UDC 539.12.01; 539.124; 539.124.6

Vacuum Instability and Pair Production in Strong QED

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Strong external electromagnetic fields make the QED vacuum unstable which decays by emitting significantly boson or fermion particle-antiparticle pairs. The recent progress in studying the particle-antiparticle pair production phenomenon is reported: 1. New exact formulas for production rates of boson and fermion pairs by a smooth potential step $\phi(\mathbf{x}) \propto \tanh kz$ in three dimensions. 2. Exact expressions for reflection and transmission coefficients, as well as for average numbers of produced pairs and for pair production intensities obtained via the studying scattering versus tunneling process by this potential. 3. On this basis, re-examining and justify the normal spin-statistics relation, a highly nontrivial task due to the vacuum instability.

Key words and phrases: QED, particle-antiparticle pair production, vacuum instability, spin-statistics relation.

1. Introduction

Electron-positron pair production due to the instability of the quantum electrodynamics (QED) vacuum in an external strong electric field is a remarkable non-perturbative prediction of QED [1–3] that has not yet been directly observed. Recent advances in laser technology have raised hopes that the required critical field strength $E_{cr} \sim 10^{18} V/m$ may soon be within experimental reach [2, 4]. Observation of this elusive phenomenon would represent a significant progress in our understanding of non-perturbative effects in quantum field theory, with potentially important lessons for related phenomena such as Unruh and Hawking radiations.

However, the above mentioned Schwinger estimation of the critical field strength has been strongly restricted to spatially homogeneous and static electric fields. The study of corrections due to spatial or temporal inhomogeneities is a nontrivial task for a generic profile of electromagnetic field, and only certain field configurations have been worked out exactly. The basic physics of the process is a "tunneling" of a particle through an energy barrier of $2mc^2$ from the negative energy levels of the Dirac "sea" to the positive ones. The efficient method to solve this problem exactly is based on the use of causal Green functions to derive the pair production probabilities by means of the asymptotic solutions of wave equations in terms of scattering data [5, 6]. As a result, the pair production rates can be expressed via the logarithm of reflection coefficients as an ordinary energy-momentum integral over the region where the particle and the hole levels overlap each other (Klein region). This allows one to connect in a most transparent and efficient way the one-particle Dirac approach suffering from the Klein paradox with the second quantized field theory in which this paradox was first solved satisfactorily.

In recent papers [7–9], we study the scattering process of a single particle satisfying the relativistic Klein-Gordon and Dirac wave equations in an external Sauter potential of the form $e\varphi(z) = v \tanh kz$ corresponding to nonuniform electric field along z-direction. The parameter k defines the inverse width of the electric field, whereas the parameter v governs its size $|E| = vk/e$, whose maximum is given by the critical value $|E_c| \equiv m^2c^3/e\hbar \simeq 1.3 \times 10^{18}$ V/m. In the transverse direction the particle propagates freely as a plane wave. The typical scattering process by this potential involves an

Received 28^{th} November, 2009.

I would like to thank V. Ivanov, H. Kleinert and A. Nikishov for interest to this work and useful discussions.

incoming particle coming in from the left which is partly reflected back to the left and partly transmitted forth to the right through the potential barrier. For this process we find both the reflection and transmission coefficients exactly.

The physically more interesting situation comes then into about when the hight of the potential barrier v becomes larger than mc^2. In this case, the transmission coefficient of boson (fermion) particles becomes negative implying the Klein paradox. As its resolution, we must find the incoming antiparticle in the right region of space instead of outgoing particle. This happens because the usual separation of positive and negative energy states occurring in the absence of external fields is no longer ensured. Instead, there can be the region where an overlap of these states is allowed. In this level-crossing region, by means of tunneling between negative and positive states, the pair production of charged particle takes place with the rate determined by the transmission probability for a particle to cross the forbidden region.

2. Particle-Antiparticle Pair Production

Taking into account the conservation of the total probability, we express the production rate of boson and fermion pairs via the logarithm of reflection coefficient as an ordinary energy-momentum integral over the level-crossing (Klein) region as

$$w_\perp = (-1)^{2s+1}\,\frac{(2s+1)}{8\pi^2\hbar^3}\int\limits_0^{(v^2-m^2)} \mathrm{d}p_\perp^2 \int\limits_{-v+\sqrt{p_\perp^2+m^2}}^{v-\sqrt{p_\perp^2+m^2}} \mathrm{d}p_0\,\ln R(p_0,\,p_\perp^2)\,, \tag{1}$$

where s being the spin of a particle and the integration is done over the Klein region in the $(p_\perp^2, p_0)$-plane as shown in Fig. 1.

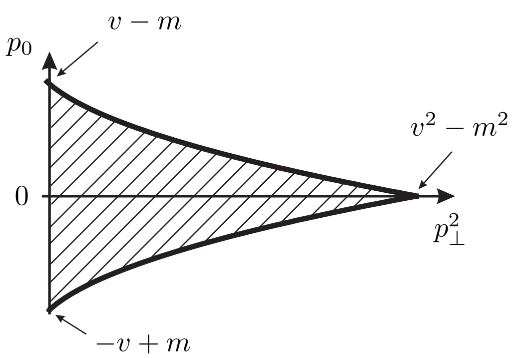

Figure 1. **In the $(p_\perp^2, p_0)$-plane, the level-crossing covers the positive region restricted by two intersecting parabolas $p_0 = v - \sqrt{p_\perp^2 + m^2}$ and $p_0 = -v + \sqrt{p_\perp^2 + m^2}$ with horizontal axes of symmetry above and below the $p_\perp^2$-axis for $v > m$**

Interchanging the order of integration, we perform the momentum integral in Eq. (1) to obtain for pair production rate the exact representation

$$w_\perp = \frac{(2s+1)}{3\pi}\,k^3\int\limits_{-\bar{\xi}}^{\bar{\xi}} \mathrm{d}\xi\, g(\xi)\,\frac{1}{e^{-2\pi(\xi-\kappa)} + (-1)^{2s}}\,, \tag{2}$$

where $\bar{\xi} \equiv \sqrt{v^2 - m^2}/\hbar k$, the parameter κ being $\kappa = \sqrt{v^2/k^2\hbar^2 - 1/4}$ for bosons and $\kappa = v/k\hbar$ for fermions respectively, and the function $g(\xi)$ is the density of boson (fermion) states

$$g(\xi) = \frac{\xi\left(\bar{\xi}^2 - \xi^2\right)^{3/2}}{\left(\bar{\xi}^2 - \xi^2 + m^2/\hbar^2 k^2\right)^{1/2}} = -g(-\xi)\,. \tag{3}$$

This function is the unique for bosons and fermions and counts the number of modes per unit ξ-interval. Contrary, the second function under the integral in Eq. (2) is different by sign for bosons and fermions. It represents the local probability for production of a single particle-antiparticle pair in a certain energy mode on this interval

$$t_s(\xi) = \frac{1}{e^{-2\pi(\xi-\kappa)} + (-1)^{2s}} = (-1)^{2s} - \frac{1}{e^{2\pi(\xi-\kappa)} + (-1)^{2s}} = (-1)^{2s}\left(1 - r_s(\xi)\right)\,, \tag{4}$$

where $r_s(\xi)$ is the local probability that no boson (fermion) pairs are created in this energy mode. This equation implies the conservation of the total probability for transition of an arbitrary initial state into all possible final states during the scattering process.

The average number of produced boson (fermion) pairs created in a certain energy mode on ξ-interval $-\bar{\xi} \leqslant \xi \leqslant \bar{\xi}$ is given by

$$\bar{n}(\xi) = \frac{t_s(\xi)}{1 + (-1)^{2s+1}\, t_s(\xi)} = \frac{t_s(\xi)}{r_s(\xi)} = \exp\left(2\pi(\xi - \kappa)\right)\,. \tag{5}$$

It represents the Bose-Einstein distribution for spin-0 particles, and the Fermi-Dirac distribution for spin-1/2 particles, respectively. This preserves therefore the normal relation between spin and statistics even though the vacuum instability invalidates the standard proof of the spin-statistics theorem of quantum field theory. In the recent paper [10], we find also the result (5) from calculating the ratio of transmission and reflection coefficients.

Finally, in the papers [7–9], we calculate the integral (2) as a series expansion in powers of small dimensionless parameter $0 < \tilde{k} < 1$. Introducing the probability rate $w \equiv w_\perp/L$, we find

$$\begin{aligned} w \;=\; & -(2s+1)\frac{(e|E|)^2}{8\pi^3\hbar^2}\times \\ & \times\left\{\mathrm{Li}_2(-e^{\tilde{\rho}}) + \tilde{k}^2\left[\frac{5\pi}{2\epsilon}\,\mathrm{Li}_1(-e^{\tilde{\rho}}) - \frac{3}{4}\,\mathrm{Li}_2(-e^{\tilde{\rho}}) - \frac{3\epsilon}{2\pi}\,\mathrm{Li}_3(-e^{\tilde{\rho}})\right] + \cdots\right\} \end{aligned} \tag{6}$$

where $\mathrm{Li}_\nu(z)$, $\nu = 1, 2, 3, \ldots$, are the polylogarithm functions, $\tilde{\rho} \simeq -\pi/\epsilon + \pi(\epsilon\tilde{k}^2)/4 + \cdots$, and $\epsilon = |E|/|E_c|$. For $\tilde{k}$ small, the leading term in Eq. (6) yields already an excellent approximation. In a constant-field limit $\tilde{k} \to 0$, we obtain from Eq. (6):

$$w \to -(2s+1)\frac{(e|E|)^2}{8\pi^3\hbar^2}\,\mathrm{Li}_2(-e^{-\pi/\epsilon})\,. \tag{7}$$

in the complete agreement with the Schwinger result [1–3].

In the paper [10], we calculate also the intensity of pair production $n \equiv n_\perp/L$ for the Sauter potential. Applying the same method of the small-$\tilde{k}$ expansion, we find

$$n = (2s+1)\frac{(e|E|)^2}{8\pi^3\hbar^2}\,e^{-\pi/\epsilon}\left[1 + \tilde{k}^2\left(\frac{\pi\epsilon}{4} - \frac{3\epsilon}{2\pi} - \frac{\pi^2}{8} - \frac{3}{8\pi^2} + \frac{33\pi}{8\epsilon} + \frac{\pi^2}{8\epsilon^2}\right) + \cdots\right]\,. \tag{8}$$

In a constant-field limit $k \to 0$, this yields

$$n^{\mathrm{f}} = 2n^{\mathrm{b}} = \frac{(e|E|)^2}{4\pi^3\hbar^2}\,e^{-\pi/\epsilon} \tag{9}$$

again in a full agreement with the Schwinger result [1–3].

3. Conclusion

We have studied the vacuum instability with the consequent production of particle-antiparticle pairs in the present of an external strong electric field of the spatial pulse shape. We have used the scattering versus tunneling method to obtain the exact formulas for pair production rates as well as for pair creation intensities of spin-0 and spin-1/2 particles. We have shown that the obtained formulas preserve the normal spin-statistics relation, and reproduce the well-known Schwinger results in a constant field limit.

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УДК 539.12.01; 539.124; 539.124.6

Нестабильность вакуума и рождение пар в КЭД

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В интенсивных внешних электромагнитных полях вакуум становится нестабильным, порождая пары частиц и античастиц. Рассматривается недавний прогресс в изучении эффекта рождения пар частиц внешним электромагнитным полем: 1. Новые точные формулы для вероятностей рождения бозонных и фермионных пар потенциалом $\phi(\mathbf{x}) \propto \tanh kz$ в трех измерениях. 2. Точные выражения для коэффициентов отражения и прохождения, а также для плотностей числа рожденных пар и для интенсивностей рождения, полученные с помощью изучения процессов рассеяния и туннелирования этим потенциалом. На основе полученных результатов сделан вывод о том, что нестабильность вакуума не приводит к изменению нормального соотношения между спином и статистикой.

Ключевые слова: КЭД, нестабильность вакуума, рождение пар частица-античастица, соотношение спина и статистики.