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Investigation of adiabatic waveguide modes model for smoothly irregular integrated optical waveguides

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Abstract. The model of adiabatic waveguide modes (AWMs) in a smoothly irregular integrated optical waveguide is studied. The model explicitly takes into account the dependence on the rapidly varying transverse coordinate and on the slowly varying horizontal coordinates. Equations are formulated for the strengths of the AWM fields in the approximations of zero and first order of smallness. The contributions of the first order of smallness introduce depolarization and complex values characteristic of leaky modes into the expressions of the AWM electromagnetic fields. A stable method is proposed for calculating the vertical distribution of the electromagnetic field of guided modes in regular multilayer waveguides, including those with a variable number of layers. A stable method for solving a nonlinear equation in partial derivatives of the first order (dispersion equation) for the thickness profile of a smoothly irregular integrated optical waveguide in models of adiabatic waveguide modes of zero and first orders of smallness is described. Stable regularized methods for calculating the AWM field strengths depending on vertical and horizontal coordinates are described. Within the framework of the listed matrix models, the same methods and algorithms for the approximate solution of problems arising in these models are used. Verification of approximate solutions of models of adiabatic waveguide modes of the first and zero orders is proposed; we compare them with the results obtained by other authors in the study of more crude models.

Key words and phrases: smoothly irregular thin–film dielectric waveguides, adiabatic waveguide modes, regularized methods for calculating field strengths

1. Introduction

The adiabatic waveguide propagation of optical radiation was previously described in optical fibers using the method of cross sections in the papers by B. Z. Katsenelenbaum [1], V. V. Shevchenko [2], M. V. Fedoruk [3], and in integrated optical waveguides using the method of adiabatic waveguide modes — in the papers by A. A. Egorov, L. A. Sevastyanov and their coauthors [4]–[6]. In the papers by A. L. Sevastyanov [7], [8], the model of adiabatic waveguide modes was substantiated.

It should be noted that in the last decade there has been an interest in the adiabatic waveguide propagation of electromagnetic radiation for the study of coherent quantum effects in atomic, molecular or condensed matter systems. These effects are difficult to investigate because of dephasing effects or fast temporal dynamics. Optical Bloch oscillations [9], quantum-mechanical analogy of dynamic mode stabilization and radiation loss suppression [10], quantum enhancement and suppression of tunneling in directional optical couplers [11], [12], as well as Landau–Zener tunneling in coupled waveguides [13] can serve as optical models of coherent quantum effects. An interesting example is the three-level system with stimulated Raman adiabatic passage (STIRAP), which vividly illustrates counterintuitive quantum effects [14]–[19].

2. Model of adiabatic waveguide modes in a multilayer waveguide

Let us specify the class of integrated optical waveguides to be considered and the electromagnetic radiation propagating through them.

1. Electromagnetic radiation is polarized, monochromatic with a given wavelength $\lambda \in [\,380; 780\,]$, nm.
2. The thickness of the guiding layer of the base thin-film waveguide is comparable to the wavelength of the propagating monochromatic electromagnetic radiation $d \sim \lambda$.
3. The surface of the additional guiding layer $(x = h(y, z))$ satisfies the following restrictions: $\left|\frac{\partial h}{\partial y}, \frac{\partial h}{\partial z}\right| \ll \frac{h k_0}{2\pi}$, $\left|\frac{\Delta\varphi}{\nabla\varphi}\right| \ll \frac{k_0}{2\pi}$.
4. The integrated optical waveguide is a material medium consisting of dielectric subregions, which together fill the entire three-dimensional space.
5. The permittivities of the subregions are different and real-valued, and the permeability is everywhere equal to that of vacuum.
6. There are no external currents and charges. Therefore, in the absence of foreign currents and charges, the induced currents and charges are zero.
7. The Cartesian coordinate system is introduced as follows: the interfaces between the dielectric media of the basic three-layer waveguide are parallel to the yOz plane. The subdomains of the space corresponding to the cover and substrate layers are infinite; the additional guiding layers are asymptotically parallel to the yOz plane. Therefore, $\varepsilon = \varepsilon(x)$.

In Cartesian coordinates associated with the geometry of the substrate (or a three-layer planar dielectric waveguide underlying a smoothly irregular integrated optical waveguide), with the introduced restrictions taken into account, the Maxwell equations have the form

$$\begin{aligned}
&\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = \frac{\varepsilon}{c}\frac{\partial E_x}{\partial t}, \quad \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -\frac{\mu}{c}\frac{\partial H_x}{\partial t},\\
&\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = \frac{\varepsilon}{c}\frac{\partial E_y}{\partial t}, \quad \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -\frac{\mu}{c}\frac{\partial H_y}{\partial t},\\
&\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = \frac{\varepsilon}{c}\frac{\partial E_z}{\partial t}, \quad \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -\frac{\mu}{c}\frac{\partial H_z}{\partial t}.
\end{aligned} \tag{1}$$

Note that variable x is fast, and variables y, z are slow with respect to the small dimensioned parameter $1/\omega$. The approximate solutions to the Maxwell equations (1) within the asymptotic method [20], [21], with the separation of slow and fast variables taken into account are sought in the form

$$\vec{E}(x,y,z,t) = \sum_{s=0}^{\infty} \frac{\vec{E}_s(x;y,z)}{(-i\omega)^{\gamma+s}} \exp\{i\omega t - ik_0\varphi(y,z)\}, \tag{2}$$

$$\vec{H}(x,y,z,t) = \sum_{s=0}^{\infty} \frac{\vec{H}_s(x;y,z)}{(-i\omega)^{\gamma+s}} \exp\{i\omega t - ik_0\varphi(y,z)\}. \tag{3}$$

Keeping in the solution (2), (3) the terms of the zero and first order of smallness leads to the model of adiabatic waveguide modes (AWMs) that describes the guided-wave propagation of a polarized optical radiation through irregular segments of smoothly irregular (multilayer) optical waveguides. In regular parts, the adiabatic waveguide modes become normal modes of a regular planar optical waveguide.

In the notation $\vec{E}_s(x;y,z)$, $\vec{H}_s(x;y,z)$, the separation by a semicolon means the following assumptions:

$$\left\|\frac{\partial \vec{E}_s(x;y,z)}{\partial y}\right\|, \left\|\frac{\partial \vec{E}_s(x;y,z)}{\partial z}\right\| \sim \frac{1}{\omega}\left\|\frac{\partial \vec{E}_s(x;y,z)}{\partial x}\right\| \tag{4}$$

and

$$\left\|\frac{\partial \vec{H}_s(x;y,z)}{\partial y}\right\|, \left\|\frac{\partial \vec{H}_s(x;y,z)}{\partial z}\right\| \sim \frac{1}{\omega}\left\|\frac{\partial \vec{H}_s(x;y,z)}{\partial x}\right\| \tag{5}$$

for each s, where $\|\|$ is the Hilbert norm of functions of x, and ω is the circular frequency of the propagating monochromatic electromagnetic radiation.

2.1. AWM model equations in the zero-order approximation

In Ref. [7] it was shown that the zero-order approximation (within the asymptotic approach) of the waveguide solution to the Maxwell equations is given by the following relations:

$$\left\{\begin{array}{c} \vec{E}(x,y,z,t) \\ \vec{H}(x,y,z,t) \end{array}\right\} = \left\{\begin{array}{c} \vec{E}_0(x;y,z) \\ \vec{H}_0(x;y,z) \end{array}\right\} \exp\{i\omega t - i\varphi(y,z)\}, \tag{6}$$

with

$$\varepsilon\frac{\partial E_0^y}{\partial x} = -ik_0\left(\frac{\partial\varphi}{\partial y}\right)\left(\frac{\partial\varphi}{\partial z}\right)H_0^y - ik_0\left(\varepsilon\mu - \left(\frac{\partial\varphi}{\partial y}\right)^2\right)H_0^z, \tag{7}$$

$$\varepsilon\frac{\partial E_0^z}{\partial x} = ik_0\left(\varepsilon\mu - \left(\frac{\partial\varphi}{\partial z}\right)^2\right)H_0^y + ik_0\left(\frac{\partial\varphi}{\partial z}\right)\left(\frac{\partial\varphi}{\partial y}\right)H_0^z, \tag{8}$$

$$\mu\frac{\partial H_0^y}{\partial x} = ik_0\left(\frac{\partial\varphi}{\partial y}\right)\left(\frac{\partial\varphi}{\partial z}\right)E_0^y + ik_0\left(\varepsilon\mu - \left(\frac{\partial\varphi}{\partial y}\right)^2\right)E_0^z, \tag{9}$$

$$\mu\frac{\partial H_0^z}{\partial x} = -ik_0\left(\varepsilon\mu - \left(\frac{\partial\varphi}{\partial z}\right)^2\right)E_0^y - ik_0\left(\frac{\partial\varphi}{\partial z}\right)\left(\frac{\partial\varphi}{\partial y}\right)E_0^z \tag{10}$$

and

$$E_0^x = -\frac{\partial\varphi}{\partial y}\frac{1}{\varepsilon}H_0^z + \frac{\partial\varphi}{\partial z}\frac{1}{\varepsilon}H_0^y, \tag{11}$$

$$H_0^x = \frac{\partial\varphi}{\partial y}\frac{1}{\mu}E_0^z - \frac{\partial\varphi}{\partial z}\frac{1}{\mu}E_0^y, \tag{12}$$

as well as

$$\left(\frac{\partial\varphi}{\partial y}(y,z)\right)^2 + \left(\frac{\partial\varphi}{\partial z}(y,z)\right)^2 = n_{\text{eff}}^2(y,z). \tag{13}$$

For a thin-film multilayer waveguide consisting of optically homogeneous layers, the conditions for matching the electromagnetic field at the interfaces between the media are valid, namely

$$\vec{n}\times\vec{E}^- + \vec{n}\times\vec{E}^+ = 0, \tag{14}$$

$$\vec{n}\times\vec{H}^- + \vec{n}\times\vec{H}^+ = 0. \tag{15}$$

In addition, the asymptotic conditions

$$E_y^0,\ E_z^0,\ H_y^0,\ H_z^0 \xrightarrow[x\to\pm\infty]{} 0 \tag{16}$$

are fulfilled.

The system of Eqs. (7)–(10), (16) for any fixed (y,z) defines the problem of finding eigenvalues $\left(\vec{\nabla}\varphi\right)_j^2(y,z)$ and eigenfunctions $\left(E_y^j, E_z^j, H_y^j, H_z^j\right)^T(y,z)$, normalized to unity:

$$\int\limits_{-\infty}^{\infty}\left|E_y^j\right|^2 dx = 1, \quad \int\limits_{-\infty}^{\infty}\left|H_y^j\right|^2 dx = 1. \tag{17}$$

2.2. AWM model equations in the first approximation

We continue to apply the approach based on the small parameter expansion and arrive at the system of equations in the first approximation of the method:

$$-\frac{\partial E_1^z}{\partial x} + \frac{ik_0}{\varepsilon}\frac{\partial\varphi}{\partial z}\left(\frac{\partial\varphi}{\partial y}H_1^z - \frac{\partial\varphi}{\partial z}H_1^y\right) + ik_0\mu H_1^y = \\ = i\omega\frac{\partial E_0^x}{\partial z} + \frac{i\omega}{\varepsilon}\frac{\partial\varphi}{\partial z}\left(\frac{\partial H_0^y}{\partial z} - \frac{\partial H_0^z}{\partial y}\right), \tag{18}$$

$$\frac{\partial E_1^y}{\partial x} - \frac{ik_0}{\varepsilon}\frac{\partial \varphi}{\partial y}\left(\frac{\partial \varphi}{\partial y}H_1^z - \frac{\partial \varphi}{\partial z}H_1^y\right) + ik_0\mu H_1^z = \\ = -i\omega\frac{\partial E_0^x}{\partial y} - \frac{i\omega}{\varepsilon}\frac{\partial \varphi}{\partial y}\left(\frac{\partial H_0^y}{\partial z} - \frac{\partial H_0^z}{\partial y}\right), \quad (19)$$

$$-\frac{\partial H_1^z}{\partial x} + \frac{ik_0}{\mu}\frac{\partial \varphi}{\partial z}\left(\frac{\partial \varphi}{\partial z}E_1^y - \frac{\partial \varphi}{\partial y}E_1^z\right) - ik_0\varepsilon E_1^y = \\ = i\omega\frac{\partial H_0^x}{\partial z} - \frac{i\omega}{\mu}\frac{\partial \varphi}{\partial z}\left(\frac{\partial E_0^y}{\partial z} - \frac{\partial E_0^z}{\partial y}\right), \quad (20)$$

$$\frac{\partial H_1^y}{\partial x} - \frac{ik_0}{\mu}\frac{\partial \varphi}{\partial y}\left(\frac{\partial \varphi}{\partial z}E_1^y - \frac{\partial \varphi}{\partial y}E_1^z\right) - ik_0\varepsilon E_1^z = \\ = -i\omega\frac{\partial H_0^x}{\partial y} + \frac{i\omega}{\mu}\frac{\partial \varphi}{\partial y}\left(\frac{\partial E_0^y}{\partial z} - \frac{\partial E_0^z}{\partial y}\right), \quad (21)$$

$$E_1^x + \frac{1}{\varepsilon}\left(\frac{\partial \varphi}{\partial y}H_1^z - \frac{\partial \varphi}{\partial z}H_1^y\right) = \frac{1}{\varepsilon}\frac{\omega}{k_0}\left(\frac{\partial H_0^y}{\partial z} - \frac{\partial H_0^z}{\partial y}\right), \quad (22)$$

$$H_1^x + \frac{1}{\mu}\left(\frac{\partial \varphi}{\partial z}E_1^y - \frac{\partial \varphi}{\partial y}E_1^z\right) = -\frac{1}{\mu}\frac{\omega}{k_0}\left(\frac{\partial E_0^y}{\partial z} - \frac{\partial E_0^z}{\partial y}\right). \quad (23)$$

The system of zero order equations (7)–(12) coincides with the system of equations (18)–(23), if in the latter we put zero into the right-hand sides (the contributions with zero-order quantities).

Substituting the solutions of system (7)–(12) into the right-hand sides of equations (18)–(23) leads to the following form of expressions for electromagnetic fields in the first (plus zero) approximation

$$\vec{E}(x;y,z) = \vec{E}_0(x;y,z) + \frac{i}{\omega}\vec{E}_1(x;y,z),$$

$$\vec{H}(x;y,z) = \vec{H}_0(x;y,z) + \frac{i}{\omega}\vec{H}_1(x;y,z).$$

These fields are necessarily complex-valued. Thus, the contributions of the first order of smallness introduce into the expressions for the AWM electromagnetic fields the characteristic features of leaky modes.

3. Implementation of numerical experiment

In Ref. [22], an hierarchy of mathematical models for the adiabatic waveguide propagation of optical radiation in integrated optical waveguides was proposed. The AWM model consists in representing the electromagnetic field in the form (6). The dependences of the field strengths on the fast variable have the form (7)–(12) in the zero approximation and (18)–(23) in the first approximation. Of course, the rigging conditions (13)–(17) of the AWM mathematical model are assumed to be fulfilled.

3.1. Algorithm for calculating the AWM electromagnetic field

A. Stage 1: reconstructing the dependence of the AWM electromagnetic field on the fast variable at fixed values of the slow variables

1. Solve the system (7)–(12) for $\vec{E}^0$, $\vec{H}^0$ describing the AWM model in the zero order of smallness in $1/\omega$, rigged with (6), (18)–(23) using the method, asymptotic with respect to δ, to obtain systems for contributions of different orders of smallness with respect to δ.
2. Solve the system (13)–(17) for $\vec{E}^1$, $\vec{H}^1$ describing the AWM model in the first order of smallness in $1/\omega$, rigged with (6), (18)–(23) using the method, asymptotic with respect to δ, to obtain systems for contributions of different orders of smallness with respect to δ.

B. Stage 2: reconstructing the dependence of the AWM electromagnetic field on the slow variables.

In Ref. [7] it is shown how the general solutions of the system of ODEs (7)–(12) and (13)–(17), represented in the form of expansion in the fundamental system of solutions with indefinite coefficients $\left(\vec{A}, \vec{B}\right)^T$, can be reduced to a homogeneous system of linear algebraic equations (SLAE) with respect to these indefinite coefficients using the conditions (14)–(16).

3. Implement stable methods of approximate solutions of the homogeneous SLAE

$$\hat{M}^0\left[(z,y), h(z,y), \varphi(z,y), \vec{\nabla}\varphi(z,y)\right]\left(\vec{A}^0(z,y), \vec{B}^0(z,y)\right)^T = \left(\vec{0}, \vec{0}\right)^T, \quad (24)$$

satisfying the conditions

$$\det\left\{\hat{M}^0\right\}\left[(z,y), h(z,y), \varphi(z,y), \vec{\nabla}\varphi(z,y)\right] = 0. \quad (25)$$

4. Implement stable methods of approximate solutions of the homogeneous SLAE

$$\hat{M}^1\left[(z,y), h(z,y), \varphi(z,y), \vec{\nabla}\varphi(z,y)\right]\left(\vec{A}^1(z,y), \vec{B}^1(z,y)\right)^T = \left(\vec{0}, \vec{0}\right)^T \quad (26)$$

satisfying the conditions

$$\det\left\{\hat{M}^1\right\}\left[(z,y), h(z,y), \varphi(z,y), \vec{\nabla}\varphi(z,y)\right] = 0. \quad (27)$$

In both cases, the solution for the field strengths depending on the fast variable x for a fixed value of the slow variables y, z makes it possible, using the rigging (6), (18)–(23), to find the dependence of the AWM electromagnetic field for all values of the slow variables (see, e.g., Ref. [8]).

Homogeneous systems of linear algebraic equations (24) and (26) are uniquely solvable under conditions (25) and (27). In both cases, these equations with respect to the derivative $\vec{\nabla}\varphi(z,y)$ are partial differential equations of the form

$$F^0\left(\vec{\nabla}\varphi(z,y);\, h(z,y), \vec{\nabla}h(z,y)\right) = 0 \tag{28}$$

and

$$F^1\left(\vec{\nabla}\varphi(z,y);\, h(z,y), \vec{\nabla}h(z,y)\right) = 0. \tag{29}$$

5. Solve Eqs. (28) and (29) numeric-symbolically using the Cauchy method (see, e.g. [23], [24]).
6. For each $\vec{\nabla}\varphi(z,y)$ calculate $\left(\vec{A}^0\left(z,y,\vec{\nabla}\varphi(z,y)\right), \vec{B}^0\left(z,y,\vec{\nabla}\varphi(z,y)\right)\right)^T$ using the Tikhonov regularization method, which consists in minimizing the Nelder–Mead functional:

$$F^0(\beta) = \left\|\hat{M}^0\left[(z,y), h(z,y), \varphi(z,y), \vec{\nabla}\varphi(z,y)\right]\left(\vec{A}^0(z,y), \vec{B}^0(z,y)\right)^T\right\|^2 +$$

$$+\alpha\left\|\left(\left(\vec{A}^0(z,y) - \vec{A}_0\left(z-\Delta z, y-\Delta y\right)\right), \left(\vec{B}^0(z,y) - \vec{B}_0\left(z-\Delta z, y-\Delta y\right)\right)\right)^T\right\|^2.$$

C. Stage 3: verifying the obtained numerical results and AWM models of the first and zero orders of smallness.

The validation of the asymptotic method of constructing AWM models is carried out by comparing solutions $\vec{E}^1$, $\vec{H}^1$ and $\vec{E}^0$, $\vec{H}^0$.

The formulation of the third condition from the set of conditions 1–7 implicitly implies the presence of the second small parameter $\delta \equiv \max\limits_{y,z} \dfrac{|\Delta\varphi|}{k_0\left|\vec{\nabla}\varphi\right|} \ll 1$ (see the beginning of the first section).

To verify the obtained approximate solutions of the zero-order model of adiabatic modes, we compare them with the results obtained by other authors using more crude models:

— matrix model of adiabatic modes in the approximation of horizontal boundary conditions (a stepped set of plates for a Luneburg thin-film generalized waveguide lens)

Such configurations are impossible in optical fibers and can be implemented in the case of adiabatic waveguide propagation of a nonparallel (converging or diverging) 2D beam of rays, normal to a nonplanar (2D) wave front.

— matrix model of comparison waveguides (passing to the horizontal boundary conditions + replacement $\beta_y \to 0$, $\beta_z \to \beta$).

Thus, three levels of making the AWM model cruder were used.

4. Discussion and conclusion

In the paper, we consider three levels of making the AWM model cruder:

— replacing the first-order AWM model with the zero-order one;
— replacing the tangential boundary conditions with the horizontal ones — the matrix model still having no name;
— replacing the tangential boundary conditions with the horizontal ones and $\beta_y \to 0$, $\beta_z \to \beta$ — the matrix model of comparison waveguides.

Two latter approximations have been used by other authors.

Within the listed matrix models, similar methods and algorithms are used for the approximate solution of problems, arising in the models. The method of studying the matrix model of adiabatic waveguide modes in the zero and first approximation of a smoothly irregular multilayer integrated optical waveguide is proposed for the first time. It allows to grade the crudeness of the approximate models used by other authors and approximate solutions in the adiabatic mode models of different order of smallness.

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Исследование модели адиабатических волноводных мод для плавно-нерегулярных интегрально-оптических волноводов

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Аннотация. Проведено исследование модели адиабатических волноводных мод плавно-нерегулярного интегрально-оптического волновода. В модели явно учтена зависимость от быстропеременной поперечной координаты и от медленно-переменных горизонтальных координат. Сформулированы уравнения для напряженностей полей АВМ в приближениях нулевого и первого порядка малости. Вклады первого порядка малости вносят в выражения электромагнитных полей АВМ деполяризацию и комлекснозначность, т.е. характерные черты вытекающих мод. Предложен устойчивый метод вычисления вертикального распределения электромагнитного поля направляемых мод регулярных многослойных волноводов, в том числе с переменным числом слоев. Описан устойчивый метод решения нелинейного уравнения в частных производных первого порядка (дисперсионного уравнения) для профиля толщины плавно-нерегулярного интегрально-оптического волновода в моделях адиабатических волноводных мод нулевого и первого порядков малости. Описаны устойчивые регуляризованные методы вычисления напряженностей полей АВМ в зависимости от вертикальных и горизонтальных координат. В рамках перечисленных матричных моделей используются одинаковые методы и алгоритмы приближенного решения задач, возникающих в этих моделях. Предложена верификация приближенных решений моделей адиабатических волноводных мод первого и нулевого порядков; проведено сравнение их с результатами других авторов, полученных при исследовании более грубых моделей.

Ключевые слова: модели квантовых измерений, возмущение дискретного спектра, комплексные собственные значения, пучки операторов