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Stochastic representation of quantum mechanics and entangled solitons

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Abstract. *Background* Using the Einstein's idea on particles-solitons, the stochastic representation of the wave function as a large sum of solitons with random phases is suggested. *Purpose* To illustrate the main idea of the stochastic representation, let us recall the lectures by N. Wiener on nonlinear problems in random theory. *Method* In these lectures Wiener used the so-called α-representation of the wave function for a nonrelativistic particle in three-dimensional space, the wave function being considered as an element of the random Hilbert space with the Gaussian dispersion. *Results* This representation appears to be equivalent, in accordance with the central limiting theorem, to the sum of many complex solitons with random phases. *Conclusions* On the basis of this representation it is possible to substantiate the Born's rule for measuring physical observables and also to explain the "spin–statistics" correlation in quantum mechanics.

Key words and phrases: Stochastic representation, Brioschi identity, "spin–statistics" correlation

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1. Introduction

It is worth-while to stress that in the period 1912–1913 the outstanding German physicist Gustav Mie published a series of works devoted to the field theory of matter [1–3]. The main idea of Mie was to dispense with the point-like charges as sources in electrodynamics, the linear Maxwell equations for the electromagnetic field being replaced with some new nonlinear ones, which would admit only regular solutions, without singularities.

This revolutionary approach by Mie promoted Einstein to discover adequate equations of General Relativity for describing gravitational fields. As a result, Einstein formulated a tremendous program of geometrizing physics, where particles were considered as pulsating extended objects, so-called solitons, that is clots of some universal "unitary field" [4–6]. This fundamental approach to creating the new physical picture of the world became popular especially within the scope of the quantum theory [7, 8]. However, the physical origin of that universal "unitary field" was unknown. It should be remarked that the important contributions to solving this enigma was made by the outstanding mathematicians Leonard Euler, Francesco Brioschi, and Norbert Wiener.

2. Euler problem on n squares and Brioschi identity

Euler formulated the following problem on n squares:

Given n real numbers $a_i,\ i = \overline{1,n}$, find n new real numbers $c_i,\ i = \overline{1,n}$, such that: 1) c_i is bilinear in a_k; 2) the following relation is valid:

$$\left(\sum_{i=1}^{n} a_i^2\right)^2 = \sum_{k=1}^{n} c_k^2.$$

In 1748 Euler knew solutions for this problem for $n = 2, 3, 4$ [9, 10]. However, the remarkable German mathematician A. Hurwitz in 1898 showed that for $n > 4$ the solution existed only for $n = 8$. That special solution was found before Hurwitz by the outstanding Italian geometrician Francesco Brioschi (1824–1897) [11], who used the complex projective coordinates (16 spinors Ψ) to study the geometry of the 8D-space. In particular, Brioschi proved the remarkable identity satisfied by any 8-spinor ψ (semispinor) [12]:

$$j^\mu j_\mu - \tilde{j}^\mu \tilde{j}_\mu = s^2 + p^2 + \vec{v}^2 + \vec{a}^2,$$

where the standard bilinear spinor quantities are used:

$$s = \bar{\psi}\psi, \quad p = \imath\bar{\psi}\gamma_5\psi, \quad \vec{v} = \bar{\psi}\vec{\lambda}\psi,$$

$$\vec{a} = \imath\bar{\psi}\gamma_5\vec{\lambda}\psi, \quad j_\mu = \bar{\psi}\gamma_\mu\psi, \quad \tilde{j}_\mu = \bar{\psi}\gamma_\mu\gamma_5\psi,$$

with Dirac matrices γ_μ, γ_5 and Pauli isotopic ones $\vec{\lambda}$ being included.

Suppose that one constructs the Lagrangian for some ψ-field model, which contains the so-called Higgs potential, the latter one monotonically depending on the invariant (2). Therefore, if one searches for stationary states in our model, the vacuum boundary condition at the space infinity should be valid:

$$\lim_{|\vec{x}|\to\infty} \psi = \psi_0.$$

Thus, fixing the vacuum value of the invariant (2), one gets, in view of (2), the field manifold

$$s^2 + p^2 + \vec{v}^2 + \vec{a}^2 = \text{const},$$

which is homeomorphic to the sphere S^7. On the other hand, the following inclusions take place:

$$S^7 \supset S^3 \supset S^2.$$

Taking into account the nontriviality of the homotopy groups

$$\pi_3(S^3) = \pi_3(S^2) = \mathbb{Z},$$

one can expect due to (2) the existence in our field model of topological solitons endowed with the corresponding topological charges of two kinds. The first one is the winding number $\deg(S^3 \to S^3)$, which can be interpreted as the baryon charge $\mathbb{B}$. The second one refers to the so-called Hopf invariant $Q_{\rm H}$ and can be interpreted as the lepton charge $\mathbb{L}$. The corresponding physical models proved their effectiveness in nuclear physics (Skyrme model [13–15]) and in the theory of lepton interactions (Faddeev model [16–18]). Topological analysis for the stability of solitons appears to be important in many physical models [19–21].

3. Stochastic representation of quantum mechanics

Wiener found a special stochastic representation of the one-particle wave function considered as an element of the random Hilbert space with the Gaussian dispersion [22–26]. Nowadays this representation is known as the stochastic integral by Wiener–Ito–Stratonowich. To show its relation to the Einstein's idea on particles-solitons, one intends to represent the wave function as a large sum of complex solitons with random phases. In accordance with the central limiting theorem [27] this sum behaves as a Gaussian random variable.

As an illustration of this effect let us consider the classical T. Young's 2-slit diffraction experiment, the photons being replaced with the Brioschi solitons $\phi(t,\vec{x}) = \phi(t,\vec{x})^*$ in the real representation. Introducing the Lagrangian density $\mathcal{L}(\phi, \partial_\mu \phi)$ and calculating the canonical momentum

$$\pi(t,\vec{x}) = \frac{\partial \mathcal{L}}{\partial(\partial_t \phi)},$$

consider the auxiliary complex function

$$\varphi(t,\vec{x}) = 2^{-1/2}\left(\nu\phi + \imath\pi/\nu\right),$$

where the number ν can be found from the normalization:

$$\hbar = \int dV\,|\varphi|^2 \approx \int\limits_{V_0} dV\,|\varphi|^2,$$

with $\hbar$ being the Planck constant and $V_0 = \ell^3$ denoting the proper volume of the soliton. Here dV stands for the elementary 3-volume.

Let us now consider an impenetrable screen with 2 slits, for which the width $d \gg \ell$ is supposed to be large with respect to the proper size ℓ of the particles-solitons. During Young's experiment the particles are pushed to the screen one by one with some fixed initial velocity. Our aim is to find the probability for observing a center $\vec{r}$ of a particle-soliton in some small volume $\Delta V(\vec{X}_0) \gg V_0$ located behind the screen at large distance from it, with $\vec{X}_0$ denoting the central point of this small volume.

Assuming now the Young's experiment with the soliton's scattering to be repeated $N \gg 1$ times, one can define the wave function as follows:

$$\psi(t,\vec{x}) = (\hbar N)^{-1/2} \sum_{j=1}^{N} \varphi_j(t,\vec{x}),$$

where $\varphi_j(t,\vec{x})$ denotes the function (3) for the j-th trial. With the intention to show that $|\psi(t,\vec{x})|^2$ plays the role of the probability density $\rho(t,\vec{x})$ let us calculate the following quantity:

$$\Delta V\,\rho = \int\limits_{\Delta V} dV\,|\psi|^2 = \frac{1}{\hbar N}\left(\sum_{i=1}^{N} a_{ii} + s\right),$$

where the denotations are used:

$$s = \sum_{i\neq k=1}^{N} a_{ik}, \quad a_{ik} = \frac{1}{2}\int\limits_{\Delta V} dV\,\left(\varphi_i^* \varphi_k + \varphi_k^* \varphi_i\right).$$

Taking into account the normalization (3), one gets

$$\sum_{i=1}^{N} a_{ii} = \hbar \Delta N, \quad \rho \Delta V = \frac{1}{\hbar N}(\hbar \Delta N + s),$$

where ΔN is the number of trials, for which the center of our soliton can be observed in the domain ΔV. Considering φ_j as independent random variables with zero mean values, one can estimate the probability for $|s|$ to surpass $\hbar\Delta N$ through applying the Chebyshev's inequality [27]:

$$P(|s| > \hbar\Delta N) \leq (\hbar\Delta N)^{-2} < s^2 >,$$

where the symbol <> denotes calculating the mean value.

Taking into account that the profiles φ_i and φ_k for $i \neq k$ can overlap only in the domain of the volume V_0, one obtains the estimate

$$< s^2 > \leq \alpha(\hbar)^2 V_0 \Delta N \frac{\Delta N}{\Delta V},$$

where $\alpha \sim 1$ stands for the effective "packing" coefficient. Thus, unifying (3) and (3), one finds the estimate:

$$P(|s| > \hbar\Delta N) < \alpha \frac{V_0}{\Delta V} \ll 1,$$

whence one concludes that with the probability close to unity the quantity $\rho = |\psi|^2$ can be interpreted as the probability density for measuring the coordinates of the soliton's center behind the screen. It means that the stochastic wave function (3) plays the role of the probability amplitude in accordance with the Born's interpretation.

4. Measuring physical observables

Let us now discuss measuring some physical observable A, which can be expressed within the scope of the Lagrangian formalism, if one follows the E. Nther's theorem in application to soliton configurations considered as island-like systems with the asymptotically flat space-time [20]. In particular, if the generator $\hat{M}_A$ of the corresponding asymptotic symmetry group is given, our physical observable takes the form:

$$A = \int dV(\pi\imath\hat{M}_A\phi) = \int dV(\varphi^*\hat{M}_A\varphi).$$

Applying (4) to the j-th trial, one can express the mean value in question as follows:

$$< A > = \frac{1}{N}\sum_{j=1}^{N} A_j = \frac{1}{N}\int dV \sum_{j=1}^{N} (\varphi_j^* \hat{M}_A \varphi_j).$$

Using the definition (3) of the stochastic wave function and taking into account the estimate (3), one can rewrite (4) with the probability close to unity in the standard quantum mechanical form (the Born's rule):

$$< A > = \int dV(\psi^*\hat{A}\psi),$$

where the Hermitian operator $\hat{A} = \hbar\hat{M}_A$ is introduced.

5. Stochastic representation for the case of several particles. "Spin–statistics" correlation

Let us consider the case of n particles-solitons. Here a new definition of the wave function in the configuration space of the dimension $3n$ should be given as the generalization of (3):

$$\psi_n\left(t, \vec{x}_1, \dots, \vec{x}_n\right) = (\hbar^n N)^{-1/2} \sum_{j=1}^{N} \prod_{k=1}^{n} \varphi_j^{(k)}\left(t, \vec{x}_k\right),$$

where $\varphi_j^{(k)}$ stands for the soliton configuration of the form (3) for the k-th particle in the j-th trial. In this case the mean value expression for some observable A_n is similar to (4) and ensues from (5):

$$< A_n > = \int dV_n \left(\psi_n^* \hat{A}_n \psi_n\right),$$

where dV_n denotes the elementary volume in the new configuration space and the operator $\hat{A}_n$ in (5) is the superposition of the one-particle generators:

$$\hat{A}_n = \sum_{k=1}^{n} \hbar \hat{M}_A^{(k)}.$$

As can be seen from (5), the wave function has the structure of the so-called entangled solitons [28].

Now it is worth-while to apply the definition (5) to explaining the well-known "spin–statistics" correlation in quantum mechanics. To this end, suppose our particles to be identical, with the soliton profiles $\varphi_j^{(k)}$ belonging to the irreducible representation D^J of the rotation group SO(3), where the weight J is the spin of our particles-solitons. Since for identical particles the profiles $\varphi_j^{(k)}$ should be equal, one can write:

$$\varphi_j^{(k)} = \varphi_j\left(\vec{x}_k - \vec{r}_k\right),$$

with $\vec{r}_k$ denoting the center of the soliton in question.

Let us now choose two arbitrary particles with the centers $\vec{r}_1$, $\vec{r}_2$ and perform their transposition. It means that our two-particle system is rotated over angle π around the bisector $\vec{b}$ of the central line $\vec{r}_1 - \vec{r}_2$. However, in this position the extended character of our particles-solitons comes into play. In fact, to restore the previous state of the system, it would be inevitable to perform proper rotations of our particles-solitons over angle π around their central axes parallel to $\vec{b}$. As can be easily seen, this latter operation appears to be equivalent to the relative rotation of our two particles over angle 2π. Due to the properties of the representation D^J the 2π-rotation implies multiplying the wave function (5) by $(-1)^{2J}$. Therefore, one concludes that the wave function should be symmetrical under transpositions of particles for the integer spin J and antisymmetrical — for the semi-integer J. This property of the wave function corresponds to the well-known "spin–statistics" correlation in quantum mechanics.

6. Results and Discussions

Motivated by the works of Gustav Mie on field theory of matter, Einstein formulated a tremendous program of geometrizing physics, where particles are represented as solitons, that is clots of some fundamental "unitary field". Basing on the central limiting theorem, one can construct the stochastic representation of the wave function as a large sum of solitons with random phases, the wave function being considered as a Gaussian random variable. This fact permits one to interpret the wave function as the probability amplitude for observing center coordinates of particles-solitons. Moreover, it's possible to substantiate the Born's rule for measuring physical observables as bilinear functionals in the wave function and also to explain the well-known "spin–statistics" correlation in quantum mechanics.

7. Conclusions

Taking into account the idea by G. Mie and A. Einstein on particles-solitons, the stochastic representation of the wave function is suggested. Using this new picture, it's possible to explain the M. Born's quantum rule for measuring physical observables and also the "spin–statistics" correlation.

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Стохастическое представление квантовой механики и запутанные солитоны

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Аннотация. *Актуальность* Используя идею Эйнштейна о частицах-солитонах, предлагается стохастическое представление волновой функции в виде большой суммы солитонов со случайными фазами. *Цель* Для иллюстрации основной идеи стохастического представления вспомним лекции Н. Винера по нелинейным задачам в теории случайных величин. *Метод* В этих лекциях Винер использовал так называемое α-представление волновой функции для нерелятивистской частицы в трёхмерном пространстве, рассматривая волновую функцию как элемент случайного гильбертова пространства с гауссовой дисперсией. *Результаты* Это представление, согласно центральной предельной теореме, оказывается эквивалентным сумме многих комплексных солитонов со случайными фазами. *Выводы* На основе этого представления можно обосновать правило Борна для измерения физических наблюдаемых величин, а также объяснить корреляцию «спин–статистика» в квантовой механике.

Ключевые слова: стохастическое представление, тождество Бриоски, корреляция спин–статистика