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Interaction of relativistic electrons with intense electromagnetic fields: ponderomotive effect, acceleration, refraction, reflection, dependence on initial conditions

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Abstract. The rigorous theory and characterization of charged-particle dynamics in high-intensity electromagnetic fields are fundamental for the development of advanced plasma-based applications. Accurate analytical models must bridge the gap between smoothed trajectories and exact particle motion to predetermine injection and energy gain. The main objective of this review is to establish a rigorous framework for the averaged relativistic motion of electrons, focusing on the strict derivation of ponderomotive forces and the impact of fast-oscillating periodic additions on dynamical variables. By making use of the Krylov–Bogoliubov–Mitropolsky averaging method to obtain the equations of motion, the study analyzes relativistic effects in laser beams and waveguides. These theoretical predictions are substantiated through numerical validation, including test-particle simulations and three-dimensional particle-in-cell simulations (PIC) of relativistic self-trapping regimes such as “laser bullet” and “bubble” structures. The review details the independence of the results on the formulation framework, the strict dependence on wave polarization, and the non-strict potential character of the relativistic ponderomotive force. The analysis demonstrates that periodic fast-oscillating additions are essential for a complete description, accurately setting initial conditions in averaged equations and enabling precise predictions of electron reflection and refraction. Simulations confirm that these fast-oscillating corrections determine electron injection and beam charge in realistic laser–plasma acceleration scenarios. The present review clearly shows that the dual framework of test-particle and PIC models is vital for probing the limits of averaged motion theory. The findings are of direct practical relevance for the optimization of radiation sources and guide the development of future theories incorporating non-adiabatic and field topology dependent effects.

Key words and phrases: averaged motion, relativistic ponderomotive forces, laser radiation, Gaussian beam, waveguides, beat wave

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1. Introduction

The dynamics of charged particles in intense electromagnetic (EM) fields is a fundamental area of research with significant implications for fields such as laser–plasma interaction, particle acceleration [1], and plasma heating [2]. The complexity of particle motion increases significantly when the EM field is inhomogeneous in space and time [3] or the relativistic effects become considerable [4]. To address this, various approximate methods, particularly averaging techniques [5], have been developed to derive simplified equations of motion for the "guiding center" or "smoothed" variables of the particle, while accounting for the "fast oscillations" or "periodic additions", as mentioned in [6–9], is treated as an intrinsic intermediate procedure.

A central concept derived from the averaging analysis is the ponderomotive force, which describes the averaged driving of a high-frequency EM field on a charged particle [10–13]. Initially derived by Gaponov and Miller [10] for nonrelativistic particles in a weak monochromatic field, this force is potential and points towards regions of lower field intensity [11, 14]. Its relativistic generalization, initially presented by Kibble [15], introduced the concept of an "effective mass" of the particle, $m^* = m\sqrt{1+A^2}$, where $A^2 = (e/mc\omega)^2\langle E^2\rangle$ is the normalized vector potential, redefined as a function of the averaged (slow) coordinates. This initial relativistic treatment also introduced the concept of mutual refraction of electrons and photons, an appealing name that captures the physical analogy of an electron's deflection in a wave field mirroring light refraction in plasmas. This powerful, physical similarity offers considerable insight into the problem, much of which has unfortunately vanished from modern discourse.

Further analyses of applicability were conducted, yielding various generalizations, particularly considering the influence of the relativistic nature of particle motion [16, 17], the superposition of multiple waves [18], and external magnetic fields. Such diversity of scenarios, field configurations, and formalisms employed leads to a wide range of significant results. Notably, the average wave-particle interaction can depend on propagation direction, polarization, spatiotemporal amplitude profile, and wave intensity, and it may not necessarily be potential. However, the absence of a unified rigorous approach to these problems often results in contradictions across studies [13, 19, 20]. Furthermore, while averaging particle motion simplifies the description of dynamics in non-uniform, rapidly oscillating fields, the rapidly oscillating components of dynamic variables, along with their initial conditions, are frequently overlooked. This omission prevents a complete and unambiguous understanding of particle motion.

The collective works [6–9] offer a rigorous and consistent application of the Krylov–Bogoliubov–Mitropolsky (KBM) [21] averaging over fast phase [22] method to analyze the relativistic motion of charged particles in a variety of intense and inhomogeneous EM fields. These studies go beyond typical derivations of ponderomotive forces by meticulously calculating oscillating additions and highlighting their crucial role, particularly in defining initial conditions for averaged equations. They refine theoretical models and uncover previously overlooked complexities in particle-wave interactions [2]. In this review, we assess the results, validity, and significance of these researches. Our goal is to provide a coherent framework that connects foundational principles with modern applications.

2. Relativistic averaged motion theory: model and methods

Classical averaging techniques, mainly applied to finding the ponderomotive force, often prove insufficient when examining the motion of charged particles in relativistic regimes. Early attempts introduced more rigorous derivations using multiple-scale perturbation theory [12], Hamiltonian

[23] and Lagrangian [24] averaging, and covariant formulations [17, 25] . These efforts primarily revealed that the relativistic ponderomotive force is not only amplitude-dependent but also sensitive to polarization and space–time structure of the EM field. The KBM averaging over the fast phase theory suggests that a correct averaging procedure first requires establishing a standard form of the equation of motion. This allows one to prove the existence of a specific change of variables that excludes time from the right-hand sides of these equations with a prescribed degree of accuracy in terms of a small parameter [26]. Here, we present a synthesis of results from theoretical manuscripts [6–9], which make use of the KBM method. These works consider a full coupling between field geometry [27], relativistic particle momentum [28], and relativistic kinematics [29], while scrutinizing the meaning of the fast oscillating terms explicitly obtained by the KBM method, thus including their influence, especially in the setting of initial conditions.

2.1. Polarization effects

The paper [6] extensively examines the relativistic motion of a charged particle in intense linearly and circularly polarized EM radiation within the geometrical optics approximation [30]. Whereas it is usually accepted that ponderomotive forces are independent of the polarization of the wave [31], research as [13] has shown that the averaged equations of motion of the particle (and consequently the expressions for the ponderomotive force) are different for circularly and linearly polarized waves. A key aspect of the work [6] is the consistent derivation of averaged relativistic equations of motion, confirming that the expressions for the ponderomotive force differ for circularly and linearly polarized waves. This directly addresses and clarifies previous contradictions between [13] and [31].

The analysis introduces two dimensionless parameters: $g = eE/mc\omega$, representing the ratio of the particle's oscillating velocity amplitude to the speed of light, and μ, associated with the space-time variations of the amplitude. For intense radiation fields, g can be comparable to or greater than unity (e.g., $g \approx 1$ for a wavelength of $1\ \mu m$ and intensity of $10^{18}\ W/cm^2$). This means that g is not generally small, making averaging via this parameter unfeasible according to the Bogoliubov-Krylov theorem.Thus averaging must be performed by expanding in the small parameter μ, which is small in the geometrical optics approximation, $\mu \approx 1/kL \approx 1/\omega T$. Let us note that all references to the parameter g in cited works [6–9] correspond to the amplitude of the normalized vector potential in the averaged description and should be identified with the standard symbol a_0 for consistency [32].

The derived expressions for the averaged relativistic force for both polarizations contain new additional small terms weakening its module. While these terms are small, their effect can be noticeable at small radiation field gradients. A critical distinction arises for linearly polarized waves: rapidly oscillating terms in the relativistic factor, specifically those oscillating with a doubled phase whose amplitude depends only on wave intensity, are not associated with the small expansion parameter. This implies that for a linearly polarized wave, the field cannot be excessively strong, as the usual binomial expansion for the relativistic factor might become invalid, complicating the averaging procedure significantly. The averaged action of a linearly polarized wave on a particle is shown to be more weakened than that of a circularly polarized wave.

2.2. The impact of spatiotemporal wave structure

The manuscript [7] focuses on ponderomotive forces in intense laser radiation fields described as the superposition of Gaussian beams of arbitrary modes in the quasi-optical approximation. This approximation is often considered more adequate for laser radiation [33] and provides a physically realistic description of the field's spatial structure analysis [30]. The small parameter here is $\mu = 2/ka$, where a is the beam waist. The ponderomotive force for circularly polarized Gaussian beams is derived. The method involves representing the laser field via a vector potential $\boldsymbol{A}(\boldsymbol{r}, t)$ and expanding its complex amplitude $A_0(r)$ in even powers of μ. The longitudinal component of the vector potential, $A_{zm}^{(1)}$, emerges in odd powers of μ from the Coulomb gauge condition $\nabla\boldsymbol{A} = 0$. Similar to the previous work [6], a generalized momentum vector, $\boldsymbol{\pi} = \boldsymbol{p} + (e/c)\boldsymbol{A}$ is employed to handle large, fast-oscillating terms. A significant finding is that relativistic effects lead to a weakening (attenuation) of the averaged force of high-power laser radiation on the particle. The ponderomotive force is proportional to the gradients of the Gaussian radiation intensity, aligning with experimental data [34]. The difficulties in averaging the longitudinal motion equations, particularly due to large rapidly oscillating components proportional to the wave field, are addressed by assuming weakly relativistic motion in the transverse plane, which is often a natural assumption for acceleration problems. The mean energy of the particle-radiation system is conserved to second-order terms when the amplitude is time-independent. Furthermore, particle acceleration or deceleration is shown to depend on its injection into a divergent or convergent Gaussian beam, respectively.

2.3. Beat waves approach to modulated wavefronts

The study in [8] extends the analysis to the relativistic motion of a charged particle in the field of a laser beat wave, formed by the superposition of two circularly polarized Gaussian beams in the fundamental mode propagating in the same direction. This is crucial for understanding mechanisms of particle acceleration [35, 36] and plasma heating [37]. A unique and significant result in this context is the demonstration that, although a relativistic generalization of the ponderomotive potential is defined, the averaged force in the field of a beat wave is not completely potential. This contrasts sharply with the potential nature of the standard form of the Gaponov-Miller force. The force is found to depend significantly on the slowly varying combination (beat wave) phase, which evolves according to its own nonlinear equation. This implies a more complex interaction than for single-wave fields. The averaging procedure here has a distinct feature: while partial phases of the constituent waves are considered “fast” and averaged over, the combination phase is “slow” and is not averaged. Relativistic effects and the diffractive spreading of the beams further weaken the averaged action on the particle. It is also shown that the transverse components of the ponderomotive force are first-order effects in their expansion, while the longitudinal component is a second-order effect. This implies that ponderomotive expulsion of particles in the radial direction (towards weaker fields) occurs faster than acceleration in the direction of wave propagation. The particle’s trajectory in the transverse plane can be approximated as a circle whose radius slowly decreases as the beat wave propagates, unlike the constant radius for a plane wave.

2.4. Oscillating additions and initial conditions

Reference [9] delves into the relativistic motion of a single electron entering a rectangular waveguide supporting an arbitrary H_{mn}-mode wave. Here, the small parameter for expansion is $g = eE/m_e c\omega$, which is typically small for waveguide fields. A salient finding is that the averaged (ponderomotive) force along the longitudinal axis of the waveguide is absent, regardless of the wave mode, meaning that no non-gradient forces are generated in this direction. As a standard effect, the transverse components of the ponderomotive force expel the charged particle from regions of high field intensity. The constant of motion, $\gamma - p_z/mc = C$, which behaves analogously to the refractive index for plane waves in a dielectric medium, is confirmed. Owing to the influence of the explicitly presented periodic additions terms, the analytical and numerical results from [9] are crucial: together, these results address and reconcile the contradiction between [19] and [38].

The crucial role of oscillating additions and Initial Conditions is studied deeper in this work. Across all the studies [6–9], a consistent and highly emphasized theme is the meticulous calculation and application of “periodic additions” to the smoothed (averaged) dynamical variables. While previous works often used these oscillating parts only for deriving averaged equations and then disregarded them, the aforesaid works rigorously derive the terms usually up to second-order expansions. The numerical simulations repeatedly demonstrate that an excellent agreement between the exact equations of motion and the averaged solutions is achieved only when the initial conditions for the averaged equations are correctly defined by incorporating the periodic additions across the entire time evolution, including the initial moment. For the initial moment onward This “initial leap” between the exact and averaged momenta, determined by the periodic additions, is critical. For example, in the waveguide case, the longitudinal averaged momentum at the initial instant may differ from zero and even be negative, which is correctly predicted by their model and verified numerically. This detailed treatment of initial conditions allows for an accurate description of phenomena like electron refraction and reflection by the waveguide field, consistent with Kibble’s [15] earlier work on mutual refraction of electrons and photons. As remarked and shown in [9] and collectively supported by [6–9], depending on injection conditions and initial phase, an electron may either penetrate or be reflected by the waveguide field, with the critical momentum being defined by the periodic additions.

2.5. Methodological constraints and physical limitations

A rigorous application of the Krylov–Bogoliubov–Mitropolskiy (KBM) averaging method to the relativistic equations of motion reveals fundamental constraints on the resulting analytical models for the ponderomotive forces. These constraints establish the foundational bounds within which the KBM-derived formalism remains both mathematically correct and physically adequate. A primary limitation arises because the Lorentz force equation in an intense field ($a_0 = eE/mc\omega \sim 1$) is not initially in the “standard form” required by the KBM formalism, which presupposes a clear separation between slow dynamics and small, fast oscillations. The right-hand side of the equation of motion contains large-amplitude terms, proportional to the field strength a_0, that oscillate at the optical frequency. Direct averaging is therefore impossible [6]. As demonstrated in the works [8], is a preliminary transformation to the particle’s canonical momentum, $\boldsymbol{\pi} = \mathbf{p} + (e/c)\mathbf{A}$ automatically absorbs and eliminates the dominant oscillatory force terms. However, this necessary step complicates the subsequent averaging of the relativistic factor $\gamma = \sqrt{1 + (\mathbf{p}/mc)^2}$. Expressed in terms of the canonical momentum $\boldsymbol{\pi}$ and the field momentum $\mathbf{p}_E = (e/c)\mathbf{A}$, γ becomes depending on the $\boldsymbol{\pi} - \mathbf{p}_E$. A consistent KBM expansion of γ is only straightforward if the oscillatory part $\mathbf{p}_E$ is the dominant momentum scale. This leads to a critical, often implicit, assumption: the smoothed transverse canonical momentum must satisfy $|\boldsymbol{\pi}_\perp| < |\mathbf{p}_E|$. When this condition holds, a binomial expansion of γ in powers of $(\boldsymbol{\pi} \cdot \mathbf{p}_E)$ is justified and allows for systematic averaging [6, 7]. Violation of this condition signals a regime where the particle’s quiver motion is no longer the primary relativistic effect, and the standard ponderomotive expansion fails.

A separate and stringent limitation concerns the longitudinal motion. For the wave phase $\theta = kz - \omega t$ to be a “fast” variable suitable for averaging, its derivative $d\theta/dt = -\omega(1 - v_z/c)$ must remain large. This requires that the quantity $G = \gamma - p_z/mc = \gamma(1 - v_z/c)$ is not too small [6]. Physically, this condition $|1 - v_z/c| \gg \mu$ (where μ is the slow-variation parameter) ensures that the Doppler-shifted frequency experienced by the particle remains high [39]. As the particle’s longitudinal velocity approaches c, this condition breaks down; the phase evolution becomes slow, the separation of time scales vanishes, and the averaging procedure is invalidated.

Finally, accounting for the finite extent in time of the EM fields is essential, particularly in laser EM fields, where attaining higher radiation intensities unavoidably entails shorter pulse durations. We note that the entire KBM approach and the adopted aprroximations(e.g. quasioptical or geometrical) rest on adiabatic assumptions, and consequently the laser pulse envelope (characterized by scales L, T, or waist a) must vary slowly compared to the optical period ($\mu \ll 1$). This assumption underpins the definition of the small expansion parameter $\mu = 1/(kL) \approx 2/(ka)$ [31]. For few-cycle or sub-cycle laser pulses, this clear scale separation collapses. In such ultra-short pulse regimes, the particle dynamics is intrinsically and strongly phase-dependent, the concept of a time-averaged ponderomotive force becomes ill-defined, and analysis must revert to fully time-resolved models or direct numerical integration of the Lorentz equations.

Let us remark that within the regime of laser intensity and particle energy considered in the articles which are the scope of this work, radiation reaction and quantum electrodynamical (QED) effects are negligible [40]. Collective plasma behavior is treated qualitatively within a phenomenological framework.

3. Numerical and experimental verification

The theory of averaged relativistic motion for single charged particles, developed in [6–9], requires systematic benchmarking against both numerical simulations and experimental data. In particular, the works [1, 41] have been instrumental — not only in validating the foundational framework — but also in extending it to more complex, less idealized scenarios of laser–plasma interaction. The theoretical predictions derived therein show strong agreement with both single-particle simulations and full particle-in-cell (PIC) results for canonical configurations, such as Gaussian laser pulses interacting with preformed or self-generated plasma channels, as well as in the relativistic self-trapping regime. Key validation metrics include trajectory fidelity over multiple laser cycles, long-term energy gain or loss, the spatial distribution of electrons expelled by the laser field, and the subsequent dynamics of these particles under quasi-static fields.

However it is crucial to recognize the boundaries of this averaged description. Beside the limitations described in section the formalism based in KBM method breaks down in stochastic regimes. Specifically, when stochasticity develops, for instance in complex field configurations where a laser field is assisted by large-amplitude plasma waves [42] the foundational KBM averaging method loses its strict applicability [43]. In such cases, the concept of a well-defined ponderomotive force is partially employed, serving primarily for qualitative estimates or as a guiding approximation and the particle dynamics must be analyzed using tools for chaotic systems, such as Lyapunov exponents, Poincaré maps, and other indicators of non-integrability and phase-space mixing.

Experimental [14, 44, 45] benchmarking remains more challenging due to the difficulty of isolating pure ponderomotive effects from competing processes such as collisional heating, space-charge fields, and instabilities. Nevertheless, combined diagnostics can enable semi-quantitative validation of predicted density modulations and EM field structures consistent with relativistic ponderomotive theory [46, 47].

3.1. Test-particle models

For controlled investigations of single-particle dynamics, test-particle models offer a complementary and highly flexible approach. In these models, prescribed EM fields — such as Gaussian laser beams, beat waves, or idealized waveguide modes — are used to integrate the relativistic equations of motion for ensembles of particles. This setup allows direct comparison with analytical expressions for the ponderomotive force and enables the isolation of effects due to field geometry [44], polarization [48], and carrier-envelope phase [49]. Future studies will employ hybrid approaches that combine test-particle trajectories with envelope-averaged field models, thereby bridging the gap between rapid quiver motion and slow drift dynamics.

Accurate modeling of the ponderomotive force, augmented by information from the periodic additions to the smoothed motion, has proven essential for predicting injection thresholds, beam quality, and energy spread. These parameters are critical for applications ranging from compact accelerators and bright radiation sources to medical therapies.

A related study of direct laser electron acceleration in plasma channels formed by ultrashort, relativistically intense pulses, both linearly and circularly polarized, examines post-injection electron dynamics governed by self-generated quasi-static fields [41]. Using test-particle simulations, this work incorporates the radial electric field, azimuthal magnetic field, and, in the case of circular polarization — an axial quasi-static magnetic field component, the latter having been first identified in PIC simulations. Through consistent application and numerical testing of the KBM averaging method, this research reveals distinct mechanisms of drift, diffusion, and acceleration, alongside detailed analyses of trajectory stability and chaotic motion.

3.2. Particle-in-Cell (PIC) simulations

Relativistic ponderomotive forces lie at the heart of modern laser-driven plasma acceleration schemes. The leading edge of an intense laser pulse expels background electrons via the ponderomotive force, creating a trailing, charge-separated cavity that sustains enormous longitudinal electric fields [50]. Particle-in-cell (PIC) simulations serve as the primary computational tool for self-consistently modeling these interactions at relativistic intensities; crucially, they capture ponderomotive effects naturally by resolving the fast oscillatory motion of particles without invoking explicit cycle averaging.

Among mechanisms associated with the formation of an ion cloud in material that moves together with the laser pulse by the effects of ponderomotive force, Relativistic self-trapping (RST) of an intense laser pulse represents one of the most efficient mechanisms for laser-driven electron acceleration, delivering extreme charge ($> 10\,\mathrm{nC}$) with high energy conversion efficiency ($\sim$ 40–50 %). This mechanism operates in two characteristic regimes: the “laser bullet”, where the pulse length $c\tau$ is comparable to the cavity diameter D ($c\tau \sim D$), and the “bubble” regime, where $c\tau \ll D$. The distinction depends on the fraction of the cavity volume filled by the laser field. A quantitative comparison of these regimes, supported by recent work [1], relies on a physical interpretation grounded in the averaged dynamics of test particles. This approach has clarified distinct electron injection mechanisms and enabled predictive scaling laws for injection efficiency.

The equilibrium size of the plasma channel (or cavity) is governed by a balance between the outward radial ponderomotive force and the inward Coulomb force of the ion core at the channel boundary [51]. The standard scaling law for the channel radius, $R \propto I_0^{1/4}$, emerges from this force balance. However, its derivation traditionally assumes a uniform transverse laser profile, where the dimensionless field amplitude a_0 is constant. In a more realistic, non-uniform transverse profile, the field amplitude at the boundary, $a_0^{(\mathrm{b})}$, is lower than the peak axial value, $a_0^{(\mathrm{axis})}$. Consequently, the proportionality constant α in the scaling law must be adjusted when the boundary field is used, resolving an ambiguity present in the literature [52].

Although full-scale particle-in-cell (PIC) simulations naturally capture collective, non-adiabatic, and stochastic effects in ultra-short laser–plasma interactions, the underlying ponderomotive dynamics can still be identified through tailored diagnostic procedures. In the analysis of [1], signatures of the averaged ponderomotive force will be extracted by examining phase-averaged particle momenta, correlating particle expulsion with local intensity gradients, and reconstructing effective force fields from simulation data. These methods allow for the assessment of the relative contribution of ponderomotive mechanisms even in strongly nonlinear regimes, providing a self-consistent bridge between single-particle averaged theory and collective plasma behavior.

It should be noted that high-resolution simulations of modern relativistic laser–plasma interactions often uncover phenomena that lie beyond conventional averaged descriptions, such as phase trapping, stochastic heating in chaotic field regions, and transient momentum kicks during the pulse rise time. These effects underscore the limitations of adiabatic approximations and motivate the development of refined, non-perturbative theories of relativistic ponderomotive dynamics [51].

4. Conclusion

The body of work exemplified by the articles [6–9] provides a comprehensive and rigorous framework for understanding the relativistic motion of charged particles in intense and inhomogeneous EM fields. Through the consistent application of the KBM averaging method, these studies yield refined expressions for the ponderomotive force, revealing its dependence on wave polarization, its non-potential character in beat-wave fields, and its complete absence along the propagation axis in certain waveguide modes.

Crucially, these works highlight the often-overlooked importance of the fast-oscillating “periodic additions” to the smoothed dynamical variables. It has been both analytically derived and numerically validated that these terms are essential for accurately setting initial conditions in the averaged equations, thereby bridging the gap between exact and approximate solutions and enabling precise predictions of phenomena such as electron reflection and refraction.

Collectively, these contributions significantly advance the theoretical understanding of charged-particle dynamics in strong fields, offering robust analytical tools for analyzing complex interactions relevant to high-intensity laser–matter physics, plasma-based accelerators, and radiation-source development. Their systematic methodology and thorough validation render them particularly valuable for future research.

Moreover, the complementary use of test-particle models and full particle-in-cell (PIC) simulations provides a powerful dual framework for probing the validity and limitations of the theory of averaged relativistic motion. While PIC simulations capture collective plasma effects and self-consistent field evolution, test-particle models isolate the fundamental single-particle physics underlying ponderomotive acceleration, injection, and transport. Together, they not only validate existing analytical models but also guide the development of next-generation theories that incorporate non-adiabatic, phase-resolved, and field-topology-dependent effects — capabilities essential for the design and optimization of next-generation laser–plasma accelerators and compact radiation sources.

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Взаимодействие релятивистских электронов с интенсивными электромагнитными полями: пондеромоторные эффекты, ускорение, преломление, отражение и зависимость от начальных условий

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Аннотация. Строгая теория и описание динамики заряженных частиц в высокоинтенсивных электромагнитных полях имеют фундаментальное значение для разработки перспективных плазменных приложений. Точные аналитические модели должны устранять разрыв между усредненными траекториями и истинным движением частиц для предварительного определения параметров инжекции и набора энергии. Основная цель данного обзора заключается в создании строгого аналитического описания усредненного релятивистского движения электронов с упором на строгий вывод пондеромоторных сил и влияние быстро осциллирующих периодических добавок на динамические переменные. С помощью метода усреднения Крылова–Боголюбова–Митропольского для получения уравнений движения в работе анализируются релятивистские эффекты в лазерных пучках и волноводах. Теоретические результаты подтверждаются в ходе численной валидации, включающей моделирование тестовых частиц в заданных полях и трехмерное моделирование плазмы в ячейках (PIC) для режимов релятивистского самозахвата, таких как структуры «лазерная пуля» и «пузырь». В обзоре подробно описаны независимость результатов от способа описания (приближение), строгая зависимость от поляризации волны и нестрого потенциальный характер релятивистской пондеромоторной силы. Анализ показывает, что периодические быстро осциллирующие добавки необходимы для полного описания, точного задания начальных условий в усреднённых уравнениях и обеспечения достоверного прогнозирования явлений отражения и преломления электронов. Моделирование подтверждает, что быстро осциллирующие добавки определяют инжекцию электронов и заряд пучка в реалистичных сценариях лазерно-плазменного ускорения. Данный обзор демонстрирует, что комбинированное использование моделей тестовых частиц и PIC-моделирования является крайне важным для исследования пределов теории усреднённого движения. Полученные результаты имеют прямую практическую значимость для оптимизации источников излучения и служат ориентиром для развития будущих теорий, учитывающих неадиабатические эффекты и эффекты, зависящие от топологии поля.

Ключевые слова: усреднённое движение, релятивистские пондеромоторные силы, лазерное излучение, гауссов пучок, волноводы, биение