UDC 3937

On Multidimensional Cosmology with Anisotropic Fluid: Asymptotical Acceleration and Zero Variation of G

A. G. Pakhomov

Institute of Gravitation and Cosmology
Peoples' Friendship University of Russia
6, Miklukho-Maklaya str., Moscow, Russia, 117198

A multidimensional cosmological model describing the dynamics of $n+1$ flat factor-spaces M_i in the presence of a one-component anisotropic fluid is offered. The pressures in all spaces are proportional to the density: $p_i = w_i \rho$, $i = 0, \dots, n$. Solutions with accelerated expansion of our 3-space M_0 and zero variation of the gravitational constant G are studied. These solutions exist for two branches of the parameter w_0: The first branch describes the matter with $w_0 > 1$, the second one may contain phantom matter with $w_0 < -1$. It is shown that these solutions are special case of more general solutions with accelerated expansion of our 3-space M_0 and asymptotically zero variation of the gravitational constant G.

The model of an ideal many-dimensional substance with three isotropic dimensions of our space, additional dimensions and time is considered. Spacelike dimensions are presented by the power metric depending on parameters of an equation of state. It is shown, that association of dynamic parameters of our three-dimensional space on additional dimensions in the open view may be expressed through coefficient of anisotropy of additional dimensions. Dependence from parameter of an equation of state of our isotropic 3-dimensional space to coefficient of anisotropy of the additional dimensions, requiring the accelerated expansion of the Universe is received in an explicit aspect. The received association is presented pictorially.

Key words and phrases: multidimensional gravitation, anisotropic fluid, accelerated expansion, variation of G.

1. Introduction

This paper deals with a possible temporal variation of gravitational constant G. This problem arose due to papers of Milne (1935) and Dirac (1937). In Russia, these ideas were developed in the 60s and 70s by K.P. Staniukovich [1, 2], who was the first to consider simultaneous variations of several fundamental constants.

The first calculations based on general relativity with a perfect fluid and a conformal scalar field [3] gave $\dot{G}/G$ at the level of $10^{-11} - 10^{-13}$ per year. The calculations in string-like [4] and multidimensional models with perfect fluid [5] led to the level 10^{-12}, those based on a general class of scalar-tensor theories [6] and simple multidimensional model with branes [7, 8] led for the present values of cosmological parameters $10^{-13} - 10^{-14}$ and 10^{-13} per year, respectively. Similar estimations were made by Miyazaki within Machian theories [9] giving for $\dot{G}/G$ the estimate 10^{-13} per year and by Fujii — on the level $10^{-14} - 10^{-15}$ per year [10]. Analysis of one more multidimensional model with two curvatures in different factor spaces gave an estimate on the level 10^{-12} .

Here we continue our studies (see [11]) on variation of G in multidimensional cosmological model with 1-component "perfect-fluid" matter. We show that the class of solutions from [11] may be enlarged by more general solutions with accelerated expansion of 3-dimensional space M_0 and asymptoticaly zero variation of the gravitational constant G.

2. The Model

We consider a cosmological model describing the dynamics of n flat spaces in the presence of a 1-component "perfect-fluid" matter [12]. The metric of the model

$$g = -\exp[2\gamma(t)]\mathrm{d}t \otimes \mathrm{d}t + \sum_{i=0}^{n} \exp[2x^i(t)]g^i \tag{1}$$

is defined on the manifold

$$M = \mathbb{R} \times M_0 \times M_1 \times \ldots \times M_n, \tag{2}$$

where M_i with the metric g^i is a flat space of dimension d_i, $i = 0, \ldots, n$; $n \geqslant 2$. The multidimensional Hilbert-Einstein equations have the form

$$R^M_N - \frac{1}{2}\delta^M_N R = \kappa^2 T^M_N,$$

where κ^2 is the gravitational constant, and the energy-momentum tensor is adopted as $(T^M_N) = \mathrm{diag}\,(-\rho,\ p_0\delta^{m_0}_{k_0}, \ldots,\ p_n\delta^{m_n}_{k_n})$, describing, in general, an anisotropic fluid.

We put pressures of this "perfect" fluid in all spaces to be proportional to the density,

$$p_i(t) = (1 - u_i/d_i)\rho(t), \tag{3}$$

where $u_i = \mathrm{const}$, $i = 0, \ldots, n$. We also put $\rho > 0$.

We impose also the following restriction on the vector $u = (u_i) \in \mathbb{R}^n$:

$$\langle u, u \rangle \neq 0. \tag{4}$$

Here, the bilinear form $\langle .,. \rangle : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ is defined by the relation

$$\langle u, v \rangle = G^{ij} u_i v_j, \tag{5}$$

$u, v \in R^{n+1}$, where

$$G^{ij} = \frac{\delta^{ij}}{d_i} + \frac{1}{2 - D}. \tag{6}$$

are components of the matrix inverse to the matrix of the minisuperspace metric [13,14]

$$G_{ij} = d_i \delta_{ij} - d_i d_j. \tag{7}$$

In (6), $D = 1 + \sum_{i=0}^{n} d_i$ is the total dimension of the manifold M from (2).

The inequality (4) reads

$$\langle u, u \rangle = \sum_{i=0}^{n} \frac{(u_i)^2}{d_i} - \frac{1}{D - 2}\left(\sum_{i=0}^{n} u_i\right)^2 \neq 0. \tag{8}$$

3. Solutions with Power-Law Scale Factors

Here, we consider a special family of "power-law" solutions from [12,15] with the metric written in the synchronous time parametrization

$$g = -\mathrm{d}t_s \otimes \mathrm{d}t_s + \sum_{i=0}^{n} a_i^2(t_s) g^i. \tag{9}$$

Here t_s is cosmological (synchronous) time variable.

A special class of solutions with a power-law behaviour of the scale factors take place for

$$\langle u^{(\Lambda)} - u, u \rangle \neq 0. \tag{10}$$

Here and below the vector

$$u_i^{(\Lambda)} = 2d_i \tag{11}$$

corresponds to the Λ-term fluid with $p_i = -\rho$.

In this case, the solutions are determined by the metric (9) with the scale factors

$$a_i = a_i(t_s) = A_i t_s^{\nu^i}, \tag{12}$$

and the density

$$\kappa^2 \rho = \frac{-2\langle u, u \rangle}{\langle u^{(\Lambda)} - u, u \rangle^2 t_s^2}. \tag{13}$$

Here

$$\nu^i = 2u^i / \langle u^{(\Lambda)} - u, u \rangle, \tag{14}$$

where $u^i = G^{ij} u_i$ and A_i are positive constants, $i = 0, \ldots, n$.

The model under consideration was integrated in [12] for $\langle u, u \rangle < 0$. The solutions from [12] were generalized in [15] to the case when a massless minimally coupled scalar field was added.

4. Acceleration and Variation of G

In this section, the metric g^0 is assumed to be 3-dimensional flat metric, i.e. $d_0 = 3$. The subspace (M_0, g^0) describes "our" 3-dimensional space and (M_i, g^i) internal factor-spaces. One may consider the special case $d_i = 1$, $i = 1, \ldots, n$, as it was done in [11]. See also [16].

We are interested in solutions with accelerated expansion of our space and small enough variations of the gravitational constant obeying the present experimental constraints, see [17]:

$$|\dot{G}/(GH)|(t_{s0}) < 0.001, \tag{15}$$

where

$$H = \frac{\dot{a_0}}{a_0} \tag{16}$$

is the Hubble parameter. We suppose that the internal spaces are compact. Hence our 4-dimensional constant is (see [5])

$$G = \mathrm{const} \cdot \prod_{i=1}^{n} (a_i^{-d_i}). \tag{17}$$

We will use the following explicit formulae for the contravariant components:

$$u^i = G^{ij} u_j = \frac{u_i}{d_i} + \frac{1}{2-D} \sum_{j=0}^{n} u_j,$$

and the scalar product reads

$$\langle u^{(\Lambda)} - u, u \rangle = -\sum_{i=0}^{n} \frac{(u_i)^2}{d_i} + \frac{2}{D-2} \sum_{i=0}^{n} u_i + \frac{1}{D-2} \left(\sum_{i=0}^{n} u_i \right)^2. \tag{18}$$

4.1. Power-Law Expansion with Acceleration

For solutions with power-law expansion, an accelerated expansion of our space takes place for

$$\nu^0 > 1. \tag{19}$$

For $D = 4$, when internal spaces are absent, we get

$$\nu^0 = 2/(6 - u_0), \tag{20}$$

$$\langle u^{(\Lambda)} - u, u \rangle = \frac{1}{6}(u_0 - 6)u_0 \neq 0, \tag{21}$$

which implies $u_0 \neq 0$ and $u_0 \neq 6$ (here $\langle u, u \rangle_* = -\frac{1}{6}u_0^2 < 0$). The condition $\nu^0 > 1$ is equivalent to $4 < u_0 < 6$, or, equivalently, $-\rho < p < -\rho/3$, which agrees with the well-known result for $D = 4$. (We note that special 5-dimensional power-law solutions (e.g., with acceleration) were considered in [18]).

For power-law solutions we get

$$\frac{\dot{G}}{G} = -\frac{\sum\limits_{j=1}^{n} \nu^i d_i}{t_s}, \quad H = \frac{\dot{a_0}}{a_0} = \frac{\nu^0}{t_s}, \tag{22}$$

and hence

$$\dot{G}/(GH) = -\frac{1}{\nu^0} \sum_{j=1}^{n} \nu^i d_i \equiv \delta. \tag{23}$$

The constant parameter δ describes variation of the gravitational constant and, according to (15),

$$|\delta| < 0.001. \tag{24}$$

It follows from the definition (14) that

$$\delta = -\frac{1}{u^0} \sum_{i=1}^{n} u^i d_i. \tag{25}$$

or, in terms of covariant components

$$\delta = -\frac{(D-4)u_0 - 2\sum\limits_{i=1}^{n} u_i}{\frac{1}{3}(5-D)u_0 + \sum\limits_{i=1}^{n} u_i}. \tag{26}$$

4.1.1. The Case of Constant G

Consider the most important case $\delta = 0$, i.e., when the variation of G is absent: $\dot{G} = 0$.

We note that according to arguments of [19], $\delta < 10^{-4}$. This constraint just follows from the identity

$$\dot{G}/G = \dot{\alpha}/\alpha, \tag{27}$$

where α is the fine structure constant.

Isotropic case. Let us consider the isotropic case when the pressures coincide in all internal spaces. This takes place when

$$u_i = v d_i, \quad i = 1, \dots, n. \tag{28}$$

For pressures in internal spaces we get from (3)

$$p_i = (1 - v)\rho, \quad i = 1, \dots, n. \tag{29}$$

Then we get from (8) and (18)

$$\langle u, u\rangle = \frac{1}{2-D}\left[-\frac{1}{3}(d-1)u_0 + 2du_0v - 2dv^2\right], \tag{30}$$

$$\langle u^{(\Lambda)} - u, u\rangle = \frac{1}{2-D}\big[2u_0 + 2dv + \frac{1}{3}(d-1)u_0^2 - 2du_0v + 2dv^2\big]. \tag{31}$$

Here we denote $d = D - 4$.

For $\delta = 0$, we get in the isotropic case

$$v = u_0/2, \tag{32}$$

or, in terms of pressures,

$$p_i = (3p_0 - \rho)/2, \quad i = 1, \dots, n. \tag{33}$$

Substituting (32) into (30) and (31) we get $\langle u, u\rangle = -u_0^2/6$, $\langle u^{(\Lambda)} - u, u\rangle = u_0(u_0 - 6)/6$.

Thus, we obtain the same relations as in $D = 4$ case (see the remark above). For our solution, we should put $u_0 \neq 0$ and $u_0 \neq 6$.

Using (28) and (32) we get $u^0 = -u_0/6$ and $u^i = 0$ for $i > 0$, hence $\nu_i = 0$ for $i = 1, \dots, n$, i.e., all internal spaces are static.

The metric (9) reads in our case

$$g = -\mathrm{d}t_s \otimes \mathrm{d}t_s + A_0^2 t_s^{2\nu^0} g^0 + \sum_{i=1}^{n} A_i^2 g^i, \tag{34}$$

where A_i are positive constants, and

$$\nu^0 = 2/(6 - u_0). \tag{35}$$

We see that the power ν^0 is the same as in case $D = 4$. For the density we get from (13)

$$\kappa^2\rho = \frac{12}{(u_0 - 6)^2 t_s^2}. \tag{36}$$

Thus the equations of state (3), with relations (28) and (32) imposed, lead to the solution (34)–(36) with flat 3-metric and n static internal flat spaces. For

$$4 < u_0 < 6 \tag{37}$$

or, equivalently, $-\rho < p_0 < -\rho/3$, we get an accelerated expansion of “our” 3-dimensional flat space.

Anisotropic case. Consider the anisotropic (w.r.t. internal spaces) case with $\delta = 0$, or, equivalently (see (26)),

$$(D - 4)u_0 = 2\sum_{i=1}^{n} u_i. \tag{38}$$

This implies

$$\langle u^{(\Lambda)} - u, u\rangle = \frac{1}{6}u_0(u_0 - 6) - \Delta, \tag{39}$$

$$\langle u, u\rangle = -\frac{1}{6}u_0^2 + \Delta, \tag{40}$$

where

$$\Delta = \sum_{i=1}^{n} \frac{u_i^2}{d_i} - \frac{1}{d}\left(\sum_{i=1}^{n} u_i\right)^2 \geqslant 0, \quad d = D - 4. \tag{41}$$

The inequality in (41) can be readily proved using the well-known Cauchy-Schwarz inequality:

$$\left(\sum_{i=1}^{n} b_i^2\right)\left(\sum_{i=1}^{n} c_i^2\right) \geqslant \left(\sum_{i=1}^{n} b_i c_i\right)^2. \tag{42}$$

$\Delta = 0$ only in the isotropic case (28). In the anisotropic case we get $\Delta > 0$. Thus Δ is anisotropy parameter.

In what follows we will use the relation

$$\langle u^{(\Lambda)} - u, u\rangle = \frac{1}{6}(u_0 - u_0^+)(u_0 - u_0^-), \tag{43}$$

where $u_0^{\pm} = 3 \pm \sqrt{9 + 6\Delta}$ are roots of the quadratic polynomial (39) obeying $u_0^- < 0$, $u_0^+ > 6$ for $\Delta > 0$. It follows from (38) that $u^0 = -u_0/6$ and hence

$$\nu^0 = -\frac{2u_0}{u_0(u_0 - 6) - 6\Delta} \tag{44}$$

(here $u_0 \neq u_0^{\pm}$).

The accelerated expansion of our space takes place when $\nu^0 > 1$, or, equivalently, when either

$$(\mathbf{A}) \quad u_0 \in (u_{0*}^-, u_0^-), \quad \text{or} \quad (\mathbf{B}) \quad u_0 \in (u_{0*}^+, u_0^+), \tag{45}$$

where $u_{0*}^{\pm} = 2 \pm \sqrt{4 + 6\Delta}$. In terms of the parameter w_0,

$$p_0 = w_0\rho, \quad w_0 = 1 - u_0/3, \tag{46}$$

these two branches read:

$$(\mathbf{A})\ w_0^- = \sqrt{1 + \frac{2}{3}\Delta} < w_0 < \frac{1}{3} + \frac{2}{3}\sqrt{1 + \frac{3}{2}\Delta} = w_{0*}^-, \tag{47}$$

$$(\mathbf{B})\ w_0^+ = -\sqrt{1 + \frac{2}{3}\Delta} < w_0 < \frac{1}{3} - \frac{2}{3}\sqrt{1 + \frac{3}{2}\Delta} = w_{0*}^+. \tag{48}$$

The range w_0 when the accelerated expansion (A, B) is realized is shown in Figure 1.

The first branch (A) describes a matter with $w_0 > 1$ and $\rho < 0$ (it follows from (13) and $\langle u, u\rangle > 0$.

The second branch (B) corresponds to matter with a broken weak energy condition (since $w_0 < -\frac{1}{3}$) and $\rho > 0$ (since $\langle u, u\rangle < 0$). This matter is phantom (i.e., $w_0 < -1$) when $\Delta \geqslant 2$.

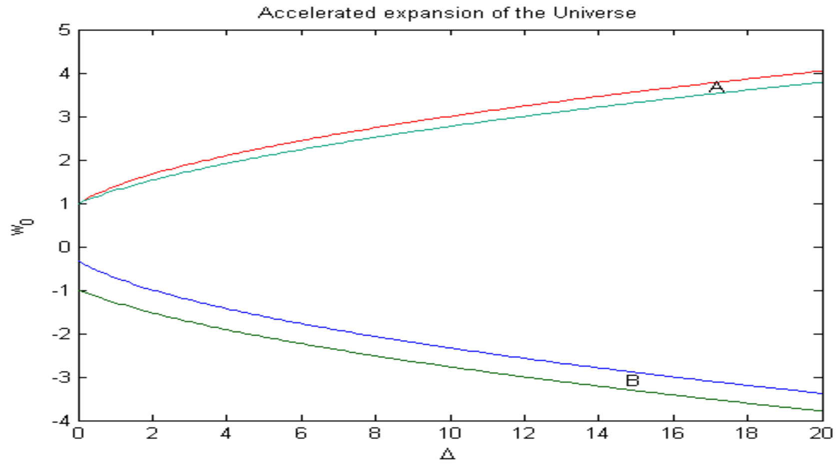

Figure 1. Dependence from parameter of an equation of state of our isotropic 3-dimensional space w_0 to coefficient of anisotropy of the additional dimensions Δ. Two branches of ranges of the solutions (A, B), realizing accelerated expansion are shown

5. Solutions with Acceleration and Asymptotically Constant G

It may be shown that power-law solutions from section 2 obeying restrictions (4), (10), $\langle u, u \rangle < 0$ and

$$\langle u, u^{\Lambda} \rangle / \langle u, u \rangle > 1 \tag{49}$$

are attractor solutions for more general class of solutions from [12] (for solutions with a massless scalar field see [15]).

So, in the case when restrictions (4), (10) and (49) take place we get

$$a_i = a_i(t_s) \sim A_i t_s^{\nu^i}, \tag{50}$$

and

$$\kappa^2 \rho \sim \frac{-2\langle u, u \rangle}{\langle u^{(\Lambda)} - u, u \rangle^2 t_s^2}, \tag{51}$$

as $t_s \to +\infty$, instead of (12) and (13).

When restriction on u_i parameters (38) is valid we get from (17)

$$G \sim G_0 = \text{const}, \tag{52}$$

as $t_s \to +\infty$.

Moreover, it may be shown that for any solution obeying restrictions (4), (10) and (49) there exists $T > 0$ such that for $t_s \in (T, +\infty)$ we get an accelerated expansion of our 3-space M_0 and small enough variation of the effective gravitational constant $G(t_s)$, obeying the observational restriction (15).

6. Conclusions

We have considered multidimensional cosmological models describing the dynamics of $n+1$ flat factor spaces M_i in the presence of a one-component anisotropic fluid with pressures in all spaces proportional to the density: $p_i = w_i \rho$, $i = 0, \dots, n$. Solutions with accelerated expansion of our 3-dimensional space M_0 and zero variation of the gravitational constant G were considered. These solutions exist for two branches of the parameter w_0: (A) and (B). Branch (A) describes superstiff matter with $w_0 > 1$ while branch (B) may contain phantom matter with $w_0 < -1$.

We have shown that these solutions are special case of more general solutions with accelerated expansion of our 3-space M_0 and asymptotically zero variation of the gravitational constant G, i.e. $G(t_s) \to \text{const}$ for $t_s \to +\infty$ (t_s is a synchronous time variable).

References

1. *Станюкович К. П.* Гравитационное поле и элементарные частицы. — М.: Наука, 1965. — 312 с. [Staniukovich K.P. Gravitational Field and Elementary Particles. — Moscow: Nauka, 1965. — (in russian).]
2. *Станюкович К. П., Мельников В. Н.* Гидродинамика, поля и константы в теории гравитации. — М.: Энергоатомиздат, 1983. — 256 с. [Staniukovich K.P., Melnikov V.N. Hydrodynamics, Fields and Constants in the Theory of Gravitation. — Moscow: Energoatomizdat, 1983. — (in russian).]
3. *Зайцев Н. А., Мельников В. Н.* Теории гравитации с переменными массами и гравитационной связью // Проюлемы теории гравитации и элементарных частиц. — М.: Атомиздат, 1979. — 131 с. [Zaitsev N. A., Melnikov V. N. Theories of Gravitation with Variable Masses and Gravitational Connection // Problems of Gravitation Theory and Particles Theory, 10th issue. — Moscow: Atomizdat, 1979. — (in russian).]
4. *Ivashchuk V. D., Melnikov V. N.* On Time Variations of Gravitations Constant in Superstring Theories // Nuovo Cim. — 1988. — Vol. B 102.
5. *Bronnikov K. A., Ivashchuk V. D., Melnikov V. N.* Time Variation of Gravitational Constant in Multidimensional Cosmology // Nuovo Cim. — 1988. — Vol. B 102. — Pp. 209–215.
6. *Bronnikov K. A., Melnikov V. N., Novello M.* Possible Time Variations of G in Scalar-Tensor Theories of Gravity // Grav. Cosmol. — 2002. — Vol. 8, Suppl. II. — Pp. 18–21.
7. *Melnikov V. N., Ivashchuk V. D.* Problems of G and Multidimensional Models // Proc. JGRG11. — Tokyo: Waseda Univ., 2002. — Vol. gr-qc/0208021. — Pp. 405–409.
8. *Melnikov V. N.* Time Variations of G in Different Models // Int. J. Theor. Phys. — 2002. — Vol. A 17. — Pp. 4325–4334.
9. *Miyazaki A.* Varying Cosmological Constant and the Machian Solution in the Generalized Scalar-Tensor Theory // Int. J. Theor. Phys. — 2001. — Vol. gr-qc/0103003. — Pp. 1–10.
10. *Fujii Y.* Varying Fine Structure Constant and the Cosmological Constant Problem // Astrophys. Space Sci. — 2003. — Vol. 283. — Pp. 559–564.
11. *Pakhomov A. G.* The Accelerated Expansion of the Universe and the Multidimensional Theory of Gravitation // Astronomical and Astrophysical Transactions. — 2005. — Vol. 24, No 5. — Pp. 441–446.
12. *Ivashchuk V. D., Melnikov V. N.* Multidimensional Cosmology with m-Component Perfect Fluid // Int. J. Mod. Phys. — 1994. — Vol. D 3. — Pp. 795–811.
13. *Ivashchuk V. D., Melnikov V. N.* Perfect-Fluid Type Solution in Multidimensional Cosmology // Phys. Lett. — 1989. — Vol. A 136. — Pp. 465–467.
14. *Ivashchuk V. D., Melnikov V. N., Zhuk A. I.* On WDW Equation in Multidimensional Cosmology // Nuovo Cim. — 1989. — Vol. B 104. — Pp. 575–581.
15. *Ivashchuk V. D., Melnikov V. N.* Multidimensional Classical and Quantum Cosmology with Perfect Fluid // Grav. Cosmol. — 1995. — Vol. 1, hep-th/9503223.
16. Multidimensional Cosmology with Anisotropic Fluid: Acceleration and Variation of G / J.-M. Alimi, V. D. Ivashchuk, S. A. Kononogov, V. N. Melnikov // Grav. Cosmol. — 2006. — Vol. 46–47, No 2–3. — Pp. 173–178.
17. *Piteva E. V.* Relativistic Effects and Solar Oblateness from Reader Observations of Planets and Spacecraft // Astron. Lett. — 2005. — Vol. 31. — Pp. 340–349.

18. *Panigrahi D., Zhang Y. Z., Chatterjee S.* Accelerating Universe as a Window to Extra Dimensions // J. Math. Phys. — 2006. — Vol. gr-qc/0604079. — Pp. 6491–6512.
19. *Baukh V., Zhuk A.* Sp-Brane Accelerating Cosmologies // Phys. Rev. — 2006. — Vol. D 73. — Pp. 104016–104031.

УДК 3937

Многомерная космологическая модель с анизотропной жидкостью: ассимптотическое ускорение и нулевая вариация G

А. Г. Пахомов

Учебно-научный институт гравитации и космологии
Российский университет дружбы народов
ул. Миклухо-Маклая, д. 6, Москва, Россия, 117198

Предлагается многомерная космологическая модель, описывающая динамику $n+1$ плоских пространств M_i в присутствии однокомпонентной анизтропной жидкости. Давление во всех пространствах пропорциональны плотности: $p_i = w_i\rho$, $i = 0, \ldots, n$. Изучаются решения с ускоренным расширением нашего трёхмерного пространства M_0 и нулевой вариацией гравитационной постоянной G. Эти решения существуют для двух ветвей параметра w_0: первая ветвь описывает материю с $w_0 > 1$, вторая может содержать фантомную материю с $w_0 < -1$. Показано, что эти решения являются частным случаем более общих решениий с ускоренным расширением нашего трёхмерного пространства M_0 и асимптотически нулевой вариацией гравитационной постоянной G.

Рассмотрена модель идеальной многомерной субстанции с тремя изотропными измерениями нашего пространства, дополнительными измерениями и временем. Пространственные измерения представлены степенной метрикой, зависящей от параметров уравнения состояния. Изучаются плоские фактор-пространства с однокомпонентной идеальной субстанцией. Получена в явном виде зависимость параметра уравнения состояния нашего изотропного 3-мерного пространства от коэффициента анизотропии дополнительных измерений, требующая ускоренного расширения Вселенной. Полученная зависимость представлена графически.

Ключевые слова: многомерная гравитация, анизотропная жидкость, ускоренное расширение, вариация G.