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Magnetic Excitations of Graphene in 8-Spinor Realization of Chiral Model

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The simplest scalar chiral model of graphene suggested earlier and based on the $SU(2)$ order parameter is generalized by including 8-spinor field as an additional order parameter for the description of spin (magnetic) excitations in graphene. As an illustration we study the interaction of the graphene layer with the external magnetic field. In the case of the magnetic field parallel to the graphene plane the diamagnetic effect is predicted, that is the weakening of the magnetic intensity in the volume of the material. However, for the case of the magnetic field orthogonal to the graphene plane the strengthening of the magnetic intensity is revealed in the central domain (at small r). Thus, the magnetic properties of the graphene prove to be strongly anisotropic.

Key words and phrases: graphene, spin excitations, chiral model, 8-spinor

1. Introduction. Scalar Chiral Model

Since the very discovery of mono-atomic carbon layers called graphenes [1,2] this material attracted deep interest of researchers due to its extraordinary properties concerning magnetism, stiffness and high electric and thermal conductivity [3–5]. The interesting connection of graphene was revealed with nano-tubes and fullerenes [6]. A very simple explanation of these unusual properties of graphene was suggested in [7], where the idea of massless Dirac-like excitations of honeycomb carbon lattice was discussed, the latter one being considered as a superposition of two triangular sub-lattices. Some phenomenological development of this idea was realized in [8, 9].

As is well known, the carbon atom possesses of four valence electrons in the so-called hybridized sp^2-states, the one of them being "free" in graphene lattice and all others forming sp-bonds with the neighbors. It appears natural to introduce scalar a_0 and 3-vector $\mathbf{a}$ fields corresponding to the s-orbital and the p-orbital states of the "free" electron respectively. These two fields can be combined into the unitary matrix $U \in SU(2)$ considered as the order parameter of the model in question, the long-wave approximation being adopted, i. e.

$$U = a_0\,\tau_0 + \imath\,\mathbf{a}\cdot\tau, \qquad (1)$$

where τ_0 is the unit 2×2-matrix and τ are the three Pauli matrices, with the $SU(2)$-condition

$$a_0^2 + \mathbf{a}^2 = 1 \qquad (2)$$

being imposed. It is convenient to construct via the differentiation of the chiral field (1) the so-called left chiral current

$$l_\mu = U^{+}\partial_\mu U, \qquad (3)$$

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the index μ running $0, 1, 2, 3$ and denoting the derivatives with respect to the time $x^0 = ct$ and the space coordinates x^i, $i = 1, 2, 3$. Then the simplest Lagrangian density reads

$$\mathcal{L} = -\frac{1}{4} I \,\mathrm{Sp}(l_\mu l^\mu) - \frac{1}{2}\lambda^2 \mathbf{a}^2 \tag{4}$$

and corresponds to the sigma-model approach in the field theory with the mass term. Here the constant model parameters I and λ are introduced. Comparing the Lagrangian density (4) with that of the Landau–Lifshits theory corresponding to the quasiclassical long-wave approximation to the Heisenberg magnetic model [10], one can interpret the parameter I in (4) as the exchange energy between the atoms (per spacing).

Inserting (1) into (3) and (4) and taking into account the condition (2), one easily finds the following Lagrangian density:

$$\mathcal{L} = \frac{1}{2} I \left(\partial_\mu a_0 \,\partial^\mu a_0 + \partial_\mu \mathbf{a} \cdot \partial^\mu \mathbf{a}\right) - \frac{1}{2}\lambda^2 \mathbf{a}^2. \tag{5}$$

For the case of small $\mathbf{a}$-excitations the equations of motion generated by (5) read as

$$\Box\, \mathbf{a} - (\lambda^2/I)\mathbf{a} = 0$$

and correspond to the dispersion law

$$\omega = k_0 c, \quad k_0^2 = \mathbf{k}^2 + \lambda^2/I,$$

which in the high-frequency approximation has the linear photon-like form.

First we begin with the static $1D$ configuration corresponding to the ideal graphene plane, the normal being oriented along the z-axis. In this case the order parameter has the form

$$U = \exp(\imath \Theta \tau_3), \quad \Theta = \Theta(z),$$

with the Lagrangian density being

$$\mathcal{L} = -\frac{I}{2}\Theta'^2 - \frac{\lambda^2}{2}\sin^2\Theta. \tag{6}$$

The Lagrangian (6) yields the equation of motion

$$2I\Theta'' - \lambda^2 \sin 2\Theta = 0. \tag{7}$$

The solution to (7) satisfying the natural boundary conditions

$$\Theta(-\infty) = \pi, \quad \Theta(+\infty) = 0$$

has the well-known kink-like (or domain-wall) form:

$$\Theta_0(z) = 2 \arctan \exp(-z/\ell), \tag{8}$$

with the characteristic thickness (length parameter)

$$\ell = \sqrt{I}/\lambda \tag{9}$$

and the energy per unit area

$$E = \frac{1}{2}\int \mathrm{d}z \left(I{\Theta_0'}^2 + \lambda^2 \sin^2\Theta_0\right) = 2\lambda\sqrt{I}.$$

2. Spinor Chiral Model of Graphene

Now we intend to include in the model the interaction with the electromagnetic field for the description of conductivity and magnetic properties. To this end, we suggest 8-spinor generalization of the scalar chiral model and use the gauge invariance principle for introducing the electromagnetic interaction. The motivation for such a generalization is the following.

For the description of spin and quasi-spin excitations in graphene, the latter ones corresponding to independent excitation modes of the two triangular sub-lattices of graphene, we introduce the two Dirac spinors ψ_1, ψ_2 and consider the combined spinor field Ψ as a new order parameter:

$$\Psi = \xi \otimes (\psi_1 \oplus \psi_2)\,, \tag{10}$$

where ξ stands for the first column of the unitary matrix (1). The Lagrangian density of the model

$$\mathcal{L} = \frac{I}{2}\overline{D_\mu \Psi} P\, D^\mu \Psi + \frac{\lambda^2}{2}\mathbf{a}^2 j_\mu j^\mu + \imath \mu_0 \mathbf{a}^2 \overline{\Psi} \sigma_{\mu\nu} F^{\mu\nu}\Psi \tag{11}$$

contains the projector $P = \gamma^\nu j_\nu$ on the positive energy states, where $j_\mu = \overline{\Psi}\gamma_\mu \psi$, $\mu = 0, 1, 2, 3$, designates the Dirac current, $\overline{\Psi} = \Psi^+\gamma_0$ and γ_μ stands for the Dirac matrix. The model contains the two constant parameters of the previous scalar model: the exchange energy I per lattice spacing and some characteristic inverse length $\sqrt{\lambda}$. The interaction with the electromagnetic field is realized through the extension of the derivative:

$$D_\mu = \partial_\mu - \imath e_0 A_\mu \Gamma_e,$$

with $e_0 > 0$ being the coupling constant and $\Gamma_e = (1 - \tau_3)/2$ being the charge operator chosen in accordance with the natural boundary condition at infinity: $a_0(\infty) = 1$. However, the additional interaction term of the Pauli type should be added to take into account the proper magnetic moments of the electrons. Here

$$\sigma_{\mu\nu} = [\gamma_\mu, \gamma_\nu]/4, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu,$$

and $\mu_0 > 0$ denotes the Bohr magneton per lattice spacing cubed.

Let us consider as an illustration the interaction of the mono-atomic carbon layer $z = 0$ with the static uniform magnetic field $\mathbf{B}_0$ oriented along the x axis. We introduce first the vector potential $A_y = A(z)$, with the intensity of the magnetic field being

$$B_x = B(z) = -A'(z)$$

and the natural boundary condition at infinity: $A \to -B_0\, z$.

The model in question admits the evident symmetry $\psi_1 \Leftrightarrow \psi_2$, γ_0 — invariance $\Psi \Rightarrow \gamma_0\Psi$ and also the discrete symmetry:

$$\psi_i \Leftrightarrow \psi_i^*; \quad a_{2,3} \Rightarrow -a_{2,3}.$$

Therefore, one can introduce the chiral angle $\Theta(z)$:

$$a_0 = \cos\Theta, \quad a_1 = \sin\Theta$$

and the real 2-spinor $\varphi(z) = \mathrm{col}(u, -u)$, where $\psi_1 = \psi_2 = \mathrm{col}(\varphi, -\varphi)$. As a result the new Lagrangian density takes the form:

$$\mathcal{L} = -2IU'^2 - 8IU^2(\Theta'^2 + e_0^2 A^2 \sin^2\Theta) - 4U\sin^2\Theta(2\lambda^2 U + \mu_0 A') - A'^2/(8\pi), \tag{12}$$

where the new variable is introduced: $U = |\varphi|^2 = 2u^2$. Taking into account that $j^2 = 16U^2$, one can deduce from (12) and the boundary conditions at infinity:

$$j^2(\infty) = 1, \quad \Theta(\infty) = 0, \quad A'(\infty) = -B_0$$

the following "energy" integral:

$$E = -2IU'^2 - 8IU^2(\Theta'^2 - e_0^2A^2\sin^2\Theta) + 8\lambda^2U^2\sin^2\Theta - A'^2/(8\pi) = -B_0^2/(8\pi),$$

that implies the Hamilton–Jacobi equation for the "action" S:

$$\frac{1}{8I}\left(\frac{\partial S}{\partial U}\right)^2 + \frac{1}{32\,I\,U^2}\left(\frac{\partial S}{\partial\Theta}\right)^2 + 2\pi\left(\frac{\partial S}{\partial A} + 4\mu_0 U\sin^2\Theta\right)^2 = \\ = \frac{B_0^2}{8\pi} + 8\,U^2\sin^2\Theta\left(\lambda^2 + I\,e_0^2A^2\right). \quad (13)$$

Here the following definitions of the Jacobi momentums are used:

$$\frac{\partial S}{\partial U} = -4\,I\,U'; \quad \frac{\partial S}{\partial\Theta} = -16\,I\,U^2\Theta'^2; \quad \frac{\partial S}{\partial A} = -4\mu_0 U\sin^2\Theta - -A'/(4\pi). \quad (14)$$

Let us study the behavior of solution to the equations (13) and (14) in the asymptotic domain $z \to \infty$, where $A \approx -B_0 z$. In the first approximation one gets:

$$S \approx \left(\frac{B_0}{4\pi} - 8\,e_0 I\,U^2\sin^2\Theta\right)A. \quad (15)$$

Inserting (15) into (14), one derives the differential equation

$$U' = 4\,U\Theta'\tan\Theta$$

with the evident integral $4\,U = \cos^{-4}\Theta$ corresponding to the boundary condition $U(\infty) = 1/4$. In view of (14) this fact permits one to obtain the equation for $\Theta(z)$:

$$\frac{2\Theta'}{\sin 2\Theta} = e_0 A \approx -e_0 B_0 z$$

with the solution of the form:

$$\tan\Theta = \tan\Theta_0\exp\left(-e_0B_0z^2/2\right), \quad (16)$$

where Θ_0 stands for the integration constant. Finally, combining (16) and the last relation in (14), one can find the magnetic field intensity in the asymptotic domain $z \to \infty$:

$$B = -A' \approx B_0 - 2\pi(e_0I - 2\mu_0)\tan^2\Theta_0\exp\left(-e_0B_0z^2\right). \quad (17)$$

As can be seen from (17), the effect of weakening of the magnetic field is revealed for the positive value of the constant $e_0I - 2\mu_0$, this effect being similar to that of London "screening" caused by the second term in the electromagnetic current:

$$J_\mu = e_0I\mathrm{Im}\left(\overline{\Psi}P\Gamma_e\partial_\mu\Psi\right) - e_0^2Ij^2(a_1^2 + a_2^2)A_\mu + 2\imath\mu_0\partial^\nu\left(\mathbf{a}^2\overline{\Psi}\sigma_{\mu\nu}\Psi\right). \quad (18)$$

The current (18) contains beyond the standard conduction term, the diamagnetic current and the Pauli magnetization-polarization one. As follows from (17), for the negative value of the constant $e_0 I - 2\mu_0$ the paramagnetic behavior of the material takes place.

3. Interaction with Magnetic Field Orthogonal to Graphene Plane

Let us now study the case with the orientation of the magnetic field $\mathbf{B}_0$ along the z-axis. Using the cylindrical coordinates r, ϕ, z, we introduce the vector potential $A_\phi = A$, with the intensity of the magnetic field being

$$B_z = \partial_r(r\,A)/r, \quad B_r = -\partial_z A,$$

and the natural boundary condition at infinity being imposed: $A(z \to \infty) = B_0 r/2$.

The model in question admits the evident symmetry $\psi_1 \Leftrightarrow \psi_2$ and γ_0 — invariance $\Psi \Rightarrow \gamma_0 \Psi$, that permits one to introduce 2-spinor φ by putting

$$\psi_1 = \psi_2 = \mathrm{col}(\varphi, \varphi), \quad \varphi = \mathrm{col}(w, u).$$

To simplify the calculations, let us suppose the smallness of the radial magnetic field:

$$B_r \ll B_z.$$

In this approximation the new discrete symmetry holds:

$$\varphi \Rightarrow -\sigma_3 \varphi, \quad w \Rightarrow -w, \quad u \Rightarrow u^*, \quad a_{2,3} \Rightarrow -a_{2,3},$$

that permits one to introduce the chiral angle Θ:

$$a_0 = \cos\Theta, \quad a_1 = \sin\Theta$$

and consider the axially-symmetric configuration:

$$u = u(r,z), \quad \Theta = \Theta(r,z).$$

As a result the new Lagrangian density takes the form:

$$\mathcal{L} = -8\,I\left[R^2(\partial_\perp\Theta)^2 + \frac{1}{4}(\partial_\perp R)^2 + e_0^2 R^2 A^2 \sin^2\Theta\right] - 8\lambda^2 R^2 \sin^2\Theta + \\ + 8\,\mu_0 R \sin^2\Theta \frac{1}{r}\partial_r(r\,A) - \frac{1}{8\pi}\left[\frac{1}{r^2}\big(\partial_r(r\,A)\big)^2 + (\partial_z A)^2\right], \quad (19)$$

where the new variable is introduced: $R = u^2$ and $\partial_\perp$ signifies the differentiation with respect to r and z. The equations of motion corresponding to (19) read:

$$I\left[\frac{1}{r}\partial_r(r\partial_r R) + \partial_z^2 R - 4\,R(\partial_\perp\Theta)^2 - 4\,e_0^2 R\,A^2\sin^2\Theta\right] = \\ = 2\sin^2\Theta\left[2\lambda^2 R - \mu_0\frac{1}{r}\partial_r(r\,A)\right], \quad (20)$$

$$I\left[\frac{2}{r}\partial_r(r\,R^2\partial_r\Theta) + 2\partial_z(R^2\partial_z\Theta) - e_0^2 R^2 A^2 \sin 2\Theta\right] =$$

$$= R\sin 2\Theta\left[\lambda^2 R - \mu_0\frac{1}{r}\partial_r(r\,A)\right], \quad (21)$$

$$\frac{1}{4\pi}\left[\frac{1}{r}\partial_r(r\partial_r A) + \partial_z^2 A - \frac{A}{r^2}\right] = 16\,I\,e_0^2 R^2 A\sin^2\Theta + 8\mu_0\partial_r(R\sin^2\Theta). \quad (22)$$

Let us now search for solutions to the equations (20), (21), (22) in the asymptotic domain $z\to\infty$, where

$$\Theta\to 0;\quad R = 1/4+\zeta,\ \ \zeta\to 0;\quad A = B_0 r/2+\alpha,\ \ \alpha\to 0.$$

Thus, the equation (21) takes the form:

$$I\left[\frac{1}{r}\partial_r(r\partial_r\Theta) + \partial_z^2\Theta - \frac{1}{4}e_0^2 B_0^2 r^2\Theta\right] = \Theta(\lambda^2 - 4\mu_o B_0),$$

and its solution can be found by separating variables:

$$\Theta = \Theta_0\exp\bigl(-\nu r^2 - \kappa z\bigr),\quad \Theta_0 = \text{const}, \quad (23)$$

with the following constant parameters:

$$\nu = e_0 B_0/4;\quad I\kappa^2 = B_0(e_0 I - 4\mu_0) + \lambda^2. \quad (24)$$

Inserting (23) into (20) and (22), one gets the inhomogeneous equations for ζ and α:

$$\frac{1}{r}\partial_r(r\partial_r\zeta) + \partial_z^2\zeta = (\partial_\perp)^2 + \left[\frac{1}{4}e_0^2 B_0^2 r^2 + \frac{1}{I}(\lambda^2 - 2\mu_0 B_0)\right]\Theta^2, \quad (25)$$

$$\frac{1}{r}\partial_r(r\partial_r\alpha) + \partial_z^2\alpha - \frac{\alpha}{r^2} = 2\pi e_0 B_0(e_0 I - 4\mu_0)r\Theta^2 \equiv \delta r\Theta^2 \quad (26)$$

with the solutions of the form:

$$\zeta = \Theta_0^2\exp\bigl(-2\nu r^2 - 2\kappa z\bigr)N(r);\quad \alpha = \delta\Theta_0^2\exp\bigl(-2\nu r^2 - 2\kappa z\bigr)K(r), \quad (27)$$

where the radial functions $N(r)$ and $K(r)$ satisfy the following equations:

$$N'' + N'\left(\frac{1}{r} - 8\nu r\right) + N\left[2\,B_0\left(e_0 - 8\frac{\mu_0}{I}\right) + 4\frac{\lambda^2}{I} + e_0^2 B_0^2 r^2\right] =$$
$$= \frac{1}{2}e_0^2 B_0^2 r^2 + e_0 B_0 + \frac{2}{I}(\lambda^2 - 3\mu_0 B_0), \quad (28)$$

$$K'' + K'\left(\frac{1}{r} - 8\nu r\right) + K\left(4\kappa^2 - 8\nu + 16\nu^2 r^2 - \frac{1}{r^2}\right) = r. \quad (29)$$

Let us now estimate the magnetic intensity:

$$B_z = B_0 + b_z,\quad b_z = \frac{1}{r}\partial_r(r\alpha),\quad B_r = b_r = -\partial_z\alpha.$$

Taking into account that due to (29) $K\approx(e_0^2 B_0^2 r)^{-1}$ as $r\to\infty$, one gets from (27):

$$b_z = -2\pi(e_0 I - 4\mu_0)\Theta_0^2\exp\bigl(-2\nu r^2 - 2\kappa z\bigr), \quad (30)$$

$$b_r = \frac{4\pi\kappa}{e_0 B_0 r}(e_0 I - 4\mu_0)\Theta_0^2 \exp\bigl(-2\nu r^2 - 2\kappa z\bigr), \tag{31}$$

However, at small $r \to 0$ one finds from (29) that $K \approx r^3/8$, and therefore the intensity of the magnetic field reads:

$$b_z = \pi e_0 B_0 (e_0 I - 4\mu_0)\Theta_0^2 r^2 \exp\bigl(-2\nu r^2 - 2\kappa z\bigr), \tag{32}$$

$$b_r = \frac{\pi\kappa}{2} e_0 B_0 (e_0 I - 4\mu_0)\Theta_0^2 r^3 \exp\bigl(-2\nu r^2 - 2\kappa z\bigr). \tag{33}$$

As can be seen from (30)–(33) , according to the sign of the multiplier $e_0 I - 4\mu_0$ our graphene material reveals diamagnetic or paramagnetic behavior. Therefore, it would be interesting to obtain numerical estimates for the parameters of the model. In view of definitions adopted one has

$$e_0 = \frac{e}{\hbar c}, \quad \mu_0 = \frac{e\hbar}{2\, m_e c\, a^3}, \quad I = \frac{E_{\text{exch}}}{a},$$

where the exchange energy is usually adopted as $E_{\text{exch}} = 2.9$ eV and the lattice spacing as $a = 3.56 \cdot 10^{-8}$ cm, with e being the absolute value of the electron charge. Finally, one can find the following numerical values:

$$e_0 I = 2 \cdot 10^3 \text{ Gauss}, \quad \mu_0 = 2 \cdot 10^2 \text{ Gauss}.$$

It means that the parameter $e_0 I - 4\mu_0$ is positive and the weakening of the magnetic field inside the graphene is predicted in accordance with (17), (30) and (31) for large r and its strengthening for small r in accordance with (32) and (33).

In view of the importance of the latter conclusion it would be desirable to investigate the magnetic field behavior in the central domain of the graphene material, i. e. at small r but arbitrary z. To this end, we consider the extrapolation of the configuration (23) to the domain wall structure of the form:

$$\Theta = 2 \arctan\left[\exp\bigl(-2\nu r^2 - 2\kappa z\bigr)\right]. \tag{34}$$

Later it will be shown that this approximation is valid in the small field limit $B_0 \to 0$. To start with, we insert (34) and $A = B_0 r/2 + \alpha$, $R \approx 1/4$ into (23), this amounting to the equation:

$$\frac{1}{r}\partial_r(r\partial_r\alpha) + \partial_z^2\alpha - \frac{\alpha}{r^2} = 2\pi r\, e_0 B_0 \sin^2\Theta[e_0 I - 4\mu_0 \tanh\bigl(\nu r^2 + \kappa z\bigr)] \equiv 2\pi r\, j. \tag{35}$$

Solution to the equation (35) satisfying boundary condition $\alpha(r = 0) = 0$ can be found by Green's function method:

$$\alpha = \int_{-\infty}^{\infty} \mathrm{d}z' \int_{-\infty}^{\infty} \mathrm{d}s\, \exp[\imath s(z - z')] \int_0^r \mathrm{d}r'\, r'\, j' \left[I_1(s\,r)K_1(s\,r') - K_1(s\,r)I_1(s\,r')\right], \tag{36}$$

where I_1 and K_1 stand for the modified Bessel functions of the imaginary argument. Taking into account their asymptotic behavior as $x \to 0$:

$$I_1(x) \approx x/2, \quad K_1(x) \approx x^{-1},$$

one finds from (35) and (36) that at small r:

$$\alpha(r,z) \approx \frac{\pi r^3 e_0 B_0}{4\cosh^2(\nu r^2+\kappa z)}\left[e_0 I - 4\mu_0 \tanh\bigl(\nu r^2+\kappa z\bigr)\right], \tag{37}$$

that confirms the paramagnetic behavior of the graphene in the central domain.

Finally, inserting (37) and $R = 1/4 + \zeta$, $A \approx B_0 r/2$ into (20), one gets the equation:

$$\frac{1}{r}\partial_r(r\partial_r\zeta) + \partial_z^2\zeta = \sin^2\Theta\left[\frac{2}{I}(\lambda^2 - 3\mu_0 B_0) + e_0 B_0\left(1 + \frac{1}{2}e_0 B_0 r^2\right)\right] \equiv j_1. \tag{38}$$

Solution to (38) can be found also by Green's function method along similar lines as for (35):

$$\zeta = \frac{1}{2\pi}\int\limits_{-\infty}^{\infty} \mathrm{d}z' \int\limits_{-\infty}^{\infty} \mathrm{d}s\, \exp[\imath s(z-z')] \int\limits_{0}^{r} \mathrm{d}r'\, r'\, j_1' \left[K_0(s\,r)I_0(s\,r') - I_0(s\,r)K_0(s\,r')\right]. \tag{39}$$

Taking into account the asymptotic behavior of the Bessel functions as $x \to 0$:

$$I_0(x) \approx 1 + x, \quad K_0(x) \approx \log[2/x],$$

one finds from (38) and (39) that in the central domain

$$\zeta(r,z) \approx -\frac{r^2}{4\cosh^2(\nu r^2+\kappa z)}\left[\frac{2}{I}(\lambda^2 - 3\mu_0 B_0) + e_0 B_0\left(1 + \frac{1}{2}e_0 B_0 r^2\right)\right]. \tag{40}$$

Using (40), one can verify the validity of the approximation (34) as solution to (21) for the central domain in the small field limit, that is if the following strong inequalities hold:

$$\lambda^2/I \gg e_0 B_0, \quad e_0 B_0 r^2 \ll 1, \quad \kappa^2 r^2 \ll 1.$$

4. Conclusions

We analyzed the two phenomenological approches to the description of the graphene: the simplest scalar chiral model and its 8-spinor generalization. The scalar model admits very simple domain-wall solution describing one layer graphene configuration. On the opposite, the 8-spinor chiral model contains all previous results of the scalar model and also permits one to describe graphene interaction with the electromagnetic field. Magnetic excitations in graphene, for the case of the external magnetic field parallel to the graphene plane, reveal the evident diamagnetic effect: the weakening of the magnetic field within the graphene sample. As for the case of the magnetic field orthogonal to the graphene plane, the strengthening of the magnetic intensity inside the material is revealed in the central domain (at small r).

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Магнитные возбуждения графена в рамках 8-спинорной реализации киральной модели

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Простейшая киральная модель графена, предложенная ранее и основанная на $SU(2)$ параметре порядка, обобщается путем введения 8-спинорного поля как дополнительного параметра порядка для описания спиновых (магнитных) возбуждений в графене. В качестве иллюстрации мы изучаем взаимодействие графенового слоя с внешним магнитным полем. В случае магнитного поля, параллельного графеновой плоскости, предсказывается диамагнитный эффект, т. е. ослабление магнитной индукции внутри образца. Однако в случае магнитного поля, ортогонального графеновой плоскости, обнаруживается усиление магнитной индукции в центральной области (при малых r). Таким образом, магнитные свойства графена оказываются сильно анизотропными.

Ключевые слова: графен, спиновые возбуждения, киральная модель, 8-спинор

Литература

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