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Mathematical model of cavitation under the influence of a single stretching pulse

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This paper describes the created mathematical model that allows you to explore the dynamics of cavitation bubbles under the influence of a single negative pressure pulse. The time dependence and coordinates of the parameters of the carrier phase, the temperature and pressure of the vapor phase, the concentration and size of the bubbles are determined numerically. It is concluded that the model created gives a good agreement between the calculated and experimental data.

Key words and phrases: numerical simulation, cavitation, mathematical model of cavitation

1. Introduction

Cavitation in a fluid is a process in which a dynamic decrease in pressure inside a fluid occurs at a constant temperature, as a result of which vapor and vapor-gas bubbles increase in the fluid caused by the evaporation of fluid into these bubbles. Bubbles (caverns) are formed in those places where the pressure in the liquid becomes below a certain critical value [1]. In a real fluid, it is approximately equal to the saturated vapor pressure at a given temperature [2].

Some features of cavitation

— Cavitation is peculiar only to liquids and does not occur under normal conditions, either in solids or in gases.

— Cavitation results from a decrease in pressure in a liquid, which means that it can be controlled by adjusting the pressure. If the pressure is maintained below a certain level for a long time, cavitation occurs.
— Cavitation is associated with the disappearance of cavities and their appearance in the liquid.
— Cavitation refers to non-stationary phenomena, since is a process of growth and collapse of cavities.
— Cavitation can occur both in the case of a moving fluid and in the case of a resting one.
— Cavitation can occur both in the volume of a liquid and on the solid boundary.

2. Cavitation equation

The equation describing the dynamics of cavitation bubbles in an incompressible fluid without taking into account the vapor pressure, surface tension and viscosity of the fluid can be represented as:

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = \frac{1}{\rho}\left[\left(p_b + \frac{2\sigma}{R_o}\right)\left(\frac{R_o}{R}\right)^{3k} - \frac{2\sigma}{R} - p_0 - p(t)\right]. \qquad (1)$$

Here: R_0 is the radius of the nucleus at $t = 0$; R — radius of the nucleus at the next time instant t; ρ — density of a liquid; σ is the surface tension of the fluid; $k = 1$ is the adiabatic index for steam in the bud; p_o is the hydrostatic pressure in a liquid ($p_o = p_b$); $\ddot{R}$ is the acceleration of the cavity wall; $\dot{R}$ is the speed of movement of the cavity wall; $2\sigma/R_0$ is the Laplace pressure; R_0/R is the amplitude of oscillations of the cavity.

3. Dynamics of a cavity under the action of single pulses of negative pressure

A single impulse is presented in the form:

$$p(t) = -\alpha\,(t + t_1)\exp\left(-\frac{t + t_1}{\tau}\right), \qquad (2)$$

where t_1 is the time of appearance of the first germ of homogeneous cavitation under the action of this pulse.

In this case

$$\frac{dp}{dt} = \left(-\frac{1}{\tau}\right)\alpha\,(t + t_1)\exp\left(-\frac{t + t_1}{\tau}\right) + \alpha\exp\left(-\frac{t + t_1}{\tau}\right). \qquad (3)$$

At $t = 0$, the rate of increase in pressure at the initial moment of time is $\alpha = -\dfrac{dp}{dt}$. From the condition $\dfrac{dp}{dt} = 0$, applied to (3) we find $t_m = \tau$, $p_m = \dfrac{\alpha\tau}{e}$, where τ is the rise time of the pressure pulse up to the maximum value p_m. Let us set the maximum pressure amplitude p_m so that it approaches 95% to the thermodynamic stability [3] of the fluid (spinodal, [4]), i.e.

$$p_m = 0.95(p_b - p_s). \tag{4}$$

Then from (4) it follows that

$$\alpha = \frac{1}{\tau} 0.95e(p_b - p_s). \tag{5}$$

Thus, the rate of pressure increase α can be calculated for a given fluid temperature using the value of $(p_b - p_s)$ for a given τ.

The time of appearance of the first nucleus t_1 is found from the condition (2), when at $t = t_1$ the pressure is $p = p_1$.

$$p_1 = -\alpha t_1 \cdot \exp\left(-\frac{t_1}{\tau}\right) = -\beta(p_b - p_s). \tag{6}$$

Here p_1 is the pressure at which one germ appears in 1 cm^3 at the moment of time t_1,

$$\beta = -\frac{p_1}{p_b - p_s} < 1. \tag{7}$$

Substituting α into (6), we get

$$\beta = e \cdot \frac{t_1}{\tau} \cdot \exp\left(-\frac{t_1}{\tau}\right) = e \cdot x \cdot \exp(-x) \tag{8}$$

where $x = t_1/\tau$.

Thus, in the equation (3), for a given value of τ, the value of α; is calculated by the formula (5) and the value of t_1 is calculated by the formula (2) for a given value of β (formula (8)).

To determine the time t_1, it is necessary to take into account the number of bubbles appearing per unit of time per unit volume of liquid:

$$\frac{dN}{V \cdot dt} = B \cdot \exp\left[-\frac{16\pi\sigma^3}{3(1-\rho_v/\rho_l)^2(p_b - p)^2}\right],$$

$$V = 1 \text{ sm}^3,$$

$$A = \frac{16\pi\sigma^3}{3(1-\rho_v/\rho_l)^2}, \tag{9}$$

$$p_b - p = \alpha t,$$

$$dN = B \cdot \exp\left(-\frac{A}{\alpha^2 t^2}\right) dt,$$

$$N = 1 = B \int_0^{t_1} \exp\left(-\frac{A}{\alpha^2 t^2}\right) dt. \tag{10}$$

By calculating α and A through the formulae (9) and (10) by integrating respectively and assuming that $B \approx 10^{33}$ cm$^{-3}s^{-1}$ at $N = 1$ the time t_1 is calculated.

Denoting by

$$C = -\frac{A}{\alpha^2},$$

we get that

$$B \cdot \int_0^{t_1} \exp\left(\frac{C}{t^2}\right) dt = 1. \tag{11}$$

Taking into account the geometric meaning of a definite integral (11), one can determine the point t_1, by numerical method knowing that the area of the figure bounded by the function $f(x)$ on the interval $[0, t_1]$ should be equal to 1.

In the basic cavitation equation (1), we substitute the expression for pressure in the form (2). Then the basic cavitation equation (Rayleigh equation) takes the form:

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = \\ = \frac{1}{\rho}\left[\left(p_b + \frac{2\sigma}{R_o}\right)\left(\frac{R_o}{R}\right)^{3k} - \frac{2\sigma}{R} - p_b - \alpha\,(t + t_1)\exp\left(-\frac{t + t_1}{\tau}\right)\right]. \tag{12}$$

The differential equation for the growth of an embryo vapor has the form:

$$\rho_l\left(R\ddot{R} + \frac{3}{2}\dot{R}^2\right) = -p(t) - \frac{2\sigma}{R}\left(1 - \frac{\rho_l}{\rho_v}\right)^{-1}, \tag{13}$$

where $p(t)$ is defined by the formula (2).

Substituting $p(t)$ from (2) into (13), we get:

$$\rho_l\left(R\ddot{R} + \frac{3}{2}\dot{R}^2\right) = \alpha(t + t_1)\cdot\exp\left(-\frac{t + t_1}{\tau}\right) - \frac{2\sigma}{R}\left(1 - \frac{\rho_l}{\rho_v}\right)^{-1}. \tag{14}$$

For the reasons described above, we will assume $t_1 \approx 0$, and then the equation (14) takes the form:

$$\rho_l\left(R\ddot{R} + \frac{3}{2}\dot{R}^2\right) = \alpha \cdot t \cdot \exp\left(-\frac{t}{\tau}\right) - \frac{2\sigma}{R}\left(1 - \frac{\rho_l}{\rho_v}\right)^{-1}. \tag{15}$$

It is solved by us numerically. For this, it is necessary to present the last equation in the form of a system of two equations in which the following

change of variables is introduced:

$$R = u(2),$$
$$\dot{R} = du(2) = u(1),$$
$$\ddot{R} = du(1).$$

The equation (15) by changing variables is converted to the form:

$$\ddot{R} = \frac{1}{\rho_l R}\left[\alpha t \exp\left(-\frac{t}{\tau}\right) - \frac{2\sigma}{R}\left(1 - \frac{\rho_l}{\rho_v}\right)^{-1}\right] - \frac{3}{2}\frac{\dot{R}^2}{R\rho_l} \tag{16}$$

or

$$du(1) = \frac{1}{\rho_l u(2)}\left[\alpha t \exp\left(-\frac{t}{\tau}\right) - \frac{2\sigma}{u(2)}\left(1 - \frac{\rho_l}{\rho_v}\right)^{-1}\right] - \frac{3}{2}\frac{[u(1)]^2}{u(2)\rho_l}. \tag{17}$$

The last equation is solved numerically by the Runge–Kutta method. The calculations were carried out for $\tau = 10^{-9}$ s.

It was believed that at the initial time, a homogeneous liquid that does not contain vapor bubbles was at a given temperature and pressure

$$t = 0, \quad p_l = p_{l0}, \quad T_l = T_{l0}, \quad N_b = 0, \quad u = 0.$$

The rate of pressure drop α_p at the point with coordinate $x = 0$ was set by the boundary conditions

$$x = 0, \quad p_l = p_{l0} - \alpha_p \cdot t, \quad \frac{\partial u}{\partial x} = 0.$$

After a bubble appeared, the following initial conditions were accepted for ordinary differential equations describing its development (12): $R = R_c$, $p_v = p_b$, $t_v = T_l$.

4. Method of numerical solution of the equation

We introduce the following notation, convenient for working with the program for the numerical solution of the basic cavitation equation in the form (1):

$$\omega R_0 = omr0, \quad \frac{2\sigma}{R_0} = sr0, \quad P_b = pb,$$
$$(\omega R_0)^2 = omr02, \quad \left(P_b + \frac{2\sigma}{R_0}\right) = pbsr0, \quad P_a = pa,$$
$$\rho(\omega R_0)^2 = rhomr, \quad \varphi_o = fio, \quad R^* = u(2), \quad z^* = u(1).$$

Then the original equation (1) takes the form of a system of two first order differential equations:

$$\begin{cases} du(1) = -\dfrac{3}{2}\dfrac{1}{omr0}\dfrac{u(1)}{u(2)} + \\ \qquad + \dfrac{1}{rhomr}\dfrac{1}{u(2)}\left[\dfrac{pbsr0}{(u(2))^{3k}} - sr0\dfrac{1}{u(2)} - pb + pa\cdot\sin(\tau + fio)\right], \\ du(2) = u(1). \end{cases}$$

The resulting system of differential equations is solved numerically by the Runge–Kutta method. It should be noted that numerical solutions were cited earlier for other cases [5–9].

5. The program for the numerical solution

We have created a program for the numerical solution of the cavitation equation in the *Compaq Visual Fortran Professional* programming language. Its work is based on the Runge–Kutta method.

Initially, the main program asks for the values of external parameters, such as fluid temperature, oscillation frequency, and others (see Fig. 1). Then the main program refers to an array of tabular data for the values of surface tension, fluid viscosity, fluid pressure, vapor pressure at a given temperature. These tabular data are discrete values and do not always correspond to a given temperature. Therefore, the main program refers to auxiliary subroutine 1, which approximates or extrapolates the table data to a given point.

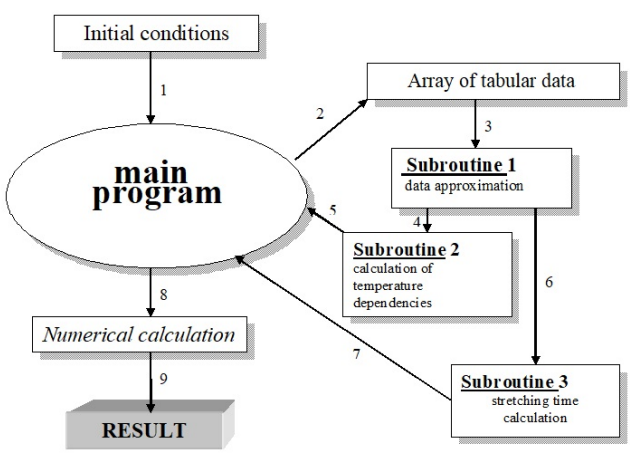

Figure 1. Block diagram

To calculate parameters such as the initial radius of the cavity, the pressure at which the first cavitation nucleus appears, the initial phase of external oscillations, the main program refers to subroutine 2, which calculates these values based on the data already calculated by subroutine 1.

Subroutine 3 then receives from subprogram 2 a task to calculate the time t_1 during which the first cavitation nucleus appears in the fluid. The required

tabular data is requested from subroutine 1. The result of the calculation is reported to the main program.

Having collected all the necessary data, the main program calculates the basic cavitation equation for the maximum amplitude of oscillations of the cavity in case of acoustic cavitation [10,11]. It should be noted that this case is also interesting in the possibility of initiating a nuclear fusion reaction [12–14], which is confirmed by the theory [15,16].

Below is a part of the main program for the numerical solution of this system of equations, written in the programming language *Fortran*:

```
external rad,res
dimension pt(5), u(2), du(2), aux(8,2)
common omr0, rmr0, rhomr, pbsr0, pk, sr0, pa, pb, pi, nk,
 ↪  fa, r0, p, p1, fi0, ky
open(1, file='p.dat')
open(2, file='r.dat')
open(3, file='bubble.txt')
print *, 'The program for calculating the dynamics of the
 ↪  bubble with acoustic effects and the frequency of Fa
 ↪  according to Runge Kutta method.'
print *, '-----------------------------------------'
print *, 'time pressure dependence P = P (t / T) is
 ↪  written to the file "p.dat", the time dependence of
 ↪  the radius R / R0 = f (t / T) is written to the file
 ↪  "r.dat".'
print *, 'THE PROGRAM WILL NOT WORK without an auxiliary
 ↪  file "bubble.txt" and also without the additional
 ↪  program "rkgs.for"!'
read (3, *) q1, q2, fa, pa, pb, p1, r0, sig, amu, ro, kt,
 ↪  ky
pi = 3.141592654
pk = 5
print *, 'The following parameters are entered here:'
print *, '1) Time step integration (wt): pt (3) =', q1
print *, '2) Integration error: pt (4) =', q2
print *, '3) Acoustic frequency: Fa =', fa, 'Hz'
print *, '4) Pressure amplitude at a given point: Pa =',
 ↪  pa, 'Pa'
print *, '5) Pressure on binodals at a given point: Pb =',
 ↪  pb, 'Pa'
print *, '6) Pressure at the point of emergence of the
 ↪  unit: P1 =', p1, 'Pa'
print *, '7) Starting radius of the nucleus: R0 =', r0,
 ↪  'm'
print *, '8) Surface tension: sigma =', sig, 'N / m'
print *, '9) Molar mass: mu =', amu, 'kg / mol'
print *, '10) The density of the fluid at this point: po
 ↪  =', ro, 'kg / m * 3'
print *, '11) Number of periods studied: n =', kt
print *, '12) Number of points for one period: N =', ky
print *, ''
```

```
      print *, 'Changing these parameters is possible only in
     ↪  the file' bubble.txt ''
      print *, 'To interrupt the program, press "Ctrl + C", for
     ↪  continuation - "Enter"'
      pause
      om=2.0*pi*fa
      omr0=om*r0
      omr02=omr0*omr0
      rhomr=ro*omr02
      rmr0=4.*amu/(r0*ro)
      sr0=2.*sig/r0
      fi0=asin((pb-p1)/pa)
      pbsr0=pb+sr0
      pt(1)=0.0
      pt(2)=2.0*pi*kt
      nk=1
 6    continue
      pt(3)=q1
      pt(4)=q2
      u(1)=0.0
      u(2)=1.
      du(1)=0.5
      du(2)=0.5
      call rkgs (pt,u,du,2,ih,rad,res,aux)
      if(ih-10)3,3,4
 3    continue
      print *,'  Error code  ',ih
      goto 5
 4    continue
      print *,'                      ATTENTION   !!!'
      print *, 'ERROR CODE SHOULD NOT EXCEED 10.'
      print *, 'Correct, please, the parameters pt(3),pt(4)'
      read(*,*)q1,q2
      goto 6
 5    continue
      stop
      end
```

Below are some results of a numerical simulation of the bubble behavior in a rarefaction wave.

6. Conclusions

The mathematical model created by us allows us to investigate the dynamics of cavitation bubbles with a change in fluid pressure. Numerical simulation made it possible to determine the time dependence and coordinates of the parameters of the carrier phase, the temperature and pressure of the vapor phase [17], the concentration and size of the bubbles. The proposed model is applicable not only to liquids, but also to metals in the liquid phase [18].

We concluded that the model created gives a good agreement between the calculated and experimental data [19], which demonstrates the applicability of the approach under consideration to the problem of rapid pressure drop.

In the studied problem, the following picture takes place: the pressure of the fluid drops in the rarefaction wave to a value below the saturation pressure, the fluid enters a metastable state [20], intense nucleation begins in the region of minimal pressure, after which the bubbles rapidly increase due to the interfacial mass transfer, which ultimately account stabilizes the pressure at a value close to the saturation pressure.

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