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Computer research of deterministic and stochastic models "two competitors—two migration areas" taking into account the variability of parameters

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Abstract. The analysis of trajectory dynamics and the solution of optimization problems using computer methods are relevant areas of research in dynamic population-migration models. In this paper, four-dimensional dynamic models describing the processes of competition and migration in ecosystems are studied. Firstly, we consider a modification of the "two competitors—two migration areas" model, which takes into account uniform intraspecific and interspecific competition in two populations as well as non-uniform bidirectional migration in both populations. Secondly, we consider a modification of the "two competitors—two migration areas" model, in which intraspecific competition is uniform and interspecific competition and bidirectional migration are non-uniform. For these two types of models, the study is carried out taking into account the variability of parameters. The problems of searching for model parameters based on the implementation of two optimality criteria are solved. The first criterion of optimality is associated with the fulfillment of such a condition for the coexistence of populations, which in mathematical form is the integral maximization of the functions product characterizing the populations densities. The second criterion of optimality involves checking the assumption of the such a four-dimensional positive vector existence, which will be a state of equilibrium. The algorithms developed on the basis of the first and second optimality criteria using the differential evolution method result in optimal sets of parameters for the studied population-migration models. The obtained sets of parameters are used to find positive equilibrium states and analyze trajectory dynamics. Using the method of constructing self-consistent one-step models and an automated stochastization procedure, the transition to the stochastic case is performed. The structural description and the possibility of analyzing two types of population-migration stochastic models are provided by obtaining Fokker–Planck equations and Langevin equations with corresponding coefficients. Algorithms for generating trajectories of the Wiener process, multipoint distributions and modifications of the Runge–Kutta method are used. A series of computational experiments is carried out using a specialized software package whose capabilities allow for the construction and analysis of dynamic models of high dimension, taking into account the evaluation of the stochastics influence. The trajectory dynamics of two types of population-migration models are investigated, and a comparative analysis of the results is carried out both in the deterministic and stochastic cases. The results can be used in the modeling and optimization of dynamic models in natural science.

Key words and phrases: one-step processes, population dynamics models, stochastic differential equations, optimality criteria, differential evolution, stochastization, trajectory dynamics, computer modeling, software package

1. Introduction

The study of mathematical models of population systems began to develop actively in the twenties of the last century, thanks to the works of A. Lotka [1] and V. Volterra [2]. Currently, this direction includes the wide class study of models taking into account various interactions between populations

(See, for example, [3–6]). Significant progress in the study is achieved due to the analysis of the dynamic stability models of ecological systems using the theory of differential equations, numerical methods and optimization methods [3, 4, 7–9].

A four-dimensional model of two competing species with migration between two ranges, taking into account the asymmetry coefficient, is considered in [10]. It is shown that the choice of the migration area is carried out depending on the value of this coefficient. The coefficient of asymmetry affects which of the habitats species migrate to first. Two-species Lotka–Volterra competition patch model is studied in [11]. It's shown that in the long time, either the competition exclusion holds that one species becomes extinct, or the two species reach a coexistence equilibrium, and the outcome of the competition is determined by the strength of the inter-specific competition and the dispersal rates.

The transition to the non-deterministic case based on the design stochastic self-consistent models (DSSM) method allows us to identify new qualitative properties of models and carry out a comparative analysis [12–16] and in the other works. For various types of population models, the DSSM method is used in [12, 17, 18]. In [18], a formalized description of the four-dimensional model "two competitors—two migration areas" and its modifications are proposed, taking into account the case when population growth coefficients are different (non-uniform reproduction of species). Using the implementation of the evolutionary algorithm, a set of parameters is obtained that ensure the coexistence of populations in the conditions of competition between two species in the general area, taking into account the migration of these species. Stochastization of the model "two competitors—two migration areas" (under conditions of non-uniform species reproduction) is carried out on the basis of the method of constructing self-consistent stochastic models. The dynamics of trajectories for deterministic and stochastic cases is studied, a comparative analysis is performed.

This paper is a continuation of [18] and contains the construction and analysis of such modifications of the model "two competitors—two migration areas", which allows us to study the influence of non-uniform migration flows and the influence of non-uniform interspecific competition on the trajectory dynamics both in the deterministic case and in the stochastic case.

In section 2 of this paper, we consider the construction of two modifications of the model "two competitors—two migration areas" with bidirectional non-uniform migration (to two refuges), taking into account the uniformity and non-uniformness of the interspecific competition coefficients. In section 3, the search for model parameters is carried out using an evolutionary algorithm taking into account different optimality criteria. In section 4, a study of the obtained deterministic four-dimensional models is carried out, two-dimensional and three-dimensional projections of phase portraits are constructed. In section 5, the transition to stochastic models "two competitors—two migration areas" is made on the basis of the constructing self-consistent stochastic models method, the dynamics of trajectories in the stochastic case is studied. The results of computer experiments are presented and the interpretation of these results is given taking into account the comparison of stochastic and deterministic models. The developed in Python [19] software package [20] is used to study the models. Section 7 discusses the results.

2. Description of the model modifications "two competitors—two migration areas" taking into account non-uniform migration

Ref. [18] describes a general four-dimensional deterministic model, which takes into account the influence of interspecific and intraspecific competition in two populations with bidirectional migration of both populations, and the non-uniform growth of population reproduction.

We describe further such a model "two competitors—two migration areas", for which the growth of population reproduction, interspecific and intraspecific competition are uniform, and migration is non-uniform. The specified model is given by a system of equations of the form

$$\begin{aligned}
\dot{x}_1 &= ax_1 - px_1^2 - rx_1x_3 + \beta x_2 - \gamma x_1,\\
\dot{x}_2 &= ax_2 - px_2^2 + \gamma x_1 - \beta x_2,\\
\dot{x}_3 &= ax_3 - px_3^2 - rx_1x_3 + \varepsilon x_4 - \delta x_3,\\
\dot{x}_4 &= ax_4 - px_4^2 + \delta x_3 - \varepsilon x_4,
\end{aligned} \tag{1}$$

where the incoming values are explained in the table 1.

Table 1

Variables and parameters of model (1)

Variable, parameter	Explanation of the variable, parameter
x_1	the competing population density of the first species in the general area
x_2	population density of the first species in the first refuge
x_3	the competing population density of the second species in the general area
x_4	population density of the second species in the second refuge
a	coefficient of natural growth
r	coefficient of interspecific competition
p	coefficient of intraspecific competition
β	coefficient of migration of the first species from the general area to the first refuge
γ	coefficient of migration of the first species from the first refuge to the general area
δ	coefficient of migration of the second species from the general area to the second refuge
ε	coefficient of migration of the second species from the second refuge to the general area

Let’s move from model (1) to a model that takes into account the non-uniformness of the coefficient of interspecific competition r. We denote the estimated parameter of the competitive impact of the second type on the first by p_{13}. Accordingly, we denote the coefficient of the impact of the first type on the second by p_{31}. Thus we obtain a system of differential equations of the following form:

$$\begin{aligned} \dot{x}_1 &= ax_1 - px_1^2 - p_{13}x_1x_3 + \beta x_2 - \gamma x_1, \\ \dot{x}_2 &= ax_2 - px_2^2 + \gamma x_1 - \beta x_2, \\ \dot{x}_3 &= ax_3 - px_3^2 - p_{31}x_1x_3 + \varepsilon x_4 - \delta x_3, \\ \dot{x}_4 &= ax_4 - px_4^2 + \delta x_3 - \varepsilon x_4, \end{aligned} \tag{2}$$

where a is the coefficient of natural growth, β is the coefficient of the first species migration from the general area to the first refuge, γ is the coefficient of the first species migration from the first refuge to the general area, δ is the coefficient of the second species migration from the general area to the second refuge, ε is the coefficient of the second species migration from the second refuge to the general area, p_{ij} $(i \neq j)$ are coefficients of interspecific competition.

In the future, we will search for optimal sets of parameters that ensure the coexistence of species in the general area and the existence of species in refuges. In addition, we will carry out a comparative analysis of the trajectory dynamics of models (1) and (2), taking into account the considered initial conditions and optimal sets of parameters, as well as construct and study stochastic models corresponding to (1) and (2).

3. Search for model parameters using differential evolution

We consider optimization problems associated with finding parameter sets that guarantee the coexistence of all species in the general area and the existence of migratory species in refuges. The sets of parameters to be searched correspond to stationary modes of the system. We use such a numerical optimization method, which reduces to the implementation of the differential evolution algorithm [21–23].

In this paper we use two optimality criteria. The first optimality criterion is associated with maximizing the integral of the product of functions characterizing population densities. The specified maximization ensures the fulfillment of the selected condition for the coexistence of populations. It should be noted that an algorithm of differential evolution is used to solve problem of maximizing the integral, a program in Python is developed [18]. The second criterion of optimality involves checking the assumption of the existence of a positive equilibrium state of the four-dimensional model [24].

Based on the use of the first and second criteria, algorithms are developed using the differential evolution method. The algorithms are aimed at obtaining optimal sets of parameters of a four-dimensional system, which makes it possible to find approximate values of the components of the vector corresponding to a positive equilibrium state and to study the trajectory dynamics near equilibrium states. It is important to note that these two types of algorithms are based on intelligent search methods [25–27]. The implementation of these methods in the case under consideration makes it possible to find the numerical values of the components of the positive equilibrium state and identify such system parameters that provide the required properties of the equilibrium states.

In [18, 24], the implementation of algorithms for modifications of the system "two competitors—two migration areas" is considered without taking into account the non-uniform competition and migration. Here we consider more complex modifications with non-uniform bidirectional migration of two populations. Table 2 shows the characteristics of models (1) and (2) taking into account the first and second optimality criteria. The set of parameters obtained using the first optimality criterion is denoted by i-I, $i = 1, 2$. The set of parameters obtained using the second optimality criterion is denoted by i-II, $i = 1, 2$.

Table 2

Summary table of characteristics of models (1) and (2) under initial conditions $(x_1(0), x_2(0), x_3(0), x_4(0)) = (0.5, 0.5, 1, 7)$.

Equilibrium States	Parameters
Model (1) with set 1-I	
$x_1 = 58.04$, $x_2 = 90.75$, $x_3 = 56.42$, $x_4 = 91.04$	$a = 10.00$, $p = 0.10$, $r = 0.10$, $\beta = 5.67$, $\delta = 9.01$, $\gamma = 7.42$, $\varepsilon = 6.48$
Model (1) with set 1-II	
$x_1 = 58.27$, $x_2 = 90.79$, $x_3 = 56.09$, $x_4 = 91.17$	$a = 10.00$, $p = 0.10$, $r = 0.10$, $\beta = 7.32$, $\delta = 9.99$, $\gamma = 9.97$, $\varepsilon = 7.03$
Model (2) with set 2-I-a	
$x_1 = 57.78$, $x_2 = 90.46$, $x_3 = 56.89$, $x_4 = 90.70$	$a = 10.00$, $p = 0.10$, $p_{13} = 0.50$, $p_{31} = 0.70$, $\beta = 5.67$, $\delta = 9.01$, $\gamma = 7.42$, $\varepsilon = 6.48$
Model (2) with set 2-I-b	
$x_1 = 58.31$, $x_2 = 90.39$, $x_3 = 56.58$, $x_4 = 90.71$	$a = 10.00$, $p = 0.10$, $p_{13} = 0.70$, $p_{31} = 0.50$, $\beta = 5.67$, $\delta = 9.01$, $\gamma = 7.42$, $\varepsilon = 6.48$
Model (2) with set 2-II-a	
$x_1 = 71.34$, $x_2 = 98.80$, $x_3 = 6.06$, $x_4 = 43.60$	$a = 10.00$, $p = 0.10$, $p_{13} = 0.50$, $p_{31} = 0.70$, $\beta = 7.32$, $\delta = 9.99$, $\gamma = 9.97$, $\varepsilon = 7.03$
Model (2) with set 2-II-b	
$x_1 = 6.16$, $x_2 = 41.57$, $x_3 = 69.73$, $x_4 = 99.63$	$a = 10.00$, $p = 0.10$, $p_{13} = 0.70$, $p_{31} = 0.50$, $\beta = 7.32$, $\delta = 9.99$, $\gamma = 9.97$, $\varepsilon = 7.03$

Table 2 presents the sets of parameters we use in the process of computer experiments related to the analysis of the trajectory dynamics of models (1) and (2).

4. Results of computational experiments and comparative analysis of trajectory dynamics for deterministic models

This section presents the results of computational experiments for models (1) and (2), taking into account the selected initial conditions and the found parameters. Figure 1 shows the trajectories of the system (2) for the set of parameters 2-I-a in comparison with the corresponding trajectories of the system (1) with the resulting set of parameters 1-I.

Figure 2 shows the trajectories of system (2) for the set of parameters 2-I-b in comparison with the corresponding trajectories of system (1) with the resulting set of parameters 1-I. According to figures 1, 2, the densities of the corresponding populations for models (1) and (2) are kept at the same level for the selected time interval. Figure 3 shows the trajectories of system (1) with sets of parameters 1-I and 1-II.

Figure 4 shows the trajectories of system (2) for a set of parameters 2-I-a in comparison with the corresponding trajectories of system (2) with the resulting set of parameters 2-II-a.

Next, we will consider some projections of the phase portraits of model (2) on the plane and in space. The projection of the phase portrait on the plane (x_1, x_2) for the system (2) taking into account $x_3 = 56.89$, $x_4 = 90.70$ is shown in figure 5. The projection of the phase portrait in space (x_1, x_2, x_3) taking into account $x_4 = 90.70$ for the model (2) is shown in figure 6.

The presented analysis of models (1) and (2) is carried out with two sets of each model parameters and is aimed at studying two modes of species coexistence for a state of equilibrium determined by the method of differential evolution. In the future, we will study and analysis of the qualitative properties of the models, taking into account the introduction of stochastics.

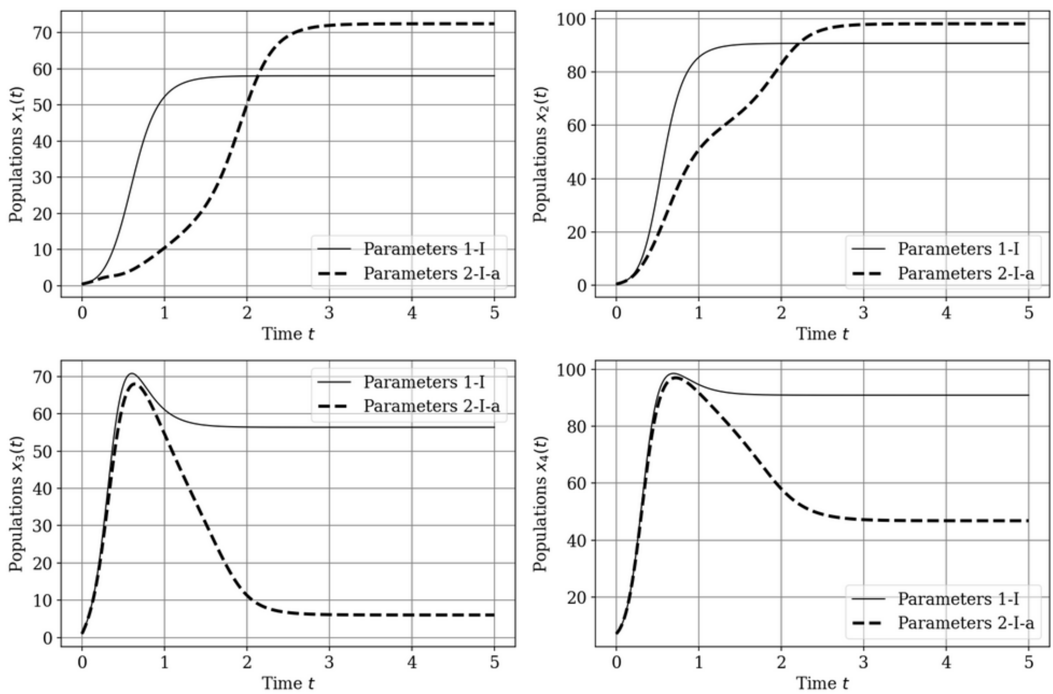

Figure 1. Trajectories of (1) and (2) at $(x_1(0), x_2(0), x_3(0), x_4(0)) = (0.5, 0.5, 1, 7)$ taking into account the parameters sets 1-I and 2-I-a

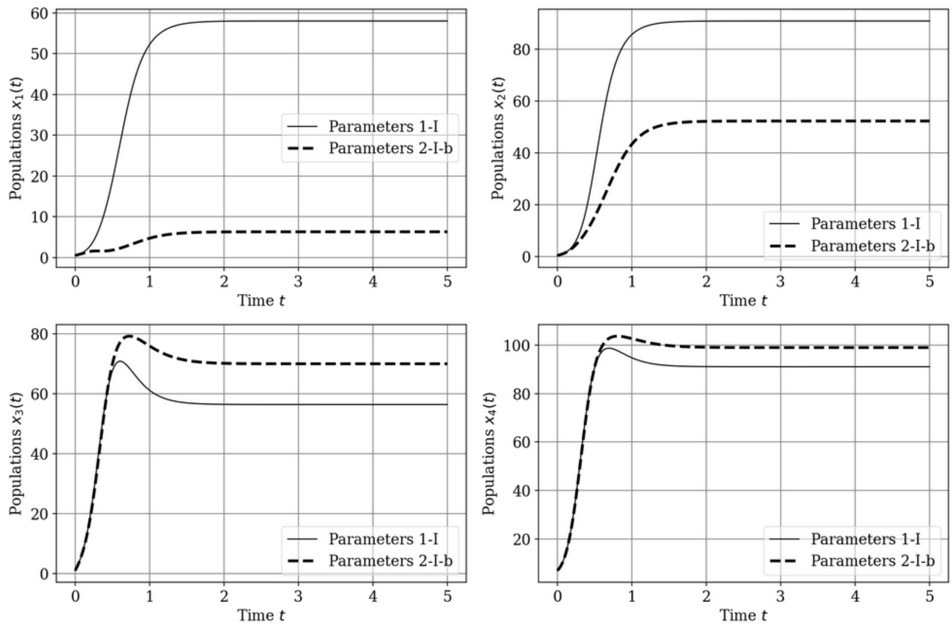

Figure 2. Trajectories of (1) and (2) at $(x_1(0), x_2(0), x_3(0), x_4(0)) = (0.5, 0.5, 1, 7)$ taking into account the parameters sets 1-I and 2-I-b

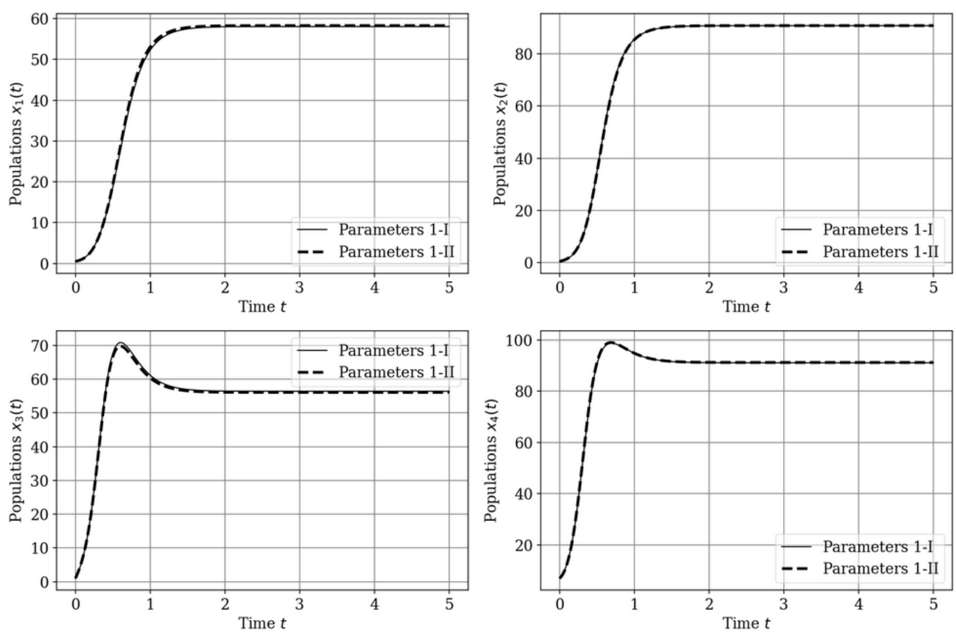

Figure 3. Trajectories of (1) and (2) at $(x_1(0), x_2(0), x_3(0), x_4(0)) = (0.5, 0.5, 1, 7)$ taking into account the parameters sets 1-I and 1-II

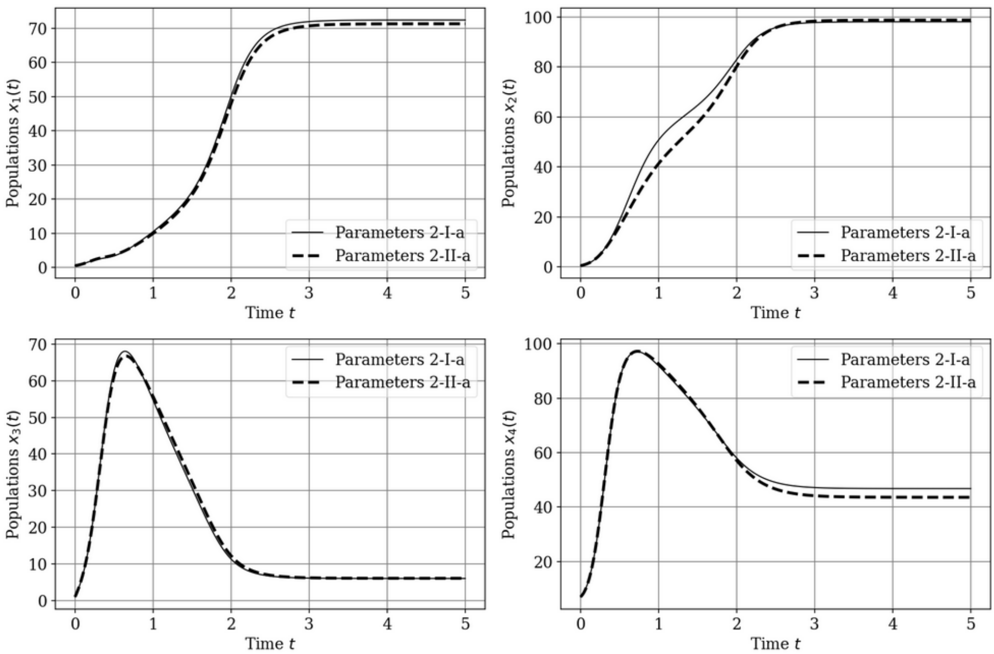

Figure 4. Trajectories of (2) at $(x_1(0), x_2(0), x_3(0), x_4(0)) = (0.5, 0.5, 1, 7)$ taking into account the parameters sets 2-I-a and 2-II-a

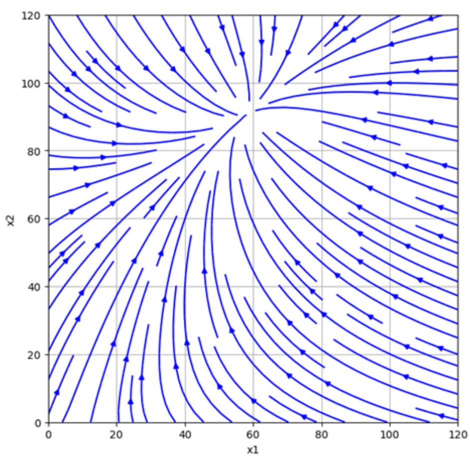

Figure 5. Projection of the phase portrait on the plane (x_1, x_2) for system (2) at $(x_1(0), x_2(0), x_3(0), x_4(0)) = (0.5, 0.5, 1, 7)$, $a = 10.00$, $p = 0.10$, $p_{13} = 0.10$, $p_{31} = 0.10$, $\beta = 2.56$, $\delta = 2.19$, $\gamma = 2.52$, $\varepsilon = 2.30$

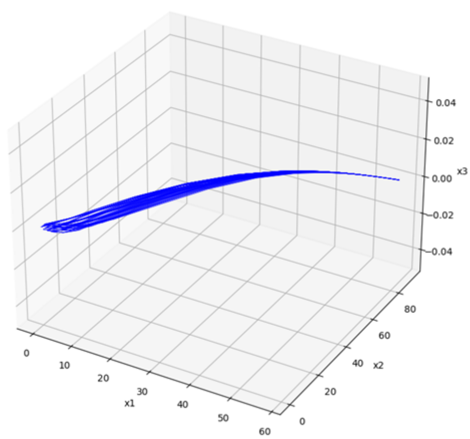

Figure 6. Projection of the phase portrait in the space (x_1, x_2, x_3) for system (2) at $(x_1(0), x_2(0), x_3(0), x_4(0)) = (0.5, 0.5, 1, 7)$, $a = 10.00$, $p = 0.10$, $p_{13} = 0.10$, $p_{31} = 0.10$, $\beta = 2.56$, $\delta = 2.19$, $\gamma = 2.52$, $\varepsilon = 2.30$

5. Modifications of the stochastic model “two competitors—two migration areas”

To construct stochastic models corresponding to models (1) and (2), it is proposed to use the DSSM method [12–16]. Using the software implementation of this method allows:

(*i*) to construct a stochastic model of a dynamic system taking into account the description of interactions;
(*ii*) to construct an appropriate deterministic model;
(*iii*) to obtain numerical solutions of ODEs and SDUs and graphical representations of solutions.

To describe a stochastic system, according to the DSSM method, it is enough to write the Fokker–Planck equation. The coefficients of the Fokker–Planck equation for models (1) and (2) are obtained using a software package and are presented respectively in figures 7, 8.

```
Ввод [19]: f = de.drift_vector(XX, k_plus_1, model_1.matr_N(), model_1.matr_M())
sp.Matrix(f)
```

Out[19]:

$$\begin{bmatrix} ax_1 - px_1^2 - rx_1x_3 - x_1\gamma + x_2\beta \\ ax_2 - px_2^2 + x_1\gamma - x_2\beta \\ ax_3 - px_3^2 - rx_1x_3 - x_3\delta + x_4\varepsilon \\ ax_4 - px_4^2 + x_3\delta - x_4\varepsilon \end{bmatrix}$$

```
Ввод [20]: g=de.diffusion_matrix(XX, k_plus_1, model_1.matr_N(), model_1.matr_M())
g
```

Out[20]:

$$\begin{bmatrix} ax_1 + px_1^2 + rx_1x_3 + x_1\gamma + x_2\beta & -x_1\gamma - x_2\beta & 0 & 0 \\ -x_1\gamma - x_2\beta & ax_2 + px_2^2 + x_1\gamma + x_2\beta & 0 & 0 \\ 0 & 0 & ax_3 + px_3^2 + rx_1x_3 + x_3\delta + x_4\varepsilon & -x_3\delta - x_4\varepsilon \\ 0 & 0 & -x_3\delta - x_4\varepsilon & ax_4 + px_4^2 + x_3\delta + x_4\varepsilon \end{bmatrix}$$

Figure 7. Coefficients of the Fokker–Planck equation for the model (1)

```
Ввод [65]: f = de.drift_vector(XX, k_plus_2, model_2.matr_N(), model_2.matr_M())
sp.Matrix(f)
```

Out[65]:

$$\begin{bmatrix} ax_1 - px_1^2 - p_{13}x_1x_3 - x_1\gamma + x_2\beta \\ ax_2 - px_2^2 + x_1\gamma - x_2\beta \\ ax_3 - px_3^2 - p_{31}x_1x_3 - x_3\delta + x_4\varepsilon \\ ax_4 - px_4^2 + x_3\delta - x_4\varepsilon \end{bmatrix}$$

```
Ввод [66]: g=de.diffusion_matrix(XX, k_plus_2, model_2.matr_N(), model_2.matr_M())
g
```

Out[66]:

$$\begin{bmatrix} ax_1 + px_1^2 + p_{13}x_1x_3 + x_1\gamma + x_2\beta & -x_1\gamma - x_2\beta & 0 & 0 \\ -x_1\gamma - x_2\beta & ax_2 + px_2^2 + x_1\gamma + x_2\beta & 0 & 0 \\ 0 & 0 & ax_3 + px_3^2 + p_{31}x_1x_3 + x_3\delta + x_4\varepsilon & -x_3\delta - x_4\varepsilon \\ 0 & 0 & -x_3\delta - x_4\varepsilon & ax_4 + px_4^2 + x_3\delta + x_4\varepsilon \end{bmatrix}$$

Figure 8. Coefficients of the Fokker–Planck equation for the model (2)

Graphs of the numerical solution for the deterministic and stochastic case taking into account the sets of parameters 1-I and 2-I-a from the table 1 are shown in the (figures 9, 10). For the numerical solution of ODE systems, we use a software implementation of standard four-order Runge–Kutta methods. To solve the corresponding SDE, we use a specially developed library a detailed description of which is contained in [13].

For sets of parameters 1-II, 2-I-a, 2-I-b, 2-II-a, numerical experiment shows that in the stochastic case the trajectories fluctuate near deterministic trajectories, similar to the one shown in figures 9, 10 taking into account sets of parameters 1-I and 2-II-b respectively. In the next section, we present a comparative analysis of the results obtained for models (1) and (2) with different sets of parameters.

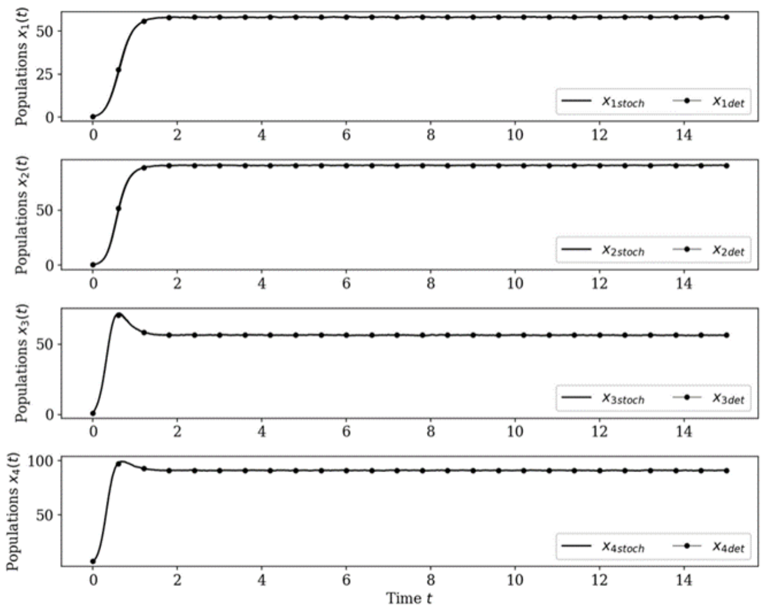

Figure 9. Trajectories of the system (1) and the corresponding stochastic system at $(x_1(0), x_2(0), x_3(0), x_4(0)) = (0.5, 0.5, 1, 7)$ taking into account parameters set 1-I: $a = 10.00$, $p = 0.10$, $r = 0.10$, $\beta = 5.67$, $\delta = 9.00$, $\gamma = 7.42$, $\varepsilon = 6.48$

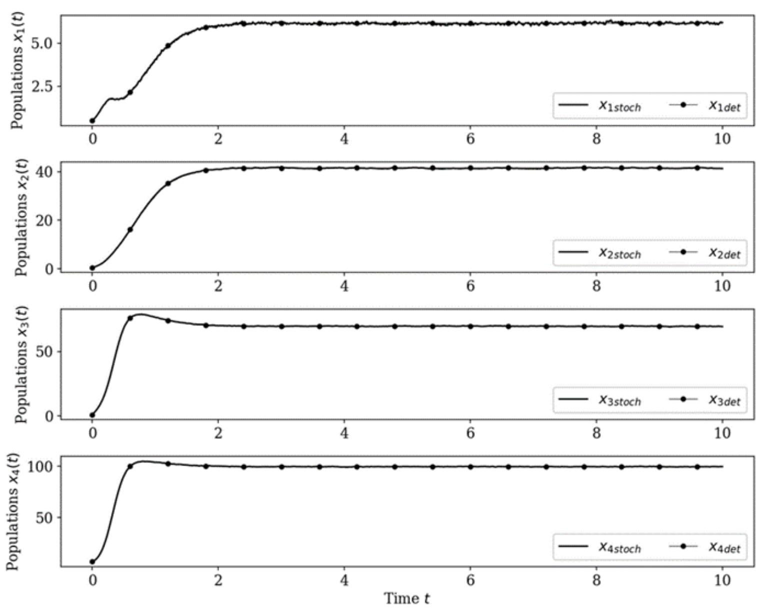

Figure 10. Trajectories of the system (2) and the corresponding stochastic system at $(x_1(0), x_2(0), x_3(0), x_4(0)) = (0.5, 0.5, 1, 7)$ taking into account parameters set 2-II-b: $a = 10.00$, $p = 0.10$, $p_{13} = 0.70$, $p_{31} = 0.50$, $\beta = 7.32$, $\delta = 9.99$, $\gamma = 9.97$, $\varepsilon = 7.03$

6. Discussion of the results

Let us first consider the results of a computational experiment for the case of parameters 1-I and parameters 2-I-a at $p_{13} < p_{31}$. A comparative analysis of the trajectories' behavior for models (1) and (2) according to figure 1 shows that:

1) there is a co-existence of all species corresponding to the stationary regime in the general area, as well as the existence of migratory species in refuges;
2) the non-uniformness of the interspecific competition coefficients affects the population density x_3 in the general area, while the population density x_3 of the model 2 decreases compared to the population density x_3 of the model 1.

Let us further consider the results for the case of parameters 1-I and 2-I-b at $p_{13} > p_{31}$. A comparative analysis of the solution trajectories of models (1) and (2) presented in figure 2 shows that:

1) there is a coexistence of all species in the general area and the existence of migratory species in refuges;
2) the population density x_1 of the model (2) decreases compared to the population density x_1 of the model (1).

Figure 3 shows the trajectories of the model (1) solutions taking into account parameters 1-I and 1-II and two optimization methods. Figure 4 shows the trajectories of solutions for model (2) taking into account the parameters 2-I-a and 2-II-a and two optimization methods. Comparison of trajectories allows us to conclude that the choice of the first or second optimality criterion does not significantly affect the population density both in the general area and in refuges. Computational experiments show the consistency of the two selected optimality criteria (the trajectories have a similar character).

Taking into account the transition to the stochastic case using the Fokker–Planck equations (figures 7, 8), computer experiments are carried out to identify trajectory dynamics. The results are presented in figures 9, 10. Computer experiments show that the introduction of stochastics has no effect on the behavior of systems described by the systems of equations (1) and (2). As in the deterministic case, solutions of stochastic differential equations reach the stationary mode. In Fig. 9, where the range from 0 to 10 corresponds to the change in population density x_1 in the general area, the fluctuating nature of the trajectory dynamics in the stochastic case is visually observed. In order to observe such a character for the other variables, it is possible to choose an enlarged scale of the drawing. For example, a fragment of the deterministic and stochastic trajectories of model (2) for the population density x_4 in the migration area, taking into account the enlarged scale, is shown in figure 11.

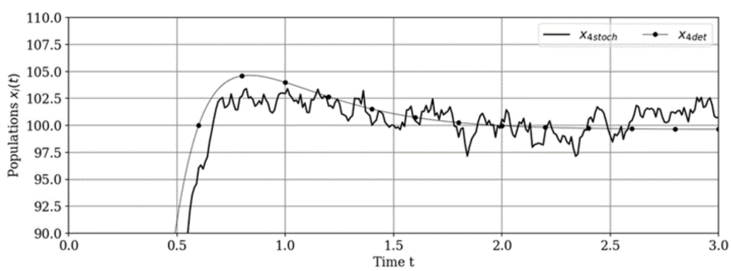

Figure 11. Fragment of deterministic and stochastic trajectories for model (2) with parameters set 2-II-b

Stochastic modeling revealed the similar nature of the trajectories of models (1) and (2) with the considered sets of parameters, and the initial set of parameters is obtained taking into account the conditions guaranteeing the coexistence of two populations in the general area and the positive abundance of each species in refuges. Thus, with the considered sets of parameters of models (1) and (2), it is sufficient to carry out a computational experiment only in the deterministic case to study the trajectory dynamics.

7. Conclusion

The paper presents a computer study of deterministic and stochastic models "two competitors—two migration areas". Computational experiments are based on the use of evolutionary algorithms and optimization methods for finding parameters taking into account their variability. The optimization problem is solved using the differential evolution method, which made it possible to find optimal parameters for population-migration models. Approximate stable equilibrium states corresponding to the obtained parameters are found for these models. The assessment of the changes influence in the coefficients of interspecific competition on the trajectory dynamics of four-dimensional models with non-uniform migration flows in the deterministic case is considered.

In this paper, the transition to stochastic models of population dynamics, taking into account competition and migration, based on the DSSM method, is carried out. The stochastization algorithm used made it possible to analyze the trajectory dynamics of stochastic models in comparison with deterministic analogues. The analysis demonstrates a negligible effect of introducing stochastics into deterministic models (1) and (2) taking into account the parameters and allows us to conclude that the computational experiment is sufficient only in the deterministic case.

As directions for further research, we can indicate the construction of population-migration models of the form "n competitors—n migration areas", where $n > 2$, as well as the study of multidimensional stochastic models taking into account the effects of additive random perturbations of the equations right-hand sides.

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Компьютерное исследование детерминированных и стохастических моделей «два конкурента—два ареала миграции» с учетом вариативности параметров

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Аннотация. Анализ траекторной динамики и решение задач оптимизации с применением компьютерных методов относится к актуальным направлениям исследования динамических популяционно-миграционных моделей. В настоящей работе изучаются четырехмерные динамические модели, описывающие процессы конкуренции и миграции в экосистемах. Во-первых, мы рассматриваем модификацию модели «два конкурента — два ареала миграции», в которой учитывается равномерная внутривидовая и межвидовая конкуренция в двух популяциях, а также неравномерная двунаправленная миграция обеих популяций. Во-вторых, мы рассматриваем модификацию модели «два конкурента — два ареала миграции», в которой внутривидовая конкуренция является равномерной, а межвидовая конкуренция и двунаправленная миграция являются неравномерными. Для указанных двух типов моделей исследование проводится с учетом вариативности параметров. Решены задачи поиска модельных параметров на основе реализации двух критериев оптимальности. Первый критерий оптимальности связан с выполнением такого условия сосуществования популяций, которое в математической форме представляет собой максимизацию интеграла от произведения функций, характеризующих плотности популяций. Второй критерий оптимальности включает в себя проверку предположения о существовании такого четырехмерного положительного вектора, который будет являться состоянием равновесия. Результатом работы алгоритмов, разработанных на основе первого и второго критериев оптимальности с применением метода дифференциальной эволюции, являются оптимальные наборы параметров изучаемых популяционно-миграционных моделей. Полученные наборы параметров используются для нахождения положительных состояний равновесия и для анализа траекторной динамики. С помощью метода построения самосогласованных одношаговых моделей и автоматизированной процедуры стохастизации выполнен переход к стохастическому случаю. Структурное описание и возможность анализа двух типов популяционно-миграционных стохастических моделей обеспечиваются получением уравнений Фоккера–Планка и уравнений в форме Ланжевена с соответствующими коэффициентами. Использованы алгоритмы генерирования траекторий винеровского процесса и многоточечных распределений и модификации метода Рунге–Кутты. Проведена серия вычислительных экспериментов с применением специализированного программного комплекса, возможности которого позволяют выполнять построение и анализ динамических моделей высокой размерности с учетом оценки влияния стохастики. Исследована траекторная динамика двух типов популяционно-миграционных моделей и выполнен сравнительный анализ результатов как в детерминированном, так и в стохастическом случае. Результаты могут найти применение в задачах моделирования и оптимизации динамических моделей естествознания.

Ключевые слова: одношаговые процессы, модели динамики популяций, стохастические дифференциальные уравнения, критерии оптимальности, дифференциальная эволюция, стохастизация, траекторная динамика, компьютерное моделирование, программный комплекс