Bulletin of PFUR Series *Mathematics. Information Sciences. Physics.* No 2 (2). 2010. Pp. 99–103

Математическое моделирование

UDC 510.676, 519.7

Mathematical Aspects of Modeling of Required Operation Modes of Multi Purpose Isochronous Cyclotrons

I. V. Amirkhanov[*], **G. A. Karamysheva**[†],
I. N. Kiyan[†], **J. Sulikowski**[‡]
[*] *Laboratory of Information Technologies*
Joint Institute for Nuclear Research
Joliot-Curie 6, 141980 Dubna, Moscow region, Russia
[†] *Dzhelepov laboratory of Nuclear Problems*
Joint Institute for Nuclear Research
Joliot-Curie 6, 141980 Dubna, Moscow region, Russia
[‡] *Institute of Nuclear Physics Polish Academy of Sciences (INP PAS)*
ul. Radzikowskiego 152, 31-342, Krakow, Poland

A mathematical and computer modelings of required operation modes for multipurpose isochronous cyclotrons is presented. The considered procedure is based on a calculation of current values in trim coils correcting the basic magnetic field ($I_j,\ j = 1, 2, \ldots, n$) for a certain current level in the main coil (I_{mc}). A series of numerical and physical experiments on modelings and testing the main operation mode for the multipurpose isochronous cyclotron AIC-144 (proton, $E_{\text{out}} \sim 60.3/60.7$ MeV, $F_{\text{rf}} = 26.155/26.25$ MHz), proved both the necessity of taking into account the estimate of the stability of the sought solution, and the possibility of accelerating proton beams in the all range of working radii from the ion source to the ion-beam extraction system for small phase losses of protons in the range of isochronization of required magnetic field.

Key words and phrases: isochronous cyclotron, mathematical model, least squares method, Gauss method, conditionality number, proton beam current.

1. Introduction

Nowadays, multipurpose isochronous cyclotrons find their application both in the field of scientific researches (radioisotopes manufacture for researches in the field of nuclear spectroscopy; electronuclear power) and in the field of medicine (radioisotopes manufacture for different kinds of tomography; the proton therapy of cancer tumors). The multipurpose isochronous cyclotron AIC-144 situated in INP PAS is intended for manufacturing radioisotopes and treatment of eye melanoma without operation. The external beam of accelerated protons with the output energy $E_{\text{out}} \geqslant 60$ MeV and the beam current in the region from $I_{\text{beam}} = 3$ nA to $I_{\text{beam}} = 10$ nA is used for the treatment. The multipurpose isochronous cyclotron AIC-144 has the spiral 4-sector magnetic structure and the twenty trim coils used for correcting the main magnetic field.

2. Mathematical Model

The functional given below formalizes the method for calculating the operation modes of multipurpose isochronous cyclotrons:

Received 28th November, 2009.
The work was performed with financial support of RFFR, the grant No 10-01-00467-a.

$$F(I_1,\ldots,I_k)=\frac{1}{B_0^2}\left\{\int\limits_0^{X_{\mathrm{septum}}}\left[\sum_{j=1}^{k}\left(\bar{B}_{\mathrm{tc},j,\max}\frac{I_j}{I_{j,\max}}\right)-\Delta\bar{B}\right]^2\mathrm{d}x+\lambda^2\sum_{j=1}^{k}\left[\frac{I_j}{I_{j,\mathrm{lim}}}\right]^{2p}\right\},$$

$$\Delta\bar{B}=\bar{B}_{\mathrm{is}}+\bar{B}_{\mathrm{bump}}+\bar{B}_{\mathrm{edge}}-\bar{B}_{\mathrm{mc}}-\sum_{l=1}^{m}\left(\bar{B}_{\mathrm{tc},l,\max}\cdot\frac{I_l}{I_{l,\max}}\right),\quad X_{\mathrm{septum}}=\frac{R_{\mathrm{septum}}}{r_0},$$

where $n = k + m$ is the number of trim coils was used; k and m are the numbers of free and bound (fixed by the user or a program for certain values) components of the solution vector I_j, $j = 1, 2, \ldots, n$, respectively; B_0, r_0 are the coefficients for transforming to dimensionless quantities; $\bar{B}_{\mathrm{tc},j,\max}$ is the mean value of the maximal contribution of the j-th trim coil from the maximal possible current in this trim coil; I_j, $I_{j,\mathrm{lim}}$ are the required and limiting admissible currents in the j-th trim coil; $\bar{B}_{\mathrm{is}}$, $\bar{B}_{\mathrm{bump}}$, $\bar{B}_{\mathrm{edge}}$, $\bar{B}_{\mathrm{mc}}$ are the isochronous magnetic field, mean magnetic fields of the central bump and the edge magnetic field, and the mean main magnetic field, respectively; I_l is the current in the l-th trim coil fixed by the user at any admissible level or by a program at the boundary of the region of admissible values; R_{septum} is the radius of the deflector septum. The penalty function in the functional allows to bring the considered components of the solution vector into the framework of the given boundary values one by one. In this case, currents in the trim coils are considered successively in the direction determined by the zones of influence of the trim coils on the main magnetic field from the centner to the edge of the cyclotron [1]. The variable λ has a particular value for each component of the solution vector fixed by the program at the boundary of the region of admissible values. The constant p is connected with the variable λ and determines its order of magnitude. The solution is found on the base of Least Squares method. The inhomogeneous system of linear algebraic equations obtained in this case is solved using method of Gauss with the choice of the main element in the matrix. The integration of the elements of the coefficient matrix and the free member vector is performed using Simpson's quadrature formula. The isochronous magnetic field is calculated on base of Gordon's method [2]

$$||a_{i,j}||\cdot||x_j||=||b_i||,\quad ||a_{i,j}||=||a^0_{i,j}||+||a^1_{i,j}||,\quad i,j=1,2,\ldots,k,$$

$$a^0_{i,j}=\frac{1}{B_0^2}\int\limits_0^{X_{\mathrm{septum}}}\left(\bar{B}_{\mathrm{tc},i,\max}\cdot\bar{B}_{\mathrm{tc},j,\max}\right)\mathrm{d}x,\quad a^1_{i,j}=\frac{1}{B_0^2}\lambda^2 p k_i^{2p}\left(\frac{I_i}{I_{i,\max}}\right)^{2p-2}\delta_{ij},$$

$$I_{i,\mathrm{lim}}=I_{i,\mathrm{low}}\ \ (-I_{i,\max}\leqslant I_{i,\mathrm{low}}<I_i<0)\quad I_{i,\mathrm{lim}}=I_{i,\mathrm{high}}\ \ (0\leqslant I_i\leqslant I_{i,\mathrm{high}}<I_{i,\max}),$$

$$k_i=\frac{I_{i,\max}}{I_{i,\mathrm{lim}}},\quad I_{i,\max}>0,\quad b_i=\frac{1}{B_0^2}\cdot\int\limits_0^{X_{\mathrm{setup}}}(\bar{B}_{\mathrm{tc},i,\max}\cdot\Delta\bar{B})\mathrm{d}x.$$

Here $I_{i,\mathrm{low}}$, $I_{i,\mathrm{high}}$ are lower and upper limiting currents in the i-th trim coil, δ_{ij} – Kronecker symbol. The operation mode for the cyclotron includes a set of current values in the main I_{mc} and trim coils I_i, $i = 1, 2, \ldots, n$, the frequency of the RF oscillator F_{rf}, and the voltage at the Dees U_d. The operation mode provides the formation of the required magnetic field with a certain precision in the range from the cyclotron centner to the ion-beam extraction system.

3. Program Complex

The Cyclotron Operator HELP 2009 program complex of five programs was created in C++ (MS Visual C++) in SDI standard. This complex includes Cyclotron Mode

(CYCMODE) 2009, which provides automatic calculation and simplifies the operator's work when switching to new operation modes in multipurpose isochronous cyclotrons. The initial data include the type of accelerated particles; the kinetic energy of particles at a certain radius (RF oscillator frequency); the harmonic number (the ratio of the RF oscillator frequency and the particle orbital frequency); the periodicity of the magnetic structure; the formation range of the required magnetic field; the radius of the working-point (intersection of the mean main and mean required magnetic fields in the range of finite acceleration radii); and the mask of the isochronous magnetic field, which determines the shape of the central bump and the edge magnetic field. The initial data also include a set of measured or calculated magnetic field maps: main magnetic fields (from the current in the main coil) and additional magnetic fields (from the maximal possible current in each separate trim coil to a certain current level in the main coil). The boundary values include the minimal and maximal current values in each of the trim coils. The calculation results present the required cyclotron operation mode. Along with the Cyclotron Mode 2009 program, the program complex includes Cyclotron Analytic Model 2005 program for calculating isochronous magnetic fields, Phase Motion Research 2004 program for calculating phase motion; Free Oscillation Research 2005 program for calculating free oscillation frequencies, and one service program for calculating maps of the additional magnetic fields. The Cyclotron Operator HELP 2009 program complex was installed on AIC-144 server.

4. Solution Stability

The stability of the required solution $I_i,\ i = 1, 2, \ldots, n$ can be improved by successive eliminating part of used trim coils from the calculation. The minimal value of the functional minimum S is used as the conventional elimination criterion. The minimal value of the product of the functional minimum and the system conditionality number $S \cdot \mathrm{cond}$ is taken as the new elimination criterion. In last case, the functional integral is limited by its maximum admissible value. Besides, a certain quantity of trim coils for elimination is defined by the minimal value of the obtained dependence of the minimal products depending on the quantity of eliminated trim coils. If the new elimination criterion is applied, the solution obtained at small perturbations of the main magnetic field satisfies the given boundary values better than if the conventional elimination criterion is applied. The formulas of the system conditionality number, the norms of coefficient matrix and solution vector, and the relative error of the obtained solution follow

$$\mathrm{cond}(A) = ||A|| \cdot ||A^{-1}||, \quad ||A|| = \max_{1 \leqslant j \leqslant k} \sum_{i=1}^{k} |a_{ij}|, \quad ||X|| = \sum_{j=1}^{k} |x_j|,$$

$$\sigma = \frac{||\delta X||}{||X||} \cdot 100\ (\%).$$

When estimating the stability of the obtained solution, one task is connected with estimating the influence of the magnetic field distortions that appear as the cyclotron switched on and in operation due to the hysteresis, heating of the main magnet iron, and so on. Another task corresponds to the assumption that the operation mode is calculated using imprecise initial data (map of the main magnetic field). The first (second) of these tasks is to find the solution if a certain number of trim coils is eliminated from the calculation using a conditionally precise (perturbed) map of the main magnetic field. To estimate the stability of the obtained solutions, it is necessary to calculate their relative error for both criterions of elimination. This can be done by introducing small perturbations into the map of the main magnetic field within its measurement precision. The evaluation of the stability of the solutions of the particular operation mode for the multipurpose isochronous cyclotron AIC-144 (proton, $E_{\mathrm{out}} \sim 60.7$ MeV, $F_{\mathrm{rf}} = 26.25$ MHz), showed that the solutions obtained using the new elimination criterion are more stable than the solutions obtained using the conventional elimination criterion (in $1.4 \div 1.5$ times) without significant deterioration of the functional minimum values [3].

5. Physical Experiment

The modeling of the main operation mode for the multipurpose isochronous cyclotron AIC-144 (proton, $E_{\text{out}} \sim 60.3$ MeV, $F_{\text{rf}} = 26.155$ MHz) was performed using the Cyclotron Operator HELP 2009 program complex. The application of interpolation both inside and between the used magnetic field maps allowed one to perform an calculation of the map of the resulting magnetic field without its measurement. This, in turn, makes it possible to calculate and estimate the cyclotron parameters (the phase motion and the free oscillation frequencies). As a result, the numerical-experimental iterations for the formation of the required magnetic field were replaced by the numerical iterations only. Thanks to it, the operation mode was obtained without stopping the cyclotron for performing additional magnetic measurements. On June 4, 2009 the obtained operation mode was tested by the physical experiment at AIC144. As a result, the proton beam was successfully accelerated and extracted from the cyclotron. Fig. 1 shows the parameters of operation mode and the measured current of accelerated proton beam.

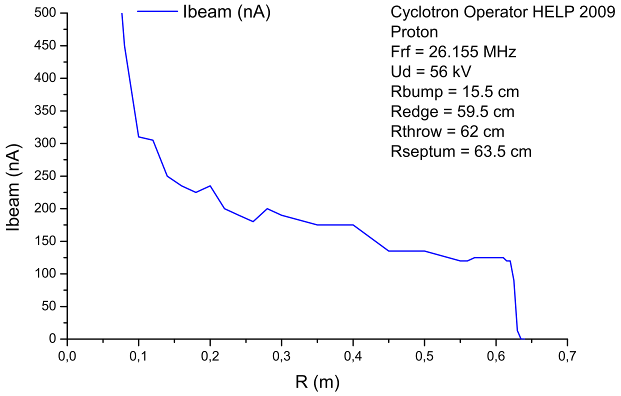

Figure 1. **The measured proton beam current**

Where R_{bump} is the final radius of the central bump; R_{edge} is the initial radius of the edge magnetic field; R_{throw} is the radius of the proton beam throw into deflector; R_{septum} is the radius of the deflector septum. It can be seen from the figure that proton beam was accelerated in the range of isochronization radii of the required magnetic field from R_{bump} to R_{edge} (and up to R_{throw}) without noticeable phase losses of protons. The phase losses of proton in the range of working radii from the ion source to the ion-beam extraction system are caused by the mean magnetic field error (by difference between mean measured and isochronous magnetic fields). Phase losses of protons grow because of the influence of magnetic channels entered in the range of final acceleration radii and compensating increase of the main coil current (from $=$ 575.4 A to $=$ 576.1 A). Some axial losses of the proton in the range of beginning acceleration radii ($R < 20\ cm$) are caused by small value of the vertical free oscillation frequency ($Q_z < 0.1$). Additional axial losses of protons in the range of final acceleration radii ($R \sim 62.5\ cm$) arise because of the influence of Walkinshow's resonance. The map of the resulting magnetic field can be measured, and the operation mode can be recalculated with the introduction of the corresponding correction into the mask of the isochronous magnetic field. So that, the phase losses of protons can be minimized in the range of isochronization radii of the required magnetic field.

6. Conclusion

On the whole, the following was done:

- The new algorithms of modeling of required operation modes for multipurpose isochronous cyclotrons were developed and the technique of modeling was improved;
- The Cyclotron Operator HELP 2009 program complex was created in C++ and installed on the server of the multipurpose isochronous cyclotron AIC-144 (INP PAS);
- The main operation mode of AIC-144 (proton, $E_{\text{out}} \sim 60.3$ MeV, $F_{\text{rf}} = 26.155$ MHz) was modeled using the Cyclotron Operator HELP 2009 program complex, and the proton beam intended for treatment of eye melanoma was successfully accelerated at AIC-144 and extracted from the cyclotron. The measured kinetic energy in the irradiation chamber has made $E_k = 60$ MeV. After five months of AIC-144 work in this operation mode the leaving of current of proton beam extracted from cyclotron has made $\sigma_{\text{Iout}} \approx 2.5$ %.

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УДК 510.676, 519.7

Математические аспекты моделирования требуемых режимов работы многоцелевых изохронных циклотронов

И. В. Амирханов[*], **Г. А. Карамышева**[†], **И. Н. Киян**[†], **Я. Суликовский**[‡]
[*] *Лаборатория информационных технологий*
Объединённый институт ядерных исследований
ул. Жолио-Кюри д.6, Дубна, Московская область, 141980, Россия
[†] *Лаборатория ядерных проблем им. В.П.Джелепова*
Объединённый институт ядерных исследований
ул. Жолио-Кюри д.6, Дубна, Московская область, 141980, Россия
[‡] *Институт ядерной физики Польской Академии Наук*
ул. Радзиковскиего, д. 152, 31-342, Краков, Польша

В данной статье рассматривается математическое и компьютерное моделирование требуемых режимов работы многоцелевых изохронных циклотронов. Рассматриваемая процедура основывается на расчёте токов в концентрических катушках коррекции основного магнитного поля (I_i, $i = 1, 2, \ldots, n$) для определённого уровня тока в главной катушке (I_{mc}). Ряд численных и физических экспериментов по моделированию и тестированию основного режима работы многоцелевого изохронного циклотрона АИЦ-144 (протон, $E_{\text{out}} \sim 60,3/60,7$ МэВ, $F_{\text{rf}} = 26,155/26,25$ МГц), подтвердил как необходимость принятия во внимание оценки устойчивости искомого решения, так и возможность ускорения пучков протонов в области рабочих радиусов от источника ионов до системы вывода пучка ионов при незначительных фазовых потерях протонов в области изохронизации требуемого магнитного поля.

Ключевые слова: изохронный циклотрон, математическая модель, метод наименьших квадратов, метод Гаусса, число обусловленности, ток пучка протонов.