong asymmetry of profile. Curve 3 shows the result obtained by the BPM. Comparison of curves 2 and 3 points to a good correspondence of the fields computed by both methods far from the turning point; at the same time, near to last, unlike the WKB method, the BPM gives a physically correct solution. Comparison with curve 1 calculated at more accurate approximation, confirms the above conclusion about a principal cause of an evaluation inaccuracy of BPM calculation of fields in waveguides with strong asymmetry of refractive index

— loss of a reflectivity phase at the boundary $x = 0$. Comparison of figures 2 and 3 proves that with grows of the refractive index step at the boundary cover-substrate this inaccuracy goes down.

Fig. 4 shows possibility of performing BPM calculations for multimode waveguides. It displays computed by the BPM field distribution of the fundamental mode of a five-mode graded-index waveguide fabricated by an ion-exchange in glass technique ($n_2 = 1.513$) [8]. The refractive index profile of the graded-index layer was evaluated on the experimental data obtained by measuring the waveguide mode effective indices and WKB calculation of the respective turning point coordinates. On this discrete data a continuous function $n(x)$ used in BPM calculations was derived by a quadratic spline interpolation. Almost pure coupling of the fundamental mode was achieved diligently selecting the beam at the waveguide input. In this case, optimal turned to be a function similar (1) but with the first term x^3.

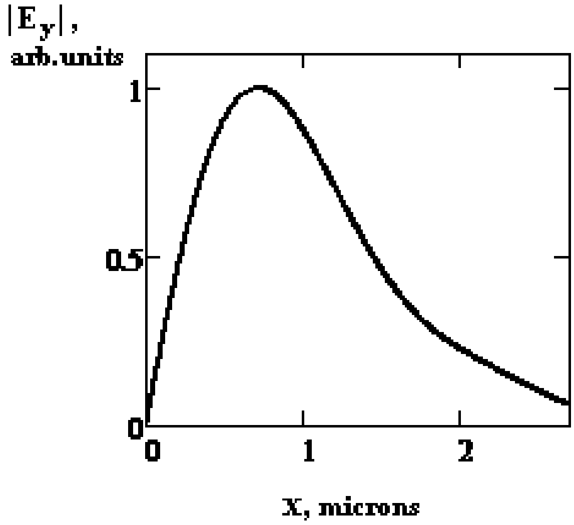

Figure 4. Electric field amplitude of the fundamental TE-mode in a multimode graded-index waveguide computed by the BPM for the refractive index profile evaluated on experimental data

Fig. 5 illustrates application of the method discussed to the analysis of nonregular waveguide structures. The figure presents the wave pattern calculated for a graded-index waveguide with two tandem sections: regular and tapered; the second is for a light coupling out into the substrate (or, on the contrary, for coupling into the waveguide from the substrate). In the model used, the nonregular section was a part of a waveguide on which the dopant density forming the graded-index layer, slowly, under the linear law decreased to zero in the z-direction. We see in the figure as the fundamental mode launched at the waveguide input $z = 0$ propagates along the regular section of 200 μm length, and then at the tapered section of the same length it radiates into the substrate in the form of a divergent light beam. The beam profile is observed at the left bottom of the figure. Evaluating by the figure the half-power beam width and taking into account distance to the beginning of the tapered section, which is 600 μm, it is possible to determine an angular divergence of the beam $\Delta\varphi \approx 3^\circ$. This value is in agreement with the experimental data [9] obtained for a tapered section of about the same length.

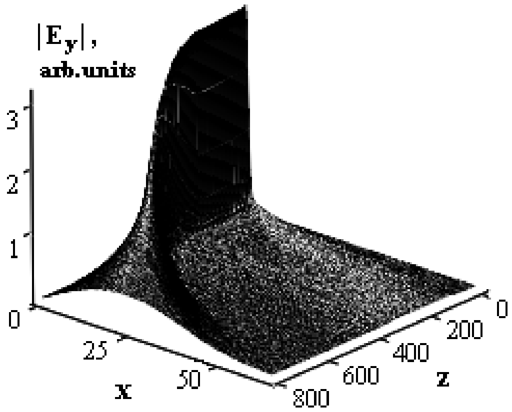

Figure 5. Field pattern of light radiation into the substrate of a graded-index single-mode waveguide with a tapered edge

4. Conclusion

Thus, by numeric computations it is shown that the BPM can be applied to analysis of planar regular graded-index waveguides and nonregular waveguide structures with a strongly asymmetric refractive index profile using approximate replacement of the profile with strong asymmetry by the respective symmetric one with the equivalent electrical wall in the plane of symmetry. For waveguides with typical exponential and Gaussian refractive indices, comparing computing results for the field of the fundamental mode with the strict and WKB solutions respectively gives the grounds to draw a conclusion, that inaccuracy at the BPM calculations is mainly due to loss of phase of reflectivity at the boundary cover — substrate at replacing the strong asymmetry profile by the symmetric one. It is numerically shown that this inaccuracy decreases as asymmetry of the profile grows. Also is demonstrated the BPM calculation of the pattern that shows coupling out a waveguide mode into the substrate by a tapered edge of a strongly asymmetric graded-index waveguide.

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Исследование плоских градиентных оптических волноводов с сильной асимметрией профиля показателя преломления методом распространяющегося пучка

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В статье анализируется возможность применения метода распространяющегося пучка (МРП) к исследованию волновых процессов в регулярных и нерегулярных градиентных волноводных структурах с сильно асимметричным профилем показателя преломления. С целью удовлетворить требованию МРП к плавности профиля показателя преломления профиль с сильной асимметрией при решении задачи приближенно заменен симметричным профилем. Для экспоненциального и гауссова профилей показателя преломления результаты вычислений с помощью МРП сравниваются со строгим и с ВКБ-решением соответственно. Продемонстрирован расчет волновой картины излучения моды в подложку на сужающемся крае волновода с сильно асимметричным профилем.

Ключевые слова: градиентный оптический волновод, метод распространяющегося пучка.