UDC 519.21

The Analysis of Queueing System with Two Input Flows and Stochastic Drop Mechanism

I. S. Zaryadov, A. V. Gorbunova

Department of Applied Probability and Informatics
Peoples' Friendship University of Russia
6, Miklukho-Maklaya str., Moscow, Russia, 117198

The queueing system with two independent flows of requests with different types of priorities is considered. The incoming flows are Poisson flows with different (non equal) rates. The service times of each type requests are independent and exponentially distributed. The priority requests at the end of its service can drop non-priority ones with probability q (renovation probability) or just leaves the system with probability $p = 1 - q$. For general case the two-dimensional Markov process is introduced and the system of equilibrium equations for steady-state probability distribution is presented. For special case, when drop probability q is equal to one, some probabilistic characteristics as the steady-state probability distribution of priority requests, the probability of idle period are obtained. Also the analytical expressions for some characteristics of non-priority requests, such as probability of being dropped (or serviced), waiting time distribution for non-priority requests (in terms of Laplace-Stieltjes transformation and generating function) and mean waiting time, are obtained.

Key words and phrases: queueing system, two input flows, renovation probability, stochastic drop mechanism, time-probability characteristics.

1. Introduction

At the present time the questions how to ensure the specified quality of service (QoS) in data networks are very important. One of the key performance indicators are the service traffic losses and the average transmission delay due to the unavoidable occurrence of bursts. Problems associated with data loss may also occur due to servers failure, receipt of a special type of data that have a negative impact on the efficiency of the service and operation of the network element (device) [1–5].

In order to regulate the intensity of the incoming data flow and to reduce the negative effects various control mechanisms to prevent network overload are introduced and under study. Thus, for example, RED-like algorithms (Random Early Detection) [6, 7] under overload situation or close thereto can control the degree of loading queue using some stochastic drop mechanism for incoming packets.

In the article the authors consider some modification of active queue management mechanism [8] as queueing system with two independent flows of requests with different priorities. The incoming flows are Poisson flows with different (non equal) rates λ_1 and λ_2. The service times of each type requests are independent and exponentially distributed, μ_1 is a service rate of priority request and μ_2 — of non-priority request.The priority requests at the end of its service can drop non-priority ones with probability q (renovation probability) or just leaves the system with probability $p = 1 - q$. In the first part of the article the general case is considered and the two-dimensional Markov process is introduced, the system of equilibrium equations for steady-state probability distribution is presented. The second part deals with the special case, when drop probability q is equal to one. For special case some probabilistic characteristics as the steady-state probability distribution of priority requests, the probability of idle period are obtained. Also the analytical expressions for some characteristics of non-priority requests, such as probability of being dropped (or serviced), waiting time distribution for non-priority requests (in terms of Laplace-Stieltjes transformation and generating function) and mean waiting time, are presented.

Received 20th February, 2015.

The reported study was supported by the Russian Foundation for Basic Research RFBR, research projects No. 13-07-00223, 13-07-00665, 14-07-00090.

2. General Queueing System Description

At first the general case of queueing system is introduced, but the main results will be for the special case. The random values $\nu_i(t), i = 1, 2$, describe the number of i-th type requests in the system at time moment t. The describing our queueing system stochastic process $\{X(t), t \geqslant 0\}$, where $X(t) = (\nu_1(t), \nu_2(t))$, is a continuous-time two-dimensional Markov process with discreet state set $\mathcal{X} = \{(i, j), i \geqslant 0, j \geqslant 0\}$.

The state (i, j) means that there are i requests of the first type (priority requests) and j requests of the second type (non-priority requests).

By $p_{i,j}$, $i \geqslant 0, j \geqslant 0$ we will denote the steady-state probability that there are i priority requests and j non-priority requests in considered queueing system. The introduced probabilities satisfy the following system of equations:

$$(\lambda_1 + \lambda_2)p_{0,0} = p\mu_1 p_{1,0} + \mu_2 p_{0,1} + q\mu_1 \sum_{j=0}^{\infty} p_{1,j}, \tag{1}$$

$$(\lambda_1 + \lambda_2 + \mu_1)p_{i,0} = \lambda_1 p_{i-1,0} + p\mu_1 p_{i+1,0} + q\mu_1 \sum_{j=0}^{\infty} p_{i+1,j} + \mu_2 p_{i,1}, \quad i \geqslant 1, \tag{2}$$

$$(\lambda_1 + \lambda_2 + \mu_2)p_{0,j} = \lambda_2 p_{0,j-1} + \mu_2 p_{0,j+1} + p\mu_1 p_{1,j}, \quad j \geqslant 1, \tag{3}$$

$$(\lambda_1 + \lambda_2 + \mu_1 + \mu_2)p_{i,j} = \lambda_1 p_{i-1,j} + \lambda_2 p_{i,j-1} + p\mu_1 p_{i+1,j} + \mu_2 p_{i,j+1}, \quad i \geqslant 1, j \geqslant 1. \tag{4}$$

Also the normalisation condition is introduced:

$$\sum_{i=0}^{\infty} \sum_{j=0}^{\infty} p_{i,j} = 1. \tag{5}$$

3. Special Case. System Description

Let's discuss the special case of earlier introduced queueing system with drop probability $q = 1$ and $p = 0$. The system of equations (1)-(4) is as follows:

$$(\lambda_1 + \lambda_2)p_{0,0} = \mu_2 p_{0,1} + \mu_1 \sum_{j=0}^{\infty} p_{1,j}, \tag{6}$$

$$(\lambda_1 + \lambda_2 + \mu_1)p_{i,0} = \lambda_1 p_{i-1,0} + \mu_1 \sum_{j=0}^{\infty} p_{i+1,j} + \mu_2 p_{i,1}, \quad i \geqslant 1, \tag{7}$$

$$(\lambda_1 + \lambda_2 + \mu_2)p_{0,j} = \lambda_2 p_{0,j-1} + \mu_2 p_{0,j+1}, \quad j \geqslant 1, \tag{8}$$

$$(\lambda_1 + \lambda_2 + \mu_1 + \mu_2)p_{i,j} = \lambda_1 p_{i-1,j} + \lambda_2 p_{i,j-1} + \mu_2 p_{i,j+1}, \quad i \geqslant 1, j \geqslant 1 \tag{9}$$

with normalisation condition (5).

Let's define the generating function:

$$B_0(z) = \sum_{j=0}^{\infty} p_{0,j} z^j, \quad 0 \leqslant z \leqslant 1. \tag{10}$$

Multiplying the equations (6) and (8) by z^j and summing over j, we will receive:

$$B_0(z) = \frac{\mu_2(1 - z)p_{0,0} - \mu_1 p_{1,\cdot} z}{\lambda_2 z^2 - (\lambda_1 + \lambda_2 + \mu_2)z + \mu_2}. \tag{11}$$

Here and elsewhere the probability $p_{i,\cdot} = \sum\limits_{j=0}^{\infty} p_{ij}$ is a steady-state probability that there are exactly i, $i \geqslant 0$, priority requests in our system (marginal steady-state probability distribution).

The denominator of the right hand side of the equation (11) is a quadratic equation in z:

$$\lambda_2 z^2 - (\lambda_1 + \lambda_2 + \mu_2)z + \mu_2 = 0$$

with the solutions

$$z_{1,2} = \frac{\lambda_1 + \lambda_2 + \mu_2 \pm \sqrt{(\lambda_1 + \lambda_2 + \mu_2)^2 - 4\lambda_2\mu_2}}{2\lambda_2}, \tag{12}$$

and $z_1 > 1$, $0 < z_2 < 1$.

The generating function $B_0(z)$ is a continuous function over the interval $0 \leqslant z \leqslant 1$, so when $z = z_2$ not only the denominator of (11) is equal to zero, but the numerator should be equal to zero:

$$\mu_2(1 - z_2)p_{0,0} - \mu_1 p_{1,\cdot} z_2 = 0,$$

so

$$p_{1,\cdot} = \frac{\mu_2(1 - z_2)}{z_2\mu_1} p_{0,0}. \tag{13}$$

The probabilities $p_{0,j}$, $j \geqslant 0$ may be expressed in terms of generating function:

$$p_{0,j} = \left.\frac{B_0^{(j)}(z)}{j!}\right|_{z=0}, \tag{14}$$

where $B_0^{(j)}$ — the value of j-th degree derivative with respect to z while $z = 0$, $j \geqslant 0$.

By summing the equilibrium equation (8) over $j = \overline{1, \infty}$, with the help of (6), and then by summing the equation (9) over $j = \overline{1, \infty}$, with the help of (7), the next steady-state system of equations is obtained:

$$\lambda_1 p_{0,\cdot} = \mu_1 p_{1,\cdot}, \quad (\lambda_1 + \mu_1)p_{i,\cdot} = \lambda_1 p_{i-1,\cdot} + \mu_1 p_{i+1,\cdot}, \quad i \geqslant 1.$$

The solution of the system:

$$p_{i,\cdot} = \left(\frac{\lambda_1}{\mu_1}\right)^i p_{0,\cdot}, \quad i \geqslant 1, \quad p_{0,\cdot} = 1 - \frac{\lambda_1}{\mu_1}. \tag{15}$$

By using the equations (15) and (13), it's easy to obtain the probability of system idle period:

$$p_{0,0} = \frac{\mu_1 - \lambda_1}{\mu_2} \cdot \frac{z_2}{1 - z_2} \cdot \frac{\lambda_1}{\mu_1}. \tag{16}$$

4. Special Case. Time and Probability Characteristics

As defined in the first part of the article non-priority requests are dropped by priority requests with probability $q = 1$, so it's important to evaluate non-priority request service probability — $p^{(\text{serv})}$. The non-priority request may be served only when there are no priority requests in queueing system:

$$p^{(\text{serv})} = \sum_{j=0}^{\infty} p_{0j} = p_{0,\cdot} = 1 - \frac{\lambda_1}{\mu_1}. \tag{17}$$

The loss probability $p^{(\mathrm{loss})}$ is $p^{(\mathrm{loss})} = 1 - p^{(\mathrm{serv})} = \dfrac{\lambda_1}{\mu_1}$.

Waiting time distribution for non-priority requests (in terms of Laplase-Stieltjes transformation) with the help of (14) and Maclaurin expansion of $B_0(z)$:

$$\omega_2^{(\mathrm{serv})}(s) = \frac{1}{p^{(\mathrm{serv})}} \sum_{j=0}^{\infty} p_{0j} \left(\frac{\mu_2}{\mu_2 + s} \right)^j = \frac{\mu_1}{\mu_1 - \lambda_1} \cdot B_0 \left(\frac{\mu_2}{\mu_2 + s} \right). \tag{18}$$

Mean waiting time for non-priority requests:

$$w_2 = (-1) \cdot (\omega_2^{(\mathrm{serv})}(s))'_{s=0} = \frac{1}{p^{(\mathrm{serv})}} \cdot \frac{\mu_2 p_{00} - (\mu_2 - \lambda_2) p_{0,\cdot}}{\mu_2 \lambda_1}, \tag{19}$$

where p_{00} are $p_{0,\cdot}$ defined by relations (16) and (15).

5. Conclusion

The one-server queuing system with two independent input flows, infinite buffer, exponential service times, renovation was considered in this work. Some steady-state probabilistic and time characteristics were derived.

References

1. A. Kreinin, Queueing Systems with Renovation, Journal of Applied Math. Stochast. Analysis 10 (4) (1997) 431–443.
2. P. P. Bocharov, I. S. Zaryadov, Probability Distribution in Queueing Systems with Renovation, Bulletin of Peoples' Friendship University of Russia. Series "Mathematics. Information Sciences. Physics" (1-2) (2007) 15–25, in Russian.
3. I. S. Zaryadov, A. V. Pechinkin, Stationary Time Characteristics of the $GI/M/n/\infty$ System with Some Variants of the Generalized Renovation Discipline, Automation and Remote Control (12) (2009) 2085–2097.
4. I. S. Zaryadov, The $GI/M/n/\infty$ Queuing System with Generalized Renovation, Automation and Remote Control (4) (2010) 663–671.
5. A. Adamu, Y. Gaidamaka, A. Samuylov, Discrete Markov Chain Model for Analyzing Probability Measures of P2P Streaming Network, in: Lecture Notes in Computer Science, Smart Spaces and Next Generation Wired/Wireless Networking 11th Internetional Conference, NEW2AN 2011, and 4th Conference on Smart Spaces, ruSMART 2011, Germany, Heidelberg, Springer-Verlag, St. Petersburg, Russia, August 22–25, 2011, pp. 428–439.
6. A. V. Korolkova, I. S. Zaryadov, The Mathematical Model of the Traffic Transfer Process with a Rate Adjustable by RED, in: ICUMT 2010 — International Conference on Ultra Modern Telecommunications, Moscow, 2010, pp. 1046–1050.
7. I. S. Zaryadov, A. V. Korolkova, The Application of Model with General Renovation to the Analysis of Characterictics of Active Queue Management with Random Early Detection (RED), T-Comm: Telecommunications and Transport (7) (2011) 84–88, in Russian.
8. I. S. Zaryadov, A. V. Korolkova, R. V. Razumchik, Mathematical Models of Active Queue Management Systems Analysis Based on Queueing System with Two Types of Traffic and Different Priorities, T-Comm: Telecommunications and Transport (7) (2012) 107–111, in Russian.

УДК 519.21

Анализ системы массового обслуживания с двумя входящими потоками и вероятностным сбросом

И. С. Зарядов, А. В. Горбунова

Кафедра прикладной информатики и теории вероятностей
Российский университет дружбы народов
ул. Миклухо-Маклая, д. 6, Москва, Россия, 117198

Рассматривается однолинейная система массового обслуживания с накопителем неограниченной ёмкости, в которую поступают два независимых пуассоновских потока заявок с различными интенсивностями и приоритетами. Заявка первого типа (приоритетная), находящаяся на приборе, может в момент окончания обслуживания либо покинуть систему с некоторой ненулевой вероятностью, либо с дополнительной вероятностью сбросить все заявки второго типа (неприоритетные заявки) из накопителя и покинуть систему. Длительности обслуживания заявок обоих типов имеют экспоненциальные распределения с различными значениями интенсивностей обслуживания. Для общего случая построен двумерный марковский процесс, получена система уравнений равновесия для стационарного распределения числа заявок обоих типов в системе. Для частного случая рассматриваемой системы (приоритетные заявки сбрасывают неприоритетные с вероятностью, равной единице) в явном виде представлено стационарное распределение числа приоритетных заявок в системе, вероятность простоя системы. Также получено выражение для вероятности простоя системы. Для неприоритетных заявок найдены вероятность ее сброса из системы приоритетной заявкой, стационарное распределение времени ожидания начала обслуживания (в терминах преобразований Лапласа-Стилтьеса и производящих функций), а также среднее время ожидания начала обслуживания.

Ключевые слова: обновление, система массового обслуживания, два входящих потока, сброс заявок, вероятностно-временные характеристики.

Литература

1. *Kreinin A.* Queueing Systems with Renovation // Journal of Applied Math. Stochast. Analysis. — 1997. — Vol. 10, No 4. — Pp. 431–443.
2. *Бочаров П. П., Зарядов И. С.* Стационарное распределение вероятностей в системах массового обслуживания с обновлением // Вестник РУДН. Серия «Математика. Информатика. Физика». — 2007. — № 1–2. — С. 15–25.
3. *Zaryadov I. S., Pechinkin A. V.* Stationary Time Characteristics of the $GI/M/n/\infty$ System with Some Variants of the Generalized Renovation Discipline // Automation and Remote Control. — 2009. — No 12. — Pp. 2085–2097.
4. *Zaryadov I. S.* The $GI/M/n/\infty$ Queuing System with Generalized Renovation // Automation and Remote Control. — 2010. — No 4. — Pp. 663–671.
5. *Adamu A., Gaidamaka Y., Samuylov A.* Discrete Markov Chain Model for Analyzing Probability Measures of P2P Streaming Network // Lecture Notes in Computer Science / Smart Spaces and Next Generation Wired/Wireless Networking 11th Internetional Conference, NEW2AN 2011, and 4th Conference on Smart Spaces, ruSMART 2011. — St. Petersburg, Russia, August 22–25: Germany, Heidelberg, Springer-Verlag, 2011. — Pp. 428–439.
6. *Korolkova A. V., Zaryadov I. S.* The Mathematical Model of the Traffic Transfer Process with a Rate Adjustable by RED // ICUMT 2010 — International Conference on Ultra Modern Telecommunications. — Moscow: 2010. — Pp. 1046–1050.
7. *Зарядов И. С., Королькова А. В.* Применение модели с обобщённым обновлением к анализу характеристик систем активного управления очередями типа Random Early Detection (RED) // T-Comm: Телекоммуникации и транспорт. — 2011. — № 7. — С. 84–88.
8. *Зарядов И. С., Королькова А. В., Разумчик Р. В.* Математические модели расчёта и анализа характеристик систем активного управления очередями с двумя входящими потоками и различными приоритетами // T-Comm: Телекоммуникации и транспорт. — 2012. — № 7. — С. 107–111.