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Design and Stability Analysis of Nondeterministic Multidimensional Populations Dynamics Models

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The multidimensional models of the population dynamics are considered in the paper. These models are the generalizations of the Lotka–Volterra model in case of interaction of the finite number of populations. The deterministic description of the models is given by the systems of the ordinary nonlinear differential equations presented in the paper in the form of the multidimensional vector differential equations. The qualitative properties of the specified models are sufficiently well studied by means of Lyapunov methods. However, the probabilistic factors influencing on the behavior of models are not taken into account at the deterministic description of models. The new approaches to the modeling and stability analysis are of theoretical and applied interest in the nondeterministic case.

In this paper, the methods for design of multidimensional nondeterministic models of interaction of populations are considered. The first method is connected with the transition from the vector nonlinear ordinary differential equation to the corresponding vector differential inclusions, fuzzy and stochastic differential equations. On the basis of the reduction principle, which makes it possible to reduce the problem of the stability of solutions of a differential inclusion to the problem of stability of solutions of other types of equations, stability conditions for the constructed models are obtained. The second method is connected with the technique of design of the self-consistent stochastic models. The scheme of interaction is received on the basis of this technique. This scheme includes a symbolical record of possible interactions between the system elements. The structure of the multidimensional stochastic Lotka–Volterra models is described, and the transition to the corresponding Fokker–Planck vector equations is carried out by means of the system state operators and the system state change operator. The rules for the transition to the multidimensional stochastic differential equation in the Langevin form are formulated. The execution of the numerical experiment with the application of the developed program complex for solving the systems of the stochastic differential equations is possible for the models which are the concretizations of the studied general models. The described approach to the modeling of the stochastic systems can be applied in the problems of comparing of the qualitative properties of the models in deterministic and stochastic cases. The obtained results are aimed at the development of the methods for the analysis of nondeterministic nonlinear models.

Key words and phrases: model of population dynamics, stability, differential inclusions, stochastic model, principle of the reduction

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1. Introduction

The stability research of the models of the population dynamics is an important problem. Some directions of the solution of the specified problem are presented in the works [1–7]. The questions of existence and stability of the solutions of the models described by the differential equations of various types were considered in [8–12] and in other works.

Lyapunov functions method is one of the widely used methods of the stability research [10, 11]. The stability of the solutions of classical and generalized models of the population dynamics by Lyapunov function method was considered in [1]. The systemic approach is described in [2, 6–9] which allows us to consider properties of stability of the models described by the differential equations of various types from the unified point of view. The specified approach is based on the transition from the deterministic description of the model to stochastic description and on the principle of reduction of the stability problem of solutions of differential inclusion to the stability problem of other types of the equations. The approach allows us to study the stability properties of solutions of differential inclusions, the fuzzy and stochastic differential equations from the unified point of view.

In this work we consider a nonlinear multidimensional model of the dynamics of the populations interaction. The determined description of model is given by system of the ordinary nonlinear differential equations. The transition from the specified model to the corresponding nondeterministic models given by means of finite-dimensional differential inclusions, the fuzzy and stochastic differential equations is performed. The stability analysis is performed on the basis of the reduction principle in this work.

It is known [5, 13–15] that in the deterministic description of the model the probabilistic factors affecting the behavior of the model are not taken into account. In this connection, an important problem is construction and study of adequate stochastic models, as well as a comparative analysis of the properties of deterministic and corresponding stochastic models.

The technique of design of the self-consistent stochastic models [14] allows us to take into account stochastics in the structure of the model without adding additive stochastic terms. In this work the structure of the multidimensional stochastic Lotka–Volterra models is described, and the transition to the corresponding Fokker–Planck vector equations is carried out by means of the system state operators and the system state change operator. The rules for the transition to the multidimensional stochastic differential equation in the Langevin form are formulated. It is shown that the used approach to construct multidimensional stochastic models can find application in problems of comparing the qualitative properties of the generalized Lotka–Volterra models.

2. Deterministic Models

We consider the model described by the system of differential equations of the form [1, 3]:

$$\dot{x_i} = x_i \left(a_i - \sum_{j=1}^{n} p_{ij} x_j \right), \quad i = 1, \ldots, n, \tag{1}$$

where x_i is density of i-th population in moment t, $\dot{x_i} = \mathrm{d}x_i/\mathrm{d}t$, a_i and p_{ii} are growth coefficients of i-th population in the absence of others, constants p_{ij} at $i \neq j$ characterize the influence of interaction between populations on the rate of growth, $P = (p_{ij})$, $i, j = 1, \ldots, n$, is interaction matrix.

The model (1) is the classical Lotka–Volterra model for the n-dimensional case. This model describes the dynamics of the biological community under the following conditions:

1) the relative growth rate of each population does not depend on the intrapopulation structure;
2) this rate depends linearly on the number of populations in the community. These conditions, characteristic of the Lotka–Volterra equation, represent a simplified hypothesis about the nature of the interactions between populations in the community.

This hypothesis, known as the principle of pair interactions, suggests the additivity of each population contribution to the relative growth rate, which is reasonably well founded biologically. However, the linear nature of this contribution is much worse in the processes occurring in biological communities, and can be taken into account in approximating the equilibrium state in some neighborhood [1]. In this connection, the study of the model (1) can be considered as an important stage preceding the study of the models that are generalizations of the model (1).

A generalization of the model (1) is a model of the following form:

$$\dot{x_i} = x_i \left(a_i - \sum_{j=1}^{n} p_{ij} f_j(x_j) \right), \quad i = 1, \ldots, n, \tag{2}$$

where $f_i(R^1 \to R^1) \in C^1$, $f_i(0) = 0$, $\dfrac{\partial f_i(x_i)}{\partial x_i} > 0$ with $x_i \geqslant 0$.

The stability conditions for solutions of the model (1), (2) on the basis of the Lyapunov functions method are obtained in [1]. The stability conditions on the basis of the divergent method are obtained in [3] for indicated models. The transition is possible from the deterministic model (1), (2) to different types of the corresponding nondeterministic models.

3. Design of Nondeterministic n-dimensional Lotka–Volterra Models and Stability Analysis Based on the Reduction Principle

The nonlinear model (1) is presented in the form of the vector equation

$$\dot{x} = f(x), \tag{3}$$

where $x = (x_1, x_2, \ldots, x_n)$, $f(x) = (f_1, f_2, \ldots, f_n) = (x_1(a_1 - p_{11}x_1 - \ldots - p_{1n}x_n), \ldots, x_n(a_n - p_{n1}x_1 - \ldots - p_{nn}x_n))$, $x \in R^n_+$, R^n_+ — n-fold Cartesian product of the set R_+ on itself, $R_+ = [0, \infty)$, $f : R^n_+ \to R^n_+$.

For the model (3) the coefficients a_i and p_{ij}, i, $j = 1, \ldots, n$, can take different values from the corresponding intervals $[\alpha_{i_1}, \alpha_{i_2}]$ and $[\gamma_{ij_1}, \gamma_{ij_2}]$ taking into account the ecological meaning accordingly. The transition from the model (3) to the finite-dimensional differential inclusion is the following

$$\dot{x}_1 \in x_1 \left(a_1 - p_{11}x_1 - \ldots - p_{1n}x_n\right), \ldots, \dot{x}_n \in x_n \left(a_n - p_{n1}x_1 - \ldots - p_{nn}x_n\right). \tag{4}$$

The model (4) in the vector form is presented as follows:

$$\dot{x} \in F(x), \tag{5}$$

where

$$F(x) = \{f(x) | a_i \in A_i, p_{ij} \in C_{ij}\}, \; A_i ::= [\alpha_{i_1}, \alpha_{i_2}], \; C_{ij} ::= [\gamma_{ij_1}, \gamma_{ij_2}], \; F : R^n_+ \to 2^{R^n_+}.$$

The introduced sets A_i and C_{ij} define the sets of values of the corresponding parameters a_i and p_{ij}.

Subsets $\{A_i\}_\alpha = \{a_i | \mu_{A_i}(a_i) \geqslant \alpha\}$ and $\{C_{ij}\}_\alpha = \{p_{ij} | \mu_{C_{ij}}(p_{ij}) \geqslant \alpha\}$ represent the narrower sets that we obtain when we take into account the additional conditions $\alpha \in (0, l]$, that affect the interaction of the components and, consequently, the stability of the model (3). Then equation (3) can be replaced by a fuzzy finite-dimensional differential equation

$$\dot{x} = F(x), \tag{6}$$

where $F : Z_+^n \to P(R_+^n)$, $P(R_+^n)$ is the set of all fuzzy subsets of R_+^n.

The differential inclusion corresponding to the equation (6) has the form $\dot{\varphi} \in F_\alpha(\varphi)$, where $\alpha \in (0, 1]$, $F_\alpha(\varphi) = \{f(\varphi(t)) | a_i \in \{A_i\}_\alpha, p_{ij} \in \{C_{ij}\}_\alpha\}$.

The following stability conditions of the differential inclusion (5) and the fuzzy equation (6) we formulate by means of the principle of reduction [7, 8] and by means of the transition from model (1) to models (5) and (6):

1) if there is a Lyapunov function V for the closed set $M \subset R_+^n$ regarding the inclusion (5), such that the inequality $D_+V(x) \leqslant 0 \ \forall x \in B(M, r)$ is satisfied, where $D_+V(x) = \sup DV(x)$ is upper derivative of Lyapunov function, set $B(M, r)$ is r-neighborhood of the set M, then the set M is stable in small regarding this inclusion;
2) if the inequality $D_+V(x) \leqslant -w(e(x, M)) \ \forall x \in B(M, r)$ is satisfied, where function $w : B(M, r) \to R$ is the continuous and positive function in $R_+^n \setminus M$, then the set M is asymptotically stable in small regarding the inclusion (5);
3) if there is a Lyapunov function V regarding the equation (6) for the closed set $M \subset P(R_+^n)$, where $P(R_+^n)$ is the set of all fuzzy subsets of R_+^n, such that the inequality $D_+V_\alpha(x) \leqslant 0 \ \forall x \in B(M_\alpha, r)$ is satisfied at $\alpha \in (0, 1]$, then the set M α-stable regarding this equation;
4) if the inequality $D_+V_\alpha(x) \leqslant -w_\alpha(e(x, M_\alpha)) \ \forall x \in B(M_\alpha, r)$ is satisfied, where $w_\alpha : (0, r) \to R$ is the continuous and positive function, then the set M is α-asymptotically stable regarding the equation (6).

In this work we consider the generalization of the model (3) to the stochastic case, namely, the transition is carried out from the equation (3) to the corresponding stochastic differential equation

$$\dot{x} = S(x), \tag{7}$$

where $S(x)$ is the stochastic function. By means of the principle of reduction we formulate the stability conditions of the fuzzy equation (6) and the stochastic equations (7).

It is shown that if the trivial solution of a fuzzy equation (6) is α-stable (asymptotically α-stable) for every $\alpha \in (0, 1]$, then the trivial solution of the corresponding stochastic equation (7) is stable on probability (asymptotically stable on probability). In addition, the conditions of almost surely stability and stability on average we give by the aid of the principle of reduction. The comparative analysis of the qualitative properties of the deterministic and stochastic models is given on the basis of the obtained sufficient stability conditions.

System (2) is represented as a nonlinear vector equation

$$\dot{x} = g(x),$$

where

$$g(x) = (g_1, g_2, \ldots, g_n) = (x_1(a_1 - p_{11}f_1(x_1) - \ldots - p_{1n}f_n(x_n)), \ldots \\ \ldots, x_n(a_n - p_{n1}f_1(x_1) - \ldots - p_{nn}f_n(x_n))),$$

$$g : R_+^n \to R_+^n.$$

We consider the transition from this vector equation to the nondeterministic models described by differential inclusion, fuzzy and stochastic differential equations. This transition is similar to the transition from the vector equation (3) to the models (5)–(7). The stability conditions of the indicated differential inclusion, fuzzy differential equation and stochastic differential equation are obtained using the reduction principle.

4. Design of the Self-Consistent n-dimensional Lotka–Volterra Stochastic Models

The synthesis of some models of population dynamics on the basis of the method of construction of self-consistent stochastic models [14] is implemented in [5,6]. According to the main idea of the method it is possible for the system under consideration to describe the scheme of interaction in the form of symbolic representation of all possible interaction between the system elements. The operators of the system state and the operator of change of the system state are used for this purpose. Then we give the intensities of transitions and master equation, for which we can obtain an approximate Fokker–Planck equation by the aids of formal series expansion. It is not difficult to transit from the Fokker–Planck equation to the equivalent stochastic differential equation in Langevin form:

$$\mathrm{d}x(t) = a(t, x(t))\mathrm{d}t + b(t, x(t))\mathrm{d}W, \tag{8}$$

where $x(t) \in R^n$ is the vector of the system state, $a(t, x(t))$ is the vector of demolition, $b(t, x(t))$ is the diffusion matrix. In addition, in equation (8) $W \in R^n$ is the standard n-dimensional Wiener process.

In practice, the stochastic differential equation can be written immediately after the representation of the interaction scheme. It is connected with the fact that for the obtained coefficients of the Fokker–Planck equation it is necessary to know only the intensities of transitions and operators of state changes.

We present the scheme of interaction elements for the system (1) in the form:

$$\begin{aligned} X_i &\xrightarrow{p_{i0}} 2X_i, \\ X_i + X_j &\xrightarrow{p_{ij}} X_j, \end{aligned} \tag{9}$$

where $i, j = 1, \ldots, n$. Thus, the scheme (9) describes the system of n species in which individuals can interact $n(n + 1)$ various ways.

The first row of the scheme of interaction corresponds to natural reproduction i-th species in the absence of other factors. The second row corresponds to intraspecific competition at $i = j$, corresponds to interspecific competition at $i \neq j$.

The operator of state change is presented in the form:

$$R = \{R_{lk},\, l = 1, \ldots, n,\, k = 0, \ldots, n\},$$

where

$$R_{ij} = \begin{cases} (0, \ldots, \overbrace{1}^{i\text{-th}}, \ldots, 0), & l = 1, \ldots, n, \quad k = 0, \\ (0, \ldots, \underbrace{-1}_{i\text{-th}}, \ldots, 0), & l = 1, \ldots, n, \quad k = 1, \ldots, n. \end{cases}$$

The state of the system can be described by means of vector $x = (x_1, \ldots, x_n)$. The following relations are given for intensities of transitions from the state x to the state $x + R$ in the unit of the time:

$$s_{l,k}(x) = p_{lk} x_l x_k,$$

where $l = 1, \ldots, n$, $k = 0, \ldots, n$, and let $x_k = 1$ if $k = 0$.

Let us present Fokker–Planck equation corresponding to the model in the form:

$$\partial_t P(x,t) = -\sum_{i=1}^{n} \partial_{x_i} \left[A_i(x) P(x,t) \right] + \frac{1}{2} \sum_{i,j=1}^{n} \partial_{x_i} \partial_{x_j} \left[B_{ij}(x) P(x,t) \right],$$

where

$$A_i(x) = \sum_{j=1}^{n} R_{ij} s_{ij}(x) = p_{i0} x_i - \sum_{j=1}^{n} p_{ij} x_i x_j,$$

$$B_{ij}(x) = \sum_{j=0}^{n} R_{ij} \left(R_{ij} \right)^T s_{ij}(x) = p_{i0} x_i + \sum_{j=1}^{n} p_{ij} x_i x_j, \text{ and } B_{ij} = 0, i \neq j.$$

We have the following relations for the coefficients of stochastic equation (8) and coefficients of Fokker–Planck equation:

$$a(x) = A(x),$$
$$b(x) = B(x) B(x)^T.$$

The obtaining of analytical solution for the constructed self-consistent stochastic Lotka–Volterra model is difficult, however, for special cases of the general n-dimensional model, it is possible to conduct a numerical experiment using the developed software package for the solution systems of stochastic differential equations [16, 17]. In the future, it is planned to conduct a numerical analysis to obtain numerical solutions for the obtained models, as well as to reveal the influence of stochastics on the behavior of the system.

The investigation of the obtained stochastic differential equation in Langevin form allows us to study the influence of stochastics on the behavior of the considered system. This approach to the construction and analysis of nonlinear models can serve to the solving of problems aimed at the comparative analysis of deterministic and stochastic models.

Note that the technique of design of the self-consistent stochastic models can be applied to the system (2), but for this it is necessary to specify the form of the functions $f_j(x_j)$ taking into account the physical sense. In the problems of constructing of self-consistent stochastic models that generalize models (1) and (2), the consideration of two-dimensional, three-dimensional and four-dimensional models is of prime interest, and it becomes necessary to compare stability properties in deterministic and stochastic cases.

5. Conclusions

The principle of reduction allowed us to obtain the conditions of stability of the multidimensional model of the population dynamics with the transition to the differential inclusion, fuzzy and stochastic differential equations. The specified transition takes into account the changing parameters of the model and allows us to perform a comparative analysis of the properties of the models based on the principle of reduction. The stability conditions can be used to study the population dynamics models. The application of self-consistent stochastic models construction method for Lotka–Volterra multidimensional systems allows us to estimate the impact of the introduction of stochastics onto the behavior of these systems.The obtained results are aimed at further development of methods of design and stability analysis of stochastic models.

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Построение и анализ устойчивости недетерминированных многомерных моделей динамики популяций

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Рассмотрены многомерные модели популяционной динамики, являющиеся обобщениями модели Лотки–Вольтерра на случай взаимодействия конечного числа популяций. Детерминистическое описание моделей даётся системами обыкновенных нелинейных дифференциальных уравнений, представленными в работе в виде многомерных векторных дифференциальных уравнений. Качественные свойства указанных моделей достаточно хорошо изучены с помощью методов Ляпунова. Однако при детерминистическом описании моделей не учитываются вероятностные факторы, влияющие на поведение моделей. В недетерминистическом случае новые подходы к моделированию и анализу устойчивости представляют теоретический и прикладной интерес.

В настоящей работе рассмотрены способы построения многомерных недетерминированных моделей взаимодействия популяций. Первый способ связан с переходом от векторного нелинейного обыкновенного дифференциального уравнения к соответствующим векторным дифференциальным включениям, нечётким и стохастическим дифференциальным уравнениям. На основе принципа редукции, позволяющего свести задачу об устойчивости решений дифференциального включения к задаче об устойчивости решений других типов уравнений, получены условия устойчивости для построенных моделей. Второй способ связан с методикой построения самосогласованных стохастических моделей. На основе этой методики получена схема взаимодействия, которая включает в себя символическую запись возможных взаимодействий между элементами системы. С помощью операторов состояния системы и оператора изменения состояния системы описана структура многомерных стохастических моделей Лотки–Вольтерра, и осуществлён переход к соответствующим векторным уравнениям Фоккера–Планка. Сформулированы правила перехода к многомерному стохастическому дифференциальному уравнению в форме Ланжевена. Для моделей, являющихся конкретизациями изучаемых общих моделей, возможно проведение численного эксперимента с применением разработанного программного комплекса для решения систем стохастических дифференциальных уравнений. Описанный подход к моделированию стохастических систем может найти применение в задачах сравнения качественных свойств моделей в детерминированном и стохастическом случаях. Полученные результаты направлены на развитие методов анализа недетерминированных нелинейных моделей.

Ключевые слова: модель популяционной динамики, устойчивость, дифференциальные включения, стохастическая модель, принцип редукции

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