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High-Order Vector Nodal Finite Elements with Harmonic, Irrotational and Solenoidal Basis Functions

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In the present paper a concept of vector nodal finite element has been introduced, algorithms of construction of the vector nodal basis functions with high approximate properties from special functional spaces are presented. Examples of high-order interpolation of harmonic, irrotational vector fields by the developed finite elements illustrate their approximate advantage in comparison with the standard Lagrange elements.

Key words and phrases: vector nodal finite elements, harmonic, irrotational, solenoidal basis functions, interpolated polynomials, approximations of high order.

1. Introduction

The finite elements as independent objects may be used for solving elliptic problems by different ways such as the finite element method, the volume and boundary integral equation methods etc. The condition of calculated solutions accuracy is peculiarity of some practical problems [1]. The known methods and approaches do not always guarantee it. In this connection, special approaches are elaborated [2, 3]. One of possible ways to get over the complications is use of special high order approximations. In the present paper we suggest a new class of finite elements for vector-functions approximations with high accuracy.

According to the classical definition [4,5], a finite element is a triple (ω,P,Φ), where $\omega \in \mathbf{R}^n$ (n=2,3) is a closed subset with a Lipschitzian boundary and with a nonempty set of inner points often called as a cell or a finite element; P is an m-dimensional space of functions defined on ω (usually this is a space of polynomials); Φ is a set of linearly independent linear functionals $F_i : P \to \mathbf{R}^1, i = 1, \ldots, m$. In the nodal finite elements $F_i(\varphi)$ is the value of a function φ at the node $x_i \in \omega$. If for a set of functions $\{N_j\}_{j=1,\ldots,m} \in P$ for each j the system of linear algebraic equations

$$F_i(N_j) = \delta_{ij}, \quad i = 1, \ldots, m, \tag{1}$$

is solvable, then any function $\varphi \in P$ can be represented in the form $\varphi(x) = \sum\limits_{i=1}^{m} F_i(\varphi) N_i(x)$. System (1) is used for finding coefficients in the representation and N_i $(1 \leqslant i \leqslant m)$ is called as basis or shape function. Accuracy of interpolation by means of basis functions may be considered as local characteristic of a finite element. Obviously that the interpolation is exact for functions from P.

It should be noted that the finite elements defined above are used for approximation of scalar functions.

Introduce the notion of a vector nodal finite element.

Definition. Define a vector nodal finite element as a triple (ω,$\mathbf{P}$,Ψ), where $\omega \in \mathbf{R}^n$ is a cell in the classical definition; $\mathbf{P}$ is an $n{\cdot}m$-dimensional space of vector-functions defined on ω. In the Cartesian coordinate system $\mathbf{P} = \sum\limits_{k=1}^{n} \mathbf{i}_k P_k$, where P_k is the $n{\cdot}m$-dimensional space of functions in the classical definition; Ψ is a set of linearly independent linear maps $\mathbf{F}_i$: $\mathbf{P} \to \mathbf{R}^n$, $i = 1, \ldots, m$.

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Let's call the maps by vector-functionals in view of the fact that $\mathbf{F}_i = \sum\limits_{k=1}^{n} \mathbf{i}_k F_i^{(k)}$, where $F_i^{(k)} : P_k \to \mathbf{R}^1$, $i = 1, \ldots, m$, $1 \leqslant k \leqslant n$. In the vector nodal finite elements $\mathbf{F}_i(\mathbf{u})$ is the value of a vector-function $\mathbf{u}$ at the node $x_i \in \omega$. If for a set of vector-functions $\{\mathbf{W}_{k,j}\}_{k=1,\ldots,n;j=1,\ldots,m} \in \mathbf{P}$ for each j the system

$$\mathbf{F}_i(\mathbf{W}_{k,j}) = \mathbf{i}_k \delta_{ij}, \quad k = 1, \ldots, n, \quad i = 1, \ldots, m,$$

is solvable, then any vector-function $\mathbf{u} \in \mathbf{P}$ is represented in the form

$$\mathbf{u}(x) = \sum_{k=1}^{n} \sum_{i=1}^{m} F_i^{(k)}(u_k) \mathbf{W}_{k,i}(x).$$

We shall call the vector-functions $\{\mathbf{W}_{k,j}\}_{k=1,\ldots,n;j=1,\ldots,m}$ by vector nodal basis functions. Each element of the vector-functions linear span is interpolated on ω exactly.

Suppose the approximate solution $\mathbf{u}$ is found in the form of decomposition on the vector basis functions $\mathbf{W}_{k,i}(x)$ inside a cell

$$\mathbf{u}(x) = \sum_{k=1}^{n} \mathbf{i}_k u_k(x) = \sum_{k=1}^{n} \sum_{i=1}^{m} u_{k,i} \mathbf{W}_{k,i}(x),$$

where $u_{k,i}$ is the value of k-th component of approximate solution at the node x_i of the cell.

Consider the problem of harmonic, irrotational and solenoidal fields approximation of high order. For mathematical classification of the introduced finite elements, it is convenient to use the corresponding functional spaces [6, 7]

$$\begin{aligned} \mathbf{Z}(\Omega) &= \{\mathbf{u} \in W_2^1(\Omega)^n : \nabla \cdot \mathbf{u} = 0, \quad \nabla \times \mathbf{u} = 0\}, \\ \mathbf{U}(\Omega) &= \{\mathbf{u} \in W_2^1(\Omega)^n : \nabla \cdot \mathbf{u} = 0, \quad \nabla \times \mathbf{u} \in L_2(\Omega)^n\}, \\ \mathbf{V}(\Omega) &= \{\mathbf{u} \in W_2^1(\Omega)^n : \nabla \cdot \mathbf{u} \in L_2(\Omega), \quad \nabla \times \mathbf{u} = 0\}, \end{aligned}$$

where $W_2^1(\Omega)^n$ is a space of vector-functions such that each component of these vector functions belongs to the Sobolev space $W_2^1(\Omega)$ [4,5]. Suppose also that each component of the basis functions $\mathbf{W}_{k,i}(x)$ is expressed by partial derivatives of scalar functions. Further, consider a case, when $n = 3$. At $n = 2$ similar results are valid.

2. Algorithms of basis functions construction, characteristics of accuracy

2.1. Harmonic fields

Let $\mathbf{u} \in \mathbf{Z}(\Omega)$. In such a case, we seek the solution in the form

$$\mathbf{u}(x) = \sum_{k=1}^{3} \sum_{i=1}^{m} u_{k,i} \mathbf{Z}_{k,i}(x).$$

Here, $\mathbf{Z}_{k,i}(x)$ is basis function from $\mathbf{Z}(\Omega)$.

Represent $\mathbf{Z}_{k,i}$ in the form

$$\mathbf{Z}_{k,i}(x) = \sum_{l=1}^{3m} a_l^{(k,i)} \nabla f_{g(l)}(x),$$

where $a_l^{(k,i)}$ is unknown coefficient, $g(l)$ is an index function, $f_{g(l)}$ is harmonic function from the set

$$M_Z = \left\{ c_{n,k} \left(\frac{r}{\rho}\right)^n \cos(k\varphi) P_n^k(\cos\vartheta),\ c_{n,k} \left(\frac{r}{\rho}\right)^n \sin(k\varphi) P_n^k(\cos\vartheta), \right.$$
$$\left. (n,k) = (0,0); (1,0), (1,1); (2,0), (2,1), (2,2); \ldots \right\}; \quad (2)$$

in the spherical coordinate system (r, ϑ, φ) ρ is radius of the circumscribed sphere, $c_{n,k} = (2n+1)(n-k)/(n+k)!$, P_n^k is the Legendre joined function. The functions from M_Z are calculated by recursion relations [8]. The unknown coefficients are obtained as a result of solving the systems

$$\sum_{l=1}^{3m} a_l^{(k,i)} \nabla f_{g(l)}(x_j) = \mathbf{i}_k \delta_{ij}; \quad f_{g(l)} \in M_Z; \quad x_j \in \omega, \quad j = 1, \ldots, m;$$
$$k = 1, 2, 3; \quad i = 1, \ldots, m. \quad (3)$$

Accuracy of approximations by the basis functions depends on degree of harmonic polynomials, gradients of which are included in (3). To construct the index function $g(l)$, a special algorithm, elaborated by the authors, is described below.

2.2. Irrotational fields

As $\mathbf{u}(x) \in \mathbf{V}(\Omega)$, the solution is found in the form

$$\mathbf{u}(x) = \sum_{k=1}^{3} \sum_{i=1}^{m} u_{k,i} \mathbf{V}_{k,i}(x),$$

where

$$\mathbf{V}_{k,i}(x) = \sum_{j=1}^{3m} b_j^{(k,i)} \nabla h_{g(j)}(x).$$

Here, $b_j^{(k,i)}$ is unknown coefficient, $g(j)$ is an index function, $h_{g(j)}$, $j = 1, \ldots, m$, are functions from the set

$$M_V = \{x_1^{k1} x_2^{k2} x_3^{k3}, \quad (k_1, k_2, k_3) = (0,0,0); (1,0,0), (0,1,0), (0,0,1);$$
$$(2,0,0), (1,1,0), (0,2,0), (1,0,1), (0,1,1), (0,0,2); \ldots\}. \quad (4)$$

The unknown coefficients are obtained as a result of solving the systems

$$\sum_{l=1}^{3m} b_l^{(k,i)} \nabla h_{g(l)}(x_j) = \mathbf{i}_k \delta_{ij}; \quad h_{g(l)} \in M_V; \quad x_j \in \omega, \quad j = 1, \ldots, m;$$
$$k = 1, 2, 3; \quad i = 1, \ldots, m. \quad (5)$$

For construction of the index function $g(l)$, the same algorithm is used. Accuracy of approximations by the basis functions depends on degree of polynomials from M_V, gradients of which are included in (5).

2.3. Solenoidal fields

Let $\mathbf{u} \in \mathbf{U}(\Omega)$ and $\mathbf{u}(x) = \nabla \times \mathbf{A}$, where $\mathbf{A} = \sum\limits_{l=1}^{3} \mathbf{i}_l A_l$ is a vector potential. Then the case of solenoidal fields is reduced to previous one in view of the formula

$$\mathbf{u}(x) = \nabla \times \mathbf{A}(x) = \sum_{l=1}^{3} (\nabla A_l \times \mathbf{i}_l) = \sum_{l=1}^{3} \left(\left(\sum_{k=1}^{3} \sum_{i=1}^{m} (\nabla A_l)_{k,i} \mathbf{V}_{k,i}(x) \right) \times \mathbf{i}_l \right),$$

where $(\nabla A_l)_{k,i}$ is the k-th component of the vector ∇A_l at the node $x_i \in \omega$ and $\mathbf{V}_{k,i}(x) \in \mathbf{V}(\Omega)$, $k = 1, 2, 3$, $i = 1, 2, \dots, m$. Accuracy of the approximations depends on degree of polynomials, gradients of which are included in the representation for $\mathbf{V}_{k,i}$.

2.4. Algorithm of index function generation

The algorithm is based on control of solvability of systems (3), (5) with the help of calculation of matrix singular values by the SVD algorithm [9]. Assume that the initial approximation is given as first numbers of functions from sets (2), (4): $2, \dots, n_0$, $2 < n_0 < 3m$. Set $q = n_0 + 1$ and $q_{\max} = 30$. The algorithm consists of three steps.

1) Form the matrix of the system and use the set of $3m$ elements

$$\{2, \dots, n_0, q, 2, \dots, 2\}$$

as values of the function g. Here, number q of the next function is located on a position with number n_0.

2) Find singular values by the SVD algorithm.

3) If there are $3m - n_0 + 1$ zeros among of the matrix singular values, then set $q = q + 1$; if $q < q_{\max}$, then go to step 1). If there are $3m - n_0$ zeros among of the matrix singular values and $n_0 \neq 3m$, then $n_0 = n_0 + 1$, $q = q + 1$ and go to step 1). If $n_0 = 3m$, then the required values set of g has been constructed.

2.5. Examples of vector nodal finite elements

Vector finite elements can include different types of cells ω and can have different location of nodes in the cells. Let us consider on 6 elements with basis functions from the spaces $\mathbf{Z}(\Omega)$, $\mathbf{V}(\Omega)$, $\mathbf{U}(\Omega)$ as an example of three-dimensional vector nodal finite elements. Information about type of the cell, number of nodes and their location, and also about structure of the sets M_Z and M_V is presented in Table 1 and in Figs. 1–3.

Table 1

Characteristics of elements

Type of cell	$\mathbf{Z}(\Omega)$			$\mathbf{V}(\Omega)$,$\mathbf{U}(\Omega)$		
	m / N	D	$D_{\max}$	m / N	D	$D_{\max}$
Tetrahedron	4 / 12	2	3	4 / 12	2	3
	10 / 30	4	5	10 / 30	3	5
	26 / 78	7	9	27 / 81	5	7
Hexahedron	8 / 24	3	5	15 / 45	3	6
	20 / 60	6	8	23 / 69	5	7
	26 / 78	7	9	45 / 135	5	9

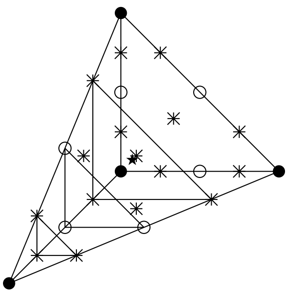

Figure 1. Cell in the form of a tetrahedron: • - 4 nodes; •, ○ - 10 nodes; •,○,∗ - 26 nodes; •,○,∗,⋆ - 27 nodes

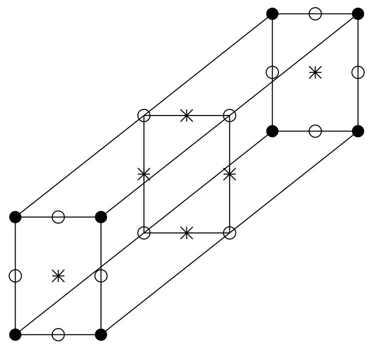

Figure 2. Cell in the form of a hexahedron : • - 8 nodes; •, ○ - 20 nodes; •,○,∗ - 26 nodes

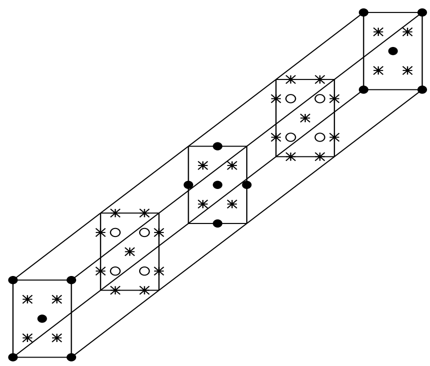

Figure 3. Cell in the form of a hexahedron : • - 15 nodes; •, ○ - 23 nodes; •,∗ - 45 nodes

In Table 1 m is the number of nodes, N is the number of vector functions from the linear span, D is the maximal order of polynomials whose gradients are approximated by the basis functions exactly, $D_{\max}$ is the maximal order of polynomials contained in the basis functions. Note that the vector nodal finite elements from $\mathbf{Z}(\Omega)$ have not inner nodes.

Let us define the space $\mathbf{P}$ from **Definition** in more detail. By $\mathbf{P}(t,N,Z)$ and $\mathbf{P}(h,N,Z)$ we denote the sets of N vector-functions from $\mathbf{Z}(\Omega)$ defined on a tetrahedron and on a hexahedron, respectively. Moreover, each element of linear spans of the sets is approximated by the basis functions exactly. Similarly, for the sets from $\mathbf{V}(\Omega)$, the notation $\mathbf{P}(t,N,V)$, $\mathbf{P}(h,N,V)$ is introduced. Then, for the finite elements in Table 1, we have

$$\mathbf{P}(t,N_1,Z)=\left\{\mathbf{v}=\sum_{i=1}^{N_1} c_i\nabla f_{g(i)},\ c_i\in\mathbf{R}^1,\ f_{g(i)}\in M_Z,\ g(i)\in I^t_{N_1}\right\},$$

$$\mathbf{P}(h, N_2, Z) = \left\{ \mathbf{v} = \sum_{i=1}^{N_2} c_i \nabla f_{g(i)},\ c_i \in \mathbf{R}^1, f_{g(i)} \in M_Z,\ g(i) \in I^h_{N_2} \right\},$$

$$\mathbf{P}(t, N_3, V) = \left\{ \mathbf{v} = \sum_{i=1}^{N_3} c_i \nabla h_{g(i)},\ c_i \in \mathbf{R}^1, h_{g(i)} \in M_V,\ g(i) \in J^t_{N_3} \right\},$$

$$\mathbf{P}(h, N_4, V) = \left\{ \mathbf{v} = \sum_{i=1}^{N_4} c_i \nabla h_{g(i)},\ c_i \in \mathbf{R}^1,\ h_{g(i)} \in M_V,\ g(i) \in J^h_{N_4} \right\},$$

for $N_1 = 12, 30, 78,\ N_2 = 24, 60, 78,\ N_3 = 12, 30, 81,\ N_4 = 45, 69, 135$. Here, the index sets $I^t_{N_1}, I^h_{N_2}, J^t_{N_3}, J^h_{N_4}$ are the sets of values for corresponding functions g. Their structure is given in Table 2 and Table 3.

Table 2

Structure of the index sets for harmonic finite elements

Set	Elements of the set
I^t_{12}	2,3, . . . 13
I^t_{30}	2,3, . . . 29,31,32
I^t_{78}	2,3, . . . ,78,82
I^h_{24}	2,3, . . . , 19,21,22,23, 25,30,34
I^h_{60}	2,3, . . . , 56,58,62,66,67,73
I^h_{78}	2,3, . . . , 75,81,82,83,84

Table 3

Structure of the index sets for irrotational finite elements

Set	Elements of the set
J^t_{12}	2,3, . . . , 10,12,15,17
J^t_{30}	2,3, . . . , 20,22,23,26, . . . , 30,32,37,42,46
J^t_{81}	2,3, . . . ,71,74,75,78,82,83,86,87,93,94,95,99
J^h_{45}	2,3, . . . , 20,22,23,24,26, . . . , 34,37, . . . ,40,42, . . . , 48,58,64,67
J^h_{69}	2,3, . . . , 56,58,60,64, . . . ,69,71,73,75,76,94,96
J^h_{135}	2,3, . . . , 56,58,. . . , 62,64, . . . , 83,86, . . . , 91, 93, . . . , 116,118,119,122, . . . , 126,130, . . . , 137,139,143,145, . . . , 148,152,177,179,181

3. Examples of interpolation by the vector nodal finite elements

Consider three examples. Let β be a number of example. Also introduce the notation

$$\delta^{(\sigma)}_\beta = \max_{x \in \omega} |\mathbf{Q} - \mathbf{I}_\sigma(\mathbf{Q})| / Q_{\max},$$

where σ defines the type of finite elements, $\mathbf{Q}$ is a vector-function under interpolation on ω, $Q_{\max}$ is its maximal value on the considered region, $\mathbf{I}_\sigma(\mathbf{Q})$ is the interpolant

$$\mathbf{I}_\sigma(\mathbf{Q}) = \sum_{k=1}^{3}\sum_{i=1}^{m} Q_{k,i}\mathbf{W}_{k,i}(x),$$

where m is the number of nodes in the cell, $Q_{k,i}$ is the value of k-th component at the node x_i, $\mathbf{W}_{k,i}$ is the vector basis function. Table 4 gives the basic notation. Here, $N_i(x)$ denotes a usual Lagrange basis function [4, 5].

Table 4

Notation

σ	Nodal finite elements	$\mathbf{W}_{k,i}(x)$
1	vector harmonic	$\mathbf{Z}_{k,i}(x)$
2	vector irrotational	$\mathbf{V}_{k,i}(x)$
3	scalar Lagrange's elements	$\mathbf{i}_k N_i(x)$

Example 1. The harmonic function interpolated on the unit cube $[-1,1]^3$ has the following form:

$$\mathbf{H}^{(7)}(x) = \nabla\left(\sum_{n=0}^{7} r^n \sum_{k=0}^{n} c_{n,k}(\cos(k\varphi)+\sin(k\varphi))P_n^k(\cos\vartheta)\right),$$

i.e. it is the sum of harmonic functions gradients from the set M_Z up to seventh order. Note that $\mathbf{H}^{(7)}$ varies strongly in the considered region: $0.5211 \leqslant |\mathbf{H}^{(7)}| \leqslant 0.11357E+04$. Then, we have $\mathbf{Q}(x) = \mathbf{H}^{(7)}(x)$, $Q_{\max} = H_{\max} = 0.11357E+04$.

Example 2. The harmonic magnetic field of two coaxial coils interpolated on the region $[40,50]^3$:

$$\mathbf{B}^S(x) = \nabla\times\frac{1}{4\pi}\int\limits_{\Omega_S}\frac{\mathbf{J}}{|x-y|}\mathrm{d}\Omega_y,$$

where, $|x-y|$ is the distance between points x and y. The coil region Ω_S is given by the following set:

$$\Omega_S = \{x=(r,\varphi,z) : 53 \leqslant r \leqslant 100;\quad 0 \leqslant \varphi \leqslant 2\pi;\quad 60 \leqslant \operatorname{sign}(z)z \leqslant 85\},$$

$\mathbf{J}(x) = \pi(116.348)\mathbf{i}_\varphi$, $x\in\Omega_S$. Here, $\mathbf{Q}(x) = \mathbf{B}^S(x)$, $Q_{\max} = B_0 = 0.2909375E+04$.

Example 3. The sum of polynomials gradients up to seventh order interpolated on the unit cube $[-1,1]^3$:

$$\mathbf{P}^{(7)}(x) = \sum_{|k|=1}^{7}\nabla(x_1^{k_1}\cdot x_2^{k_2}\cdot x_3^{k_3});\quad |k| = k_1+k_2+k_3,\quad k_i \geqslant 0,\quad i=1,2,3.$$

Here, $\mathbf{Q}(x) = \mathbf{P}^{(7)}(x)$, $Q_{\max} = P_{\max} = 0.3637307E+03$.

Interpolation of the vector-functions from Examples 1– 3 has been performed by the finite elements of different types. Comparison of the obtained results is shown in Table 5.

It should be noted that $\mathbf{H}^{(7)}\in\mathbf{Z}((-1,1)^3)$, $\mathbf{B}^S\in\mathbf{Z}((40,50)^3)$ and $\mathbf{P}^{(7)}\in\mathbf{V}((-1,1)^3)$. As one would expect, for Examples 1,2, the best interpolations have been obtained by

Table 5

Comparison of interpolations

σ	m / N	$\delta_1^{(\sigma)}$	$\delta_2^{(\sigma)}$	$\delta_3^{(\sigma)}$
1	8/24	1.0632E+00	1.3927E-03	4.4309E-01
	20/60	2.1227E-03	2.5963E-04	6.8534E-01
	26/78	1.5206E-15	8.6087E-06	3.1766E-01
2	15/45	1.1059E+00	1.3085E-03	2.1898E-01
	23/69	1.1654E+00	4.2181E-04	6.3755E-02
	45/135	5.6569E-01	1.7243E-04	1.1083E-02
3	27/27	6.7378E-01	7.9872E-04	1.8074E-01
	64/64	2.4368E-01	1.0137E-04	6.4156E-02
	125/125	6.4104E-02	9.5325E-06	1.5832E-02

means of the harmonic element ($\sigma = 1,\ m = 26,\ N = 78$). The same it is necessary to tell concerning the irrotational nonharmonic element ($\sigma = 2,\ m = 45,\ N = 135$) in Example 3. Moreover, for these interpolations the smaller number of nodes is required in two-four times, than the comparable in accuracy interpolations with the help of the usual Lagrange one.

4. Conclusion

To obtain high-order approximations for vector fields, we have introduced a concept of vector nodal finite element. In particular, the theoretical basis and the algorithms for constructions of the new class of finite elements have been elaborated for harmonic, irrotational and solenoidal fields. The new formulas for representation of approximations and special choice of basis functions from the corresponding functional spaces provide high order in this approach.The numerical examples, presented in the paper, show that harmonic and irrotational nonharmonic vector nodal finite elements for vector-functions from corresponding classes give the best approximations with smaller number of nodes than the Lagrange elements. This corresponds to the finite element method theory because the space of vector-functions $\mathbf{P}$ in **Definition** has dimension of $n{\cdot}m$ whereas in the Lagrange elements the space P is an m-dimensional space of functions.

The basis functions of the suggested finite elements with computer accuracy satisfy to homogeneous equations for intensity of magnetic field in air, ferromagnetic and current regions. This property can essentially reduce computing work for solving the magnetostatic problem in comparison, for example, with the standard hp-version of the finite element method. To obtain algebraic systems of equations, the principles of the discontinuous Galerkin schemes construction or the methods from [10] may be used. Note that enough efficient algorithms for solvers are given in [11].

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Векторные узловые конечные элементы высокого порядка с гармоническими, безвихревыми и соленоидальными базисными функциями

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В настоящей работе вводится понятие векторного узлового конечного элемента, представлены алгоритмы построения векторных узловых базисных функций с высокими аппроксимационными свойствами из специальных функциональных пространств. Примеры интерполяции с высоким порядком точности гармонических, безвихревых полей с помощью разработанных конечных элементов иллюстрируют их аппроксимационные преимущества по сравнению со стандартными лагранжевыми элементами.

Ключевые слова: векторные узловые конечные элементы, гармонические, безвихревые, соленоидальные базисные функции, интерполяционные многочлены, аппроксимации высокого порядка.