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Construction of the Mathematical Model of Pricing for Telecommunication Services with Allowance for Congestion in Networks

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This paper considers a model of dynamic pricing in the telecommunications market incomplete competition and taking into account overloads in multiservice networks. The model consists in the use of mathematical modeling methods, game theory and queueing theory. It is assumed that telecommunication companies agree on the rules of incoming and outgoing traffic charging in pairs, and this charging is built as a function of the tariffs that companies offer their subscribers for service. Companies are limited the agreement on mutual rules of reciprocal proportional charging for access traffic at first, which subsequently determine the tariffs for the multiservice network users. The reciprocity of the rules means that companies are subject to the same rules for the entire time interval during which the agreement is in force. Taking into account imperfect competition in the telecommunications market and using profit optimization method for each company the equilibrium tariffs and the volume of services are found with subject to congestion in multi-service networks.

Key words and phrases: queueing theory, game theory, optimization methods, probability theory, industrial market theory, economic and mathematical modeling

Introduction

Methods of mathematical modeling in the economy of telecommunications are being actively developed [1–7]. Jean Tirole considers the impact of telecommunication technologies on competition in services and goods markets [8–12]. In 2014 he was awarded the Nobel Memorial Prize in Economic Sciences for his analysis of market power and regulation.

In paper [13], Se-Hak Chuna considered optimal access charges for the provision of telecommunication network, mobile commerce, and cloud services. Using theoretical analysis, Se-Hak Chuna investigated, when a regulator can set rational access pricing, considering the characteristics of access demand. Se-Hak Chuna demonstrated that optimal access prices depend on whether the final products or services are independent strategies or substitute strategies. The results have applications for policy makers setting optimal access charges that maximize social welfare.

In this article a mathematical model of pricing for telecommunications services with overloads in networks is built. It generalizes the model that was built earlier [14, 15].

It is assumed that telecommunications companies agree in pairs on the rules of charging for access traffic to the network of the other company, and it is considered as a function of the tariffs that companies offer their consumers (subscribers) for services. Thus, these companies have contracts at the first stage by agreements on reciprocal proportional access charge rules (RPACR), which subsequently allow them to determine the subscription rates. The ambiguity of the rules means that companies are subject to the same rules for the entire time interval during which the agreement is valid.

RPACR may be seen as analogous to the regulatory policy of the state of the telecommunications industry. If telecommunication services, provided by different companies, are close substitutes, the use of RPACR by companies leads to competitive prices in

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industry. However, if it is assumed that competing companies follow the policy of services differentiation, then intervention of the state is required to preclude the use by companies of monopoly power.

It is also assumed that the utility function of subscribers consists of deterministic and stochastic parts. The deterministic part allows to find a linear function of subscribers demand for telecommunications services, which has a constant price elasticity. It allows to avoid unlimited growth of consumption of telecommunication services by subscribers at aspiration the corresponding tariffs to zero and ensures the existence of a saturation point, i.e., for example, there are time limits that the subscriber uses for using telecommunication services. The Weibull distribution is used for the stochastic component of the utility function, which is convenient for further analysis. It is possible to find equilibrium tariffs and equilibrium demand for telecommunication services. This equilibrium is equilibrium in pure strategies and it always exists, and the subscription rates are calculated explicitly.

1. The Model of the Telecommunications Industry in the Case of Multiservice Network

Let's consider a network NW ($NW = \bigcup\limits_{i=1}^{n} NW_i$) consisting of n equivalent multiservice network (numbered in a certain order multiservice network $SR = \bigcup\limits_{s=1}^{m} SR_s$) belonging to different telecommunication companies T_i $(i = \overline{1,n})$, and it is assumed that in between all the networking companies there are switching nodes.

Let $t \in \{1, 2, \ldots, T_{\max}\}$ be time intervals (for example, the time period equals a week, a month or a year) equal to the length of time periods during which companies T_i independently decide on pricing for their services, and $t_{\max}$ is the maximum planning horizon.

Let's assume that the network NW consists of a set of nodes $J^t = \bigcup\limits_{i=1}^{s_j} J_i^t$ and a set of channels $L^t = \bigcup\limits_{i=1}^{s_l} L_i^t$, and $NW = J^t \cup l^t$.

In the time period t each network NW_i of the company T_i $(i = \overline{1,n})$ is represented by the set of nodes J_{ij}^t $(j = 1, \ldots, s_i^J)$ and channel set L_{ij}^t $(j = 1, \ldots, s_i^L)$, numbered in a certain way, where $J_i^t = \bigcup\limits_{j=1}^{s_i^J} J_{ij}^t$, $L_i^t = \bigcup\limits_{k=1}^{s_i^L} L_{ik}^t$ and $NW_i = J_i^t \cup L_i^t$, and the total number of nodes is $S_{NW}^J(t) = \sum\limits_{i=1}^{n} s_i^J$, and the total number of channels is $S_{NW}^L(t) = \sum\limits_{i=1}^{n} s_i^L$ for network NW.

Let H_{ij}^t be a capacity (bits/sec) of j-node $(j = \overline{1, J_{s_i^J}})$, and S_{ik}^t a throughput (bits/sec) k-link $(k = \overline{1, L_{s_i^L}})$ T_i of network NW_i company T_i in the time period t.

Two-point connections can be established to transmit information flows between the network nodes of network NW. Each connection is characterized by a route, i.e. a set of network links NW, through which connections are established.

Let $s = \{1, \ldots, m\}$ be a set of services that offer companies for potential consumers (subscribers) during the period $t \in \{1, 2, \ldots, T_{\max}\}$. Let b $(b \in (1, 2, \ldots, B^t))$ be a set of consumers, who want to use the telecommunications services in the market.

Let's assume that the individual consumer demand function for the service $s = \{1, \ldots, m\}$ has the form:

$$D_{bs}^t(p_s^t) = \frac{r_{bs}^t - p_s^t}{2s_{bs}^t} = a_{bs}^t - b_{bs}^t p_s^t, \quad a_{bs}^t = \frac{r_{bs}^t}{2s_{bs}^t}, \quad b_{bs}^t = \frac{1}{2s_{bs}^t}, \tag{1}$$

$D_{bs}^t(p_s^t)$ is a linear function of the price p_s^t, and $r_{bs}^t > 0$ and $s_{bs}^t > 0$ is positive coefficients, which are determined from the market research services SR in the period t.

A consumer b generates the traffic loading or the load using the service s in the period t. Let Y_{bs}^t be an individual traffic volume of a consumer b, and let $Y_{bs}^t = \bar{\lambda}_{bs}^t h_{bs}^t$ be the average value of Y_{bs}^t, where the parameter $\bar{\lambda}_{bs}^t$ is the average intensity of the flow of requests and the parameter h_{bs}^t is the average duration of service in the period t.

We assume that the average load is generated by the consumer b when using the service s in the period t, linearly depends on the corresponding demand function for this service s

$$Y_{bs}^t = \bar{\lambda}_{bs}^t h_{bs}^t = \theta_s D_{bs}^t(p_s^t) = \theta_s \left(a_{bs}^t - b_{bs}^t p_s^t \right), \tag{2}$$

where θ_s is the proportionality factor for the s service. It links the consumer demand for telecommunication services and the amount of traffic generated by this consumer in the network.

The total network traffic volume that is created by a consumer in the period t during using the service s, is the sum of consumers network traffic volumes

$$\begin{gathered} Y_s^t = \sum_{b=1}^{B_t} Y_{bs}^t = \sum_{b=1}^{B_t} \theta_s \left(a_{bs}^t - b_{bs}^t p_s^t \right) = \bar{A}_s^t - \bar{B}_s^t \bar{p}_s^t, \\ \bar{A}_s^t = \sum_{b=1}^{B_t} \theta_s a_{bs}^t, \quad \bar{B}_s^t = \sum_{b=1}^{B_t} \theta_s b_{bs}^t, \end{gathered} \tag{3}$$

where $\bar{a}_s^t$, $\bar{B}_s^t$ are parameters of the function Y_s^t.

The total consumers demand for the service s during the time t is the sum of all demand functions for the service s of all:

$$\begin{gathered} D_{bs}^t(p_s^t) = \sum_{b=1}^{B^t} D_{bs}^t(p_s^t) = \sum_{b=1}^{B^t} \left(a_{bs}^t - b_{bs}^t p_s^t \right), \\ D_{bs}^t(p_s^t) = \left(a_s^t - b_s^t p_s^t \right), \quad a_s^t = \sum_{b=1}^{B^t} a_{bs}^t, \quad b_s^t = \sum_{b=1} B_t b_{bs}^t, \end{gathered} \tag{4}$$

where the parameters $a_s^t \geqslant 0$ and $b_s^t \geqslant 0$ are determined from market research of services in the period t.

We can get a link between the network traffic volume $Y_s^t(p_s^t)$ and the demand function $D_{bs}^t(p_s^t)$ of the service s during the period t:

$$Y_s^t(p_s^t) = Q_{bs}^t(p_s^t) \theta_s D_{bs}^t(p_s^t) = \theta_s \left(a_s^t - b_s^t p_s^t \right) = A_s^t - B_s^t p_s^t, \tag{5}$$

where $Y_s^t(p_s^t)$ is linear price functions and $A_s^t = \theta_s a_s^t$, $B_s^t = \theta_s b_s^t$ are coefficients.

We can get the network traffic volume that is associated with the consumer b ($b = \overline{1, B^t}$)

$$\begin{gathered} Y_b^t = \sum_{s=1}^{m} Y_{bs}^t = \sum_{s=1}^{m} \theta_s \left(a_{bs}^t - b_{bs}^t p_s^t \right) \leqslant \bar{A}_b^t - \bar{B}_b^t \bar{p}^t, \\ \bar{A}_b^t = \sum_{s=1}^{m} \theta_s \, a_{bs}^t, \; \bar{B}_b^t = \sum_{s=1}^{m} b_{bs}^t, \quad \bar{p}^t = \sum_{s=1}^{m} p_s^t, \quad \bar{B}_b^t \bar{p}^t \leqslant \sum_{s=1}^{m} \theta_s b_{bs}^t p_s^t. \end{gathered} \tag{6}$$

where $\bar{A}_b^t \geqslant 0$, $\bar{B}_b^t \geqslant 0$ are parameters load functions Y_b^t associated with the consumer b, and a parameter $\bar{p}^t$ is a tariff for services SR (service package) during the time period t.

A consumer's b ($b = \overline{1, B^t}$) demand for SR-services in the considered time period t has the form:

$$\begin{gathered} Q_b^t(p_b^t) = \sum_{s=1}^{m} D_{bs}^t(p_s^t) = \sum_{s=1}^{m} \left(a_{bs}^t - b_{bs}^t p_s^t\right) \leqslant \left(a_b^t - b_b^t \bar{p}^t\right), \\ a_b^t = \sum_{s=1}^{m} a_{bs}^t, \quad b_b^t = \sum_{s=1}^{m} b_{bs}^t, \quad b_b^t \bar{p}^t \leqslant \sum_{s=1}^{m} b_{bs}^t p_s. \end{gathered} \tag{7}$$

Aggregating the network traffic volume $Y_s^t(p_s^t)$ from (5) for all services $s = \{1, \ldots, m\}$, we can get the total network traffic volume $Y(t)$ for the period t in the form:

$$\begin{gathered} Y(t) = \sum_{s=1}^{m} Y_s^t(p_s^t) = \sum_{s=1}^{m} \left(a_s^t - b_s^t p_s^t\right) = \sum_{s=1}^{m} \theta_s \left(a_s - b_s^t p_s^t\right) = \bar{A}^t - \bar{B}^t \bar{p}^t, \\ \bar{A}^t = \sum_{s=1}^{m} \theta_s a_s^t, \quad \bar{B}^t \bar{p}^t \geqslant \sum_{s=1}^{m} \theta_s b_s^t p_s^t, \quad \bar{B}^t = \sum_{s=1}^{m} \theta_s b_s^t, \end{gathered} \tag{8}$$

where $\bar{A}^t \geqslant 0$ and $\bar{B}^t \geqslant 0$ are aggregated parameters of function $Y(t)$, and where function of aggregated demand for services SR (service package) has the form:

$$\begin{gathered} D(t) = \sum_{s=1}^{m} \left(a_s^t - b_s^t p_s^t\right) = \bar{a}^t - \bar{b}^t \bar{p}^t, \\ \bar{a}^t = \sum_{s=1}^{m} a_s^t, \quad \bar{b}^t \bar{p}^t \geqslant \sum_{s=1}^{m} b_s t\, p_s^t, \quad \bar{b}^t = \sum_{s=1}^{m} b_s^t, \end{gathered} \tag{9}$$

where the parameters $\bar{a}^t \geqslant 0$ and $\bar{b}^t \geqslant 0$ are aggregated parameters of the demand function $D(t)$.

We can assume that for each company T_i ($i = \overline{1, n}$) there exists a function of consumer demand for services SR (service package) during the time period t. Let D_{sii} ($i \in \{1, \ldots, n\}$) be a demand function of services $SR = \bigcup_{s=1}^{m} SR_s$ provided by the company T_i using its NW_i network resource only, and let D_{sij}^t ($i, j \in \{1, \ldots, n\}$, $i \neq j$) be a demand function of services provided together with a network NW_i of a company T_i and a network NW_j of a company T_j ($i, j \in \{1, \ldots, n\}$, $i \neq j$). Thus, there is a question of access of one company to resources of a network of the other company.

We assume that the companies T_i and T_j ($i, j \in \{1, \ldots, n\}$, $i \neq j$) agree on the charges $\hat{a}_{ij}^t$ and $\hat{a}_{ji}^t$, where $\hat{a}_{ij}^t$ is a charge, which company T_i pays the company T_j ($i, j \in \{1, \ldots, n\}$, $i \neq j$) for the use of its network resources in connection with the service of $s \in \{1, \ldots, m\}$ (traffic from the network NW_i to the network NW_j or outgoing traffic for the company T_i and incoming traffic for the company T_j), and $\hat{a}_{ji}^t$ is a corresponding charge at which the company T_j pays the company T_i ($i, j \in \{1, \ldots, n\}$, $i \neq j$) for the use of network resources in connection with the provision of a similar service $s \in \{1, \ldots, m\}$ (traffic from the network NW_j to the network NW_i or outgoing traffic for the company T_j and incoming traffic for the company T_i) during the time period t.

Suppose that any two companies T_i and T_j $(i, j \in \{1, \ldots, n\},\ i \neq j)$ charges $\hat{a}_{ij}^t$ and $\hat{a}_{ji}^t$ depend on tariffs $\bar{p}_i^t$ and $\bar{p}_j^t$, and $\hat{a}_{ij}^t = a_i^t(\bar{p}_i^t, \bar{p}_j^t)$ for any $(i, j \in \{1, \ldots, n\},\ i \neq j)$ and $s \in \{1, \ldots, m\}$ at any time $t \in \{1, 2, \ldots, T_{\max}\}$.

We assume that there is the proportional dependence between $\hat{a}_{ij}^t$ and $\bar{p}_i^t$, then $\hat{a}_{ij}^t = a_i^t \bar{p}_i^t$, where the proportionality factor is $0 \leqslant a_i^t \leqslant 1$ for $i \in \{1, \ldots, n\}$ and $s \in \{1, \ldots, m\}$.

2. Multiservice Demand Function

Suppose that each consumer can use telecommunication multiservice network of companies T_i $(i \in \{1, \ldots, n\})$ at any time period t . Let's assume that each consumer has individual tastes and preferences in relation to these services SR. We assume that the consumer b $(b \in \{1, \ldots, B^t\})$, which is ready to choose one service from the set $s \in \{1, \ldots, m\}$ of the company T_i $(i \in \{1, \ldots, n\})$, has the following utility function:

$$u_{ibs}^t = U_{ibs}^t e^{\eta_s \epsilon_{ibs}^t} = U_{bs}^t(Q_{bs}^t(p_{is}^t), p_{is}) e^{\eta_s \epsilon_{ibs}^t}, \tag{10}$$

$$U_{ibs}^t = \left[r_{bs}^t - s_{bs}^t Q_{bs}^t(p_s^t)\right] Q_{bs}^t(p_s^t) - p_s^t Q_{bs}^t(p_s^t),$$

where the random parameter ϵ_{ibs}^t characterizes individual tastes and preferences of the consumer. Let's consider that ϵ_{ibs}^t has a Weibull distribution. The value of η_s gives the characteristic measures of the dispersion of tastes and preferences of the consumers, that is, η_s allows us to estimate the substitutability telecommunication services $s \in \{1, \ldots, m\}$ that provide companies T_i and T_j $(i, j \in \{1, \ldots, n\},\ i \neq j)$. The services $s \in \{1, \ldots, m\}$ of companies become total substitutes with $\eta_s \to 0$, and it is total complementary with $\eta_s \to \infty$.

Let's assume that each consumer b $(b \in \{1, \ldots, B^t\})$ chooses the company T_i and rejects the company T_j $(i, j \in \{1, \ldots, t\},\ i \neq j)$ at the period t then there is inequality

$$U_{ibs}^t e^{\eta_s \epsilon_{ibs}} \geqslant U_{jbs}^t e^{\eta_s \epsilon_{jbs}}.$$

Thus, the probability P_{ibs}^t that the consumer b gives preference to the company T_i and rejects the company T_j $(i, j \in \{1, \ldots, n\},\ i \neq j)$ equals to

$$P_{ibs}^t = \mathrm{P}\{U_{ibs}^t e^{\eta_s \epsilon_{ibs}} > U_{jbs}^t e^{\eta_s \epsilon_{jbs}}\}. \tag{11}$$

Since the values ϵ_{ibs} are independent and have a Weibull distribution we have that

$$P_{ibs}^t = \frac{1}{1 + \left(\frac{U_{ibs}^t}{U_{jbs}^t}\right)^{\frac{1}{\eta_s}}} = \frac{(r_{bs}^t p_{is}^t)_s^\tau}{(r_{bs}^t p_{is}^t)_s^\tau + (r_{bs}^t - p_{js}^t)_s^\tau}, \tag{12}$$

where $\tau_s = 2/\eta_s$. Similarly for the company T_j we have the same

$$P_{jbs}^t = \frac{1}{1 + \left(\frac{U_{jbs}^t}{U_{ibs}^t}\right)^{\frac{1}{\eta_s}}} = \frac{\left(r_{bs}^t p_{js}^t\right)_s^\tau}{\left(r_{bs}^t p_{js}^t\right)_s^\tau + (r_{bs}^t - p_{is}^t)_s^\tau}. \tag{13}$$

Thus, each consumer chooses one service s in the company T_i with probability p_{ibs} and in the company T_j with probability p_{jbs}.

We can generalize this approach for the case when the consumer chooses one company T_i from the set of companies $\{T_1, \ldots, T_n\}$ to obtain the service s, and we can get the probability in case the consumer gives preference to the company T_i:

$$P^t_{ibs} = \frac{(r^t_{bs} - p^t_{is})^\tau_s}{\sum\limits_{j=1}^{n} (r^t_{bs} - p^t_{js})^\tau_s}. \tag{14}$$

The probability that the consumer chooses one company T_i from a set of companies $\{T_1, \ldots, T_n\}$ to receive service package SR has the form:

$$P^t_{ib} = \frac{\sum\limits_{s=1}^{m} (r^t_{bs} - p^t_{is})^\tau_s}{\sum\limits_{s=1}^{m} \sum\limits_{j=1}^{n} (r^t_{bs} - p^t_{js})^\tau_s}. \tag{15}$$

The expected value of consumers $b_i(t)$ who chooses a company T_i is determined by the probability P^t_{ib}, which can be considered as the market share m^t_i of a company T_i, and has the form

$$m^t_i = P^t_{ib} = \frac{\sum\limits_{s=1}^{m} \left(r^t_{bs} - p^t_{is}\right)^\tau_s}{\sum\limits_{s=1}^{m} \sum\limits_{j=1}^{n} \left(r^t_{bs} - p^t_{js}\right)^\tau_s}, \quad \sum_{i=1}^{n} m^t_i = 1. \tag{16}$$

The demand of consumers for services $s \in \{1, \ldots, m\}$ of the company T_i ($i \in \{1, \ldots, n\}$) has the form:

$$D^t_{ibs}(p^t_{is}) = \frac{B^t P^t_{ib}}{2s^t_{bs}} \left(r^t_{bs} - p^t_{is}\right) = \frac{B^t m^t_i}{2s^t_{bs}} \left(r^t_{bs} - p^t_{is}\right). \tag{17}$$

Demand function of the consumers D^t_{sii} who have plan to use the service SR of a company T_i, which may be implemented within network NW_i, and demand function of the consumer D^t_{ij} who has plan to use the service SR implemented with resources of the networks NW_i and NW_j, have the form:

$$D^t_{sii} = \frac{B^t m^{t2}_i}{2s^t_{bs}} \left(r^t_{bs} - p^t_{is}\right), \; D^t_{ijs} = \frac{B^t m^t_i m^t_j}{2s^t_{bs}} \left(r^t_{bs} - p^t_{is}\right), \tag{18}$$

where the aggregated s-service demand D^t_{is} has the form:

$$D^t_{is} = D^t_{sii} + \sum_{j=1}^{n} D^t_{sij} = \frac{B^t m^{t2}_i}{2s^t_{bs}} \left(r^t_{bs} - p^t_{is}\right) + \sum_{j=1; i \neq j}^{n} \frac{B^t m^t_i m^t_j}{2s^t_{bs}} \left(r^t_{bs} - p^t_{is}\right), \tag{19}$$

and the total network traffic volume demand D^t_i for company T_i has the form:

$$D^t_i = \sum_{s=1}^{m} \left[D^t_{sii} + \sum_{j=1}^{n} D^t_{sij} \right] = \sum_{s=1}^{m} \left[\frac{B^t m^{t2}_i}{2s^t_{bs}} (r^t_{bs} - p^t_{is}) + \sum_{j=1; i \neq j}^{n} \frac{B^t m^t_i m^t_j}{2s^t_{bs}} \left(r^t_{bs} - p^t_{is}\right) \right],$$

where

$$D_{ii}^t = \sum_{s=1}^{m} D_{sii}^t,\ D_{ij}^t = \sum_{s=1}^{m} D_{sij}^t,$$

and the total network traffic volume for a company T_i has the form:

$$Y_i^t = \theta D_i^t = \sum_{s=1}^{m} \theta_s D_{is}^t = \sum_{s=1}^{m} \theta_s \left[\frac{B^t\, m_i^{t2}}{2s_{bs}^t} \left(r_{bs}^t - p_{is}^t \right) + \sum_{\substack{j=1;\\ i\neq j}}^{n} \frac{B^t\, m_i^t m_j^t}{2s_{bs}^t} \left(r_{bs}^t - p_{is}^t \right) \right], \quad (20)$$

where θ is an "average" linking parameter for function Y_i^t and D_i^t.

Revenue function TR_i^t of companies T_i ($i \in \{1, \ldots, n\}$) at the period t ($t = 1, 2, \ldots, T_{\max}$) has the form:

$$TR_i^t = \sum_{\substack{i,j=1;\\ i\neq j}}^{n} \left[\bar{p}_i^t D_{ii}^t \left(\bar{p}_i^t \right) + \left(\bar{p}_i^t - \delta_{ij}^t \bar{p}_j^t \right) D_{ij}^t \left(\bar{p}_i \right) + \delta_{ij}^t \bar{p}_i^t D_{ji}^t \left(\bar{p}_j^t \right) \right], \quad (21)$$

where $\delta_{ij}^t \in [0, 1]$ is a parameter to be defined during negotiations between companies T_i and T_j. We assume that the cost of an access service to the competitor's network is a value proportional to the cost of servicing by this company of its consumers. Profit function Π_i^t of companies T_i ($i \in \{1, \ldots, n\}$) at the period t ($t = 1, 2, \ldots, T_{\max}$) has the form:

$$\begin{gathered} \Pi_i^t = TR_i^t - TC^t \left({w_J}_{ik}^t, H_{ik}^t, {w_L}_{ik}^t, c_{ik}^t, F^t \right), \\ TC^t = \left(\sum_{k=1}^{s_i^J} {w_J}_{ik}^t H_{ik}^t + \sum_{k=1}^{s_i^L} {w_L}_{ik}^t c_{ik}^t \right) + F^t, \end{gathered} \quad (22)$$

where TC^t is a total costs function and F^t is a fix cost.

3. Profit Company Control Problem and Overloads in Networks

We can formulate an optimization problem for each company T_i ($i \in \{1, \ldots, n\}$) at any time $t \in \{1, 2, \ldots, T_{\max}\}$:

$$\begin{cases} \partial \Pi_i^t / \partial p_i^t & = 0; \\ \partial^2 \Pi_i^t / \partial p_i^{t2} & < 0. \end{cases} \quad (23)$$

The following theorem holds true.

Theorem 1. *Provided that the parameters $\theta_s > 0$, $\bar{a}^t > 0$, $\bar{b}^t > 0$, $\delta_{ij}^t \in [0, 1]$, ${w_J}_{ij}^t \geqslant 0$, ${w_L}_{ij}^t \geqslant 0$, $F^t \geqslant 0$, there is a unique solution of the problem (23) in the form of the equilibrium value of the tariff for the use of services SR of company $i \in \{1, \ldots, n\}$ during the period t:*

$$\bar{p}_t^* = \left(m_i^t + \sum_{j=1; i\neq j}^{n} \delta_{ij}^t m_j^t \right) \frac{\bar{a}^t}{2\bar{b}^t}.$$

Proof. Let's write out the profit function of i company in the form of:

$$\Pi_i^t = \sum_{\substack{i,j;\\ i\neq j}}^{n} \left[\bar{p}_i^t m_i^{t2}\left(\bar{a}^t - \bar{a}^t\bar{p}_i^t\right) + m_i^t m_j^t\left(\bar{p}_i^t - \delta_{ij}^t\bar{p}_j^t\right)\left(\bar{a}^t - \bar{b}^t\bar{p}_i^t\right) + \right.$$

$$\left. + \delta_{ij}^t m_j^t m_i^t \bar{p}_i^t\left(\bar{a}^t - \bar{b}^t\bar{p}_j^t\right)\right] - \left(\sum_{k=1}^{s_i^J} {w_J}_{ik}^t H_{ik}^t + \sum_{k=1}^{s_i^L} {w_L}_{ik}^t c_{ik}^t\right) - F^t,$$

We can calculate the derivatives of $\bar{p}_i^t$ and equal them to zero, thus we obtain a system of algebraic equations of the form:

$$m_i^t\left(\bar{a}^t - 2\bar{b}^t\bar{p}_i^t\right) + \sum_{j=1;j\neq i}^{n}\left[m_j^t\left(\bar{a}^t - 2\bar{b}^t\bar{p}_i^t + \delta_{ij}^t\bar{b}^t\bar{p}_j^t\right) + \delta_{ij}^t m_j^t\left(\bar{a}^t - \bar{b}^t\bar{p}_j^t\right)\right] = 0,$$

and the equilibrium value of the tariff has the form:

$$\bar{p}_t^* = \left(m_i^t + \sum_{j=1;j\neq i}^{n}\delta_{ij}^t m_j^t\right)\frac{\bar{a}^t}{2\bar{b}^t}.$$

We can obtain for $\partial^2\Pi_i^t/\partial\bar{p}_i^{t2}$,

$$\frac{\partial^2\Pi_i^t}{\partial\bar{p}_i^{t2}} = \sum_{i,j;i\neq j}\left[-m_i^{t2}2\bar{b}^t - m_i^t m_j^t 2\bar{b}^t - \delta_{ij}^t m_j^t m_i^t\bar{b}^t\bar{p}_j^t\right] < 0.$$

The theorem is proved. □

We can formulate an optimization problem for each company T_i $(i \in \{1,\dots,n\})$ at any time $t \in \{1,2,\dots,T_{\max}\}$ for the tariff value $\bar{p}_t^*$:

$$\begin{cases}\partial\Pi_i^t(\bar{p}_t^*,\delta_{ij}^t)/\partial\delta_{ij}^t = 0;\\ \partial^2\Pi_i^t(\bar{p}_t^*,\delta_{ij}^t)/\partial\delta_{ij}^{t2} < 0;\end{cases}$$

which allows maximizing the profit of each company of T_i using the parameter δ_{ij}^t.

After substituting the corresponding equilibrium tariffs $\bar{p}_t^*$ in the profit function, we obtain the following equation

$$\Pi_i^t = \sum_{\substack{i,j;\\ i\neq j}}^{n}\frac{\bar{a}^{t2}m_i^t\left[m_i^t + m_j^t\right]}{2\bar{b}^t}\left(m_i^t + \sum_{\substack{j=1;\\ j\neq i}}^{n}\delta_{ij}^t m_j^t\right)\left(1 - 0.5\left(m_i^t + \sum_{\substack{j=1;\\ j\neq i}}^{n}\delta_{ij}^t m_j^t\right)\right) -$$

$$- \left(\sum_{k=1}^{s_i^J} {w_J}_{ik}^t H_{ik}^t + \sum_{k=1}^{s_i^L} {w_L}_{ik}^t C_{ik}^t\right) - F^t,$$

and differentiating by δ_{ij}^t and equaling to zero, we have a system of algebraic equations, by solving which, we obtain an equilibrium value of $\delta_t^* = 0.5$.

The equilibrium tariff $\bar{p}_t^*$ for the services of company T_i, taking into account the optimal value $\delta_t^* = 0.5$ during the period t, has the form:

$$\bar{p}_t^* = \left(m_i^t + 1\right) \frac{\bar{a}^t}{4\bar{b}^t},$$

The equilibrium demand function for the company T_i $(i \in \{1, \ldots, n\})$ services SR at any t can be represented as follows:

$$D_{it}^* \left(\bar{p}_t^*\right) = m_i^t D_t \left(\bar{p}_t^*\right) = 0.25 \cdot m_i^t \bar{a}_t \left(3 - m_i^t\right),$$

and the total network traffic volume for a company T_i with the equilibrium tariff has the form:

$$Y_i^t = \theta D_{it}^* = 0.25 \cdot \theta m_i^t \bar{a}_t \left(3 - m_i^t\right).$$

The total equilibrium market demand function D_t^* and the total equilibrium traffic volume Y_t^* for services SR at any t has the form:

$$D_t^* = \bar{a}^t \left(3 - \sum_{i=1}^n m_i^{t2}\right), \quad Y_t^* = \theta \bar{a}^t \left(3 - \sum_{i=1}^n m_i^{t2}\right)$$

and we can show that with a uniform distribution of customers between all companies T_i $(i \in \{1, \ldots, n\})$ the total equilibrium traffic volume for services SR reaches maximum.

If the network bandwidth of companies is less than the traffic volume that subscribers generate, then companies can manage the overload by creating such tariffs that reduce the overload on the network.

Conclusions

In this paper a mathematical model of the telecommunications market is constructed taking into account overloads in networks. The analysis of equilibrium tariffs for telecommunications services for this type of market is carried out.

The most important result of this paper is the following: when the companies follow the reciprocal proportional access charge rules (PACR) then there always exist equilibrium tariffs for services. The applied value of the model is that the use of PACR telecommunication companies does not require detailed information market telecommunications, as the number of parameters of the model is minimized. This model proved to be effective in analysing the dynamics of the telecommunications market, as it allows companies to respond flexibly to external changes, which allows to change the strategy at every moment of time. The proposed model can serve as a tool for analyzing the existence of collusion between companies in the telecommunications industry market.

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Построение математической модели ценообразования на телекоммуникационные услуги с учётом перегрузок в сетях

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В работе строится модель динамического ценообразования на рынке телекоммуникаций при условии ограниченной конкуренции и с учётом перегрузок в мультисервисных сетях. Для построения и исследования модели был применён комплексный подход, заключающийся в использовании методов экономико-математического моделирования и теории массового обслуживания. В предлагаемой модели предполагается, что телекоммуникационные компании попарно договариваются о правилах тарификации входящего и исходящего трафика, причём эта тарификация строится как функция от тарифов, которые компании предлагают своим абонентам за обслуживание. Таким образом, эти компании ограничиваются на первом шаге договорённостями по обоюдным правилам пропорциональной тарификации за доступ трафика (ОППТДТ), которые впоследствии позволяют определить тарифы для пользователей услуг мультисервисных сетей, которыми владеют компании. Обоюдность правил означает, что компании подчиняются одним и тем же правилам на всем интервале времени, в течение которого действует договорённость. С учётом несовершенной конкуренции на рынке телекоммуникаций и при условии максимизации прибыли каждой компанией, которая является поставщиком услуг, в рамках построенной модели были найдены равновесные тарифы на эти услуги с учётом перегрузок в мультисервисных сетях, а также объёмы этих услуг.

Ключевые слова: теория массового обслуживания, теория игр, методы оптимизации, теория вероятностей, теория отраслевых рынков, экономико-математическое моделирование

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