

---

# Физика

UDC 530.122:531.266.3

## Spherically Symmetric Solution of the Weyl–Dirac Theory of Gravitation and its Consequences

O. V. Babourova, B. N. Frolov, P. E. Kudlaev, E. V. Romanova

*Moscow Pedagogical State University, Moscow, Russia*

The Poincaré and Poincaré–Weyl gauge theories of gravitation with Lagrangians quadratic on curvature and torsion in post-Riemannian spaces with the Dirac scalar field is discussed in a historical aspect. The various hypotheses concerning the models of a dark matter with the help of a scalar field are considered. The new conformal Weyl–Dirac theory of gravitation is proposed, which is a gravitational theory in Cartan–Weyl spacetime with the Dirac scalar field representing the dark matter model. A static spherically symmetric solution of the field equations in vacuum for a central compact mass is obtained as the metrics conformal to the Yilmaz–Rosen metrics. On the base of this solution one considers a radial movement of an interplanetary spacecraft starting from the Earth. Using the Newton approximation one obtains that the asymptotic line-of-sight velocity in this case depends on the parameters of the solution, and therefore one can obtain, on basis of the observable data, the values of these parameters and then the value of a rest mass of the Dirac scalar field.

**Key words and phrases:** Dark matter, Dirac scalar field, Weyl–Dirac theory of gravitation, Cartan–Weyl spacetime, Yilmaz–Rosen metrics, spacecraft Earth flyby

### 1. Introduction

In [1] a gauge principle has been applied to Poincaré group that has resulted in construction a gauge theory of gravitation in a post-Riemannian space with curvature and torsion — a Riemann–Cartan space. In [1] a curvature scalar (generalized on a Riemann–Cartan space) was used as a Lagrangian of the theory. Such generalized theory of gravitation has been named the Einstein–Cartan theory of gravitation. The similar gauge theory of gravitation was proposed in [2–5] where it was offered to use as a Lagrangian along with a curvature scalar also quantities quadratic on curvature and torsion. Later in [6] the most general Lagrangian of a such kind in a Riemann–Cartan space was constructed containing ten arbitrary connection constants. A gauge theory of gravitation with quadratic Lagrangians has received the name the Poincaré gauge theory of gravitation, see Refs. in [7–9].

Then in [10, 11] it was advanced a conformally invariant generalization of the Poincaré gauge theory of gravitation was proposed which uses the method offered by Dirac in his well-known work [12]. The given method is based on using in a Lagrangian of the theory an additional scalar field which in [10] was named as a Dirac scalar field.

In [13, 14] the gauge theory of gravitation was constructed, proceeding from the requirement of the gauge invariance of the theory concerning the Poincaré–Weyl group, supplementing the Poincaré group by a group of spacetime stretching and compression (dilatations). It was shown that from this requirement spacetime obtains a geometrical structure of Cartan–Weyl space. Besides, in this theory a requirement appears of necessary existence of the additional scalar field having so fundamental geometrical status, as well as the metrics. The further development of the theory has shown, that the given scalar field coincides by the properties with the scalar field entered by Dirac in [12].

Further on the basis of the given result the theory of gravitation in a Cartan–Weyl space has been constructed [15–18], which generalizes the Einstein–Cartan and the Poincaré gauge theories of gravitation in presence of nonmetricity of the Weyl’s type and uses the

Dirac scalar field for supporting conformance of the theory. This generalized theory of gravitation is pertinent to be named *the Weyl–Dirac theory of gravitation*.

According to Gliner [19], the cosmological constant in the Einstein equation determines a vacuum energy density (dark energy). In the Weyl–Dirac conformal theory of gravitation, an effective cosmological constant appears, which value is determined by the Dirac scalar field. Application of the Weyl–Dirac theory to the early universe cosmology has allowed to find the solution of a well-known cosmological constant problem [18, 20–22], which represents the important problem of modern physics [23, 24].

Matos with co-authors [25] within the framework of a Riemann geometry in GR advanced a cosmological SFDM model, in which the dark matter was modelled with the help of a scalar field using a special kind of a potential. The full solution of a cosmological scenario was obtained. To the hypothesis that the scalar field can carry out the same problems, which are assigned to a dark matter, Capozziello with coauthors have joined [26]. In [26] the Yukawa interaction between the scalar field and substance is used.

In the monography [18] within the framework of the Weyl–Dirac theory of gravitation, the hypothesis has been stated that the Dirac scalar field in Cartan–Weyl space not only determines a size of the effective cosmological constant (dark energy density), but also plays a role of the basic component of a dark matter. Then the spherically symmetric solution of the Weyl–Dirac theory for the central mass in vacuum [18, 27, 28] was found.

Thus, the hypothesis about a possible modeling of dark matter by a scalar field is fruitful idea, which now is developed by some modern researchers. In the present work a new method of deriving the spherically symmetric solution of the conformal Weyl–Dirac theory of gravitation is elaborated, and also possible influence of dark matter on movement of space vehicles within the limits of Solar system is found out.

## 2. Lagrangian Density and Field Equations in Weyl–Dirac Theory

Let us consider [18] a connected 4D oriented differentiable manifold  $\mathcal{M}$  equipped with a metric  $\hat{g}$  of the index 3, a linear connection  $\Gamma^a_b$  and a volume 4-form  $\eta$ . Then a Cartan–Weyl space  $CW_4$  is defined as such manifold equipped with a curvature 2-form  $\mathcal{R}^a_b$ , a torsion 2-form  $\mathcal{T}^a$  and a nonmetricity 1-form  $\mathcal{Q}_{ab}$  obeying the Weyl condition

$$\mathcal{Q}_{ab} = \frac{1}{4}g_{ab}\mathcal{Q}. \quad (1)$$

Here  $\mathcal{Q}_{ab} = -\mathcal{D}g_{ab}$ , and  $\mathcal{D} = d + \Gamma \wedge \dots$  is the exterior covariant differential.

In [13, 14] the Poincaré–Weyl gauge theory of gravitation (PWTG) has been developed. The gauge field introduced by the subgroup of dilatations is named the dilatation field, its vector-potential is the Weyl 1-form, and quanta of this field can have nonzero rest mass. An additional scalar field  $\beta(x)$  is introduced in PWTG as an essential geometrical addendum to the metric tensor, the tangent space metrics being the form,

$$g_{ab} = \beta^{-2}g_{ab}^M, \quad (2)$$

where  $g_{ab}^M$  are the constant components of the Minkowski metric tensor.

The properties of the field  $\beta(x)$  coincide with those of the scalar field introduced by Dirac [12]. Some terms of the Dirac scalar field Lagrangian have structure of the Higgs Lagrangian and can cause an appearance of nonzero rest masses of particles [11].

On the basis of PWTG, the conformal theory of gravitation in Cartan–Weyl spacetime with Dirac scalar has been developed [15–18] with the Lagrangian density 4-form (in

exterior form formalism) [20, 21],

$$\mathcal{L} = \mathcal{L}_G + \mathcal{L}_{\text{mat}} + \beta^4 \Lambda^{ab} \wedge \left( \mathcal{Q}_{ab} - \frac{1}{4} g_{ab} \mathcal{Q} \right), \quad (3)$$

$$\begin{aligned} \mathcal{L}_G = 2f_0 \left[ \frac{1}{2} \beta^2 \mathcal{R}^a{}_b \wedge \eta_a{}^b - \beta^4 \Lambda \eta + \frac{1}{4} \lambda \mathcal{R}^a{}_a \wedge * \mathcal{R}^b{}_b + \rho_1 \beta^2 \mathcal{T}^a \wedge * \mathcal{T}_a + \right. \\ \left. + \rho_2 \beta^2 (\mathcal{T}^a \wedge \theta_b) \wedge * (\mathcal{T}^b \wedge \theta_a) + \rho_3 \beta^2 (\mathcal{T}^a \wedge \theta_a) \wedge * (\mathcal{T}^b \wedge \theta_b) + \right. \\ \left. + \xi \beta^2 \mathcal{Q} \wedge * \mathcal{Q} + \zeta \beta^2 \mathcal{Q} \wedge \theta^a \wedge * \mathcal{T}_a + l_1 d\beta \wedge * d\beta + \right. \\ \left. + l_2 \beta d\beta \wedge \theta^a \wedge * \mathcal{T}_a + l_3 \beta d\beta \wedge * \mathcal{Q} \right]. \quad (4) \end{aligned}$$

here  $\mathcal{L}_G$  is the gravitational field Lagrange density,  $\mathcal{L}_{\text{mat}}$  is the matter Lagrange density. The first term in  $\mathcal{L}_G$  is the Gilbert–Einstein Lagrangian density generalized to the Cartan–Weyl space ( $\eta_a{}^b = *(\theta_a \wedge \theta^b)$ ,  $f_0 = c^4/16\pi G$ ), the second term is a generalized cosmological term describing vacuum energy ( $\Lambda$  is the Einstein cosmological constant).

We use the exterior form variational formalism on the base of the Lemma on the commutation rule between variation and Hodge star dualization [29]. The independent variables are the nonholonomic connection 1-form  $\Gamma^a{}_b$ , the basis 1-form  $\theta^a$ , the Dirac scalar field  $\beta(x)$ , and the Lagrange multipliers  $\Lambda^{ab}$ .  $\Lambda$ -equation yields the Weyl's condition (1). The variational field equations of the theory ( $\Gamma$ -equation,  $\theta$ -equation and  $\beta$ -equation) can be found in [18, 21].

These variational field equations have been solved for the very early stage of evolution of universe for the scale factor  $a(t)$  and the field  $\beta(t)$ , when the matter density is very small [18, 20–22]. This solution realizes exponential diminution of the field  $\beta$ , and thus sharp exponential decrease of physical vacuum energy (dark energy) by many orders. Thus this result can explain the exponential decrease in time at very early Universe of the dark energy being described by the effective cosmological constant. This can give way to solving one of the fundamental problems of the modern theoretical physics — the problem of the cosmological constant (see [23, 24]) — as a consequence of fields dynamics at the early Universe.

### 3. Spherically Symmetric Solution of the Weyl–Dirac Theory

Now a static spherically symmetric solution of the field equations in vacuum (in case of  $\Lambda = 0$ ,  $\lambda = 0$ ) is obtained for a central compact mass  $m$  [18, 27, 28].

In the spherically symmetric case the torsion 2-form is,  $\mathcal{T}^a = (1/3)\mathcal{T} \wedge \theta^a$ ,  $\mathcal{T} = *(\theta_a \wedge * \mathcal{T}^a)$ , where  $\mathcal{T}$  is a torsion trace 1-form.

As a consequence of the  $\Gamma$ -equation, one can conclude that the torsion 1-form  $\mathcal{T}$  and the nonmetricity 1-form  $\mathcal{Q}$  can be realized as  $\mathcal{T} = sdU$ ,  $\mathcal{Q} = qdU$ ,  $U = \log \beta$ , where  $s$  and  $q$  are arbitrary constants.

We shall find a static spherically symmetric solution with a metrics of the form,

$$ds^2 = e^{-2U(r)} \left[ e^{-\mu(r)} dt^2 - e^{\mu(r)} (dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2)) \right]. \quad (5)$$

After calculation the  $\Gamma$ -,  $\theta$ - and  $\beta$ -equations with the help of this metrics, one can conclude that these equations are reduced to the following equations,

$$\mu'' + \frac{2}{r} \mu' = 0, \quad U' = \pm \frac{k}{2} \mu', \quad k = \sqrt{k^2} > 0, \quad (6)$$

with  $q = -8$ ,  $s = -6$ , and  $k^{-2} = l_1$  under some conditions on the coupling constants of the Lagrangian density 4-form (4).

The equations (6) have solutions,

$$\mu = \frac{r_0}{r}, \quad \beta(r) = \beta_0 e^{\pm \frac{kr_0}{2r}}, \quad (7)$$

which lead to the metrics,

$$ds^2 = e^{\mp \frac{kr_0}{r}} ds_{\text{YR}}^2, \quad (8)$$

$$ds_{\text{YR}}^2 = e^{-\frac{r_0}{r}} dt^2 - e^{\frac{r_0}{r}} (dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2)). \quad (9)$$

With the help of the conformal transformation

$$\check{g}_{\alpha\beta} = e^{\mp \frac{kr_0}{r}} g_{\alpha\beta}, \quad (10)$$

the metrics (8) can be transformed to the metrics (9), the Cartan–Weyl space being transformed to the Riemann–Cartan space.

If one puts  $r_0 = r_g = 2Gm/c^2$ , the metrics (9) is known as the Yilmaz–Rosen (YR) metrics [30–32]. In this case this metrics in the post-Newtonian approximation at large distances gives the same results as the Schwarzschild metrics. The metrics (9) belongs to the Majumdar–Papapetrou class of metrics [33, 34].

The metrics (8) will be named the generalized Yilmaz–Rosen metrics. In the simplest case the constant  $k$  can be chosen as  $k = 1/\sqrt{l_1}$ , where  $l_1$  is the coupling constant in the Lagrangian density (4).

#### 4. Possible Influence of Dark Matter on the Interplanetary Spacecraft Motion

Let's consider a radial motion of a test body under the influence of the metrics (8). The  $t$ -component of the geodesic equation has the first integral,

$$e^{-(1\pm k)\frac{r_g}{r}} \frac{cdt}{ds} = E_0 = \text{const}. \quad (11)$$

Let us divide (8) by  $ds^2$  and put  $d\theta = 0$ ,  $d\phi = 0$ . Then after some transformations we shall obtain for radial movement the following functional dependence between the velocity  $v$  of a test body and the radial coordinate  $r$ ,

$$\frac{v^2}{c^2} = e^{-\frac{2r_g}{r}} \left( 1 - \frac{1}{E_0^2} e^{-(1\pm k)\frac{r_g}{r}} \right). \quad (12)$$

This equation yields the identity,

$$\frac{v_{\text{inf}}^2}{c^2} = 1 - \frac{1}{E_0^2}, \quad (13)$$

where  $v_{\text{inf}}$  is an asymptotical value of the test body velocity at infinity.

Let's apply the equalities (12) and (13) to the motion of interplanetary spacecraft starting from the Earth (ignoring its rotation). If we use the Newton approximation in

this case, then we obtain the approximate equality,

$$\frac{v_{\text{inf}}^2 - v_{\text{inf}/0}^2}{c^2} \approx \mp k \left( \frac{r_g}{R} \right)_{\text{Earth}}, \quad (14)$$

where  $v_{\text{inf}/0}$  is the value of the test body velocity at infinity calculated under the condition  $k = 0$ .

The data on Galileo, Cassini and other Earth flyby of the interplanetary spacecrafts show the increase  $\Delta v_{\text{inf}}$  in the asymptotic line-of-sight velocity  $v_{\text{inf}}$ , of the order of  $1\text{--}10 \frac{\text{mm}}{\text{s}}$  [35]. Therefore the value of (14) is not zero. From this fact one can make two conclusions. First, we need to choose the second sign in the solutions (7), (9). Second, the formula (14) allows to estimate the values of  $k$  and  $l_1$ , and thus the value of the Dirac field rest mass. In [11] one can see, how a scalar field, which is a model of dark matter, can obtain a rest mass by the Higgs machinery.

## 5. Conclusions

As a consequence of the Poincaré–Weyl gauge theory of gravitation, the Dirac scalar field, which has an equally fundamental status as the metrics, should exist in Nature, and spacetime has a geometrical structure of the Cartan–Weyl space. We have named such gravitational theory as the Weyl–Dirac theory of gravitation. In this theory we derive a static spherically symmetric solution of the field equations in vacuum for a central mass. With this solution we consider a radial motion of an interplanetary spacecraft starting from the Earth. Using the Newton approximation we obtain that the asymptotic line-of-sight velocity  $v_{\text{inf}}$  in this case depends on the parameter  $k$  of the solution. Using the observable data, one can obtain the value of this parameter and then the value of the Dirac field rest mass.

The results were obtained within the framework of performance of the State Task No 3.1968.2014/K of the Ministry of Education and Science of the Russian Federation.

## References

1. T. W. B. Kibble, Lorentz Invariance and the Gravitational Field, *Journal of Mathematical Physics* 2 (1961) 212–221.
2. B. N. Frolov, Principle of Local Invariance and Noether Theorem, *Vestnik Moskovskogo Universiteta* 6 (1963) 48–58, (in Russian).
3. B. N. Frolov, Principle of Local Invariance and Noether Theorem, in: *Proceedings, Modern Problems of Gravitation, 2nd Soviet grav. conf.*, Publ. House Tbilisi Univ, Tbilisi, 1967, pp. 270–278, (in Russian).
4. B. N. Frolov, Tetrad Palatini Formalism and Quadratic Lagrangians in the Gravitational Field Theory, *Acta Physica Polonica B* 9 (1978) 823–829.
5. B. N. Frolov, On Foundations of Poincaré-gauged Theory of Gravity, *Gravitation and Cosmology* 6 (2004) 116–120.
6. K. Hayashi, Gauge Theories of Massive and Massless Tensor Fields, *Progress of Theoretical Physics* 39 (1968) 494–515.
7. F. W. Hehl, J. L. McCrea, E. W. Mielke, Y. Ne’eman, Metric-Affine Gauge Theory of Gravity: Field Equations, Noether Identities, World Spinors, and Breaking of Dilaton Invariance, *Physics Reports* 258 (1995) 1–171.
8. B. N. Frolov, *Poincaré Gauge Theory of Gravitation*, MPGU, Moscow, 2003, (in Russian).
9. M. Blagojević, F. W. Hehl, *Gauge Theory of Gravitation*, WSPC, Imperial College Press, London, 2013.

10. B. N. Frolov, *Gravitatsiya i elektromagnetizm*, Universitetskoe, Minsk, 1992, Ch. Generalized-conformal Poincaré-gauged Quadratic Theory of Gravity, pp. 174–178, (in Russian).
11. B. N. Frolov, *Gravity, Particles and Spacetime*, World Scientific, Singapore, 1996, Ch. Generalized Conformal Invariance and Gauge Theory of Gravity, pp. 113–144, (in Russian).
12. P. A. M. Dirac, Long Range Forces and Broken Symmetries, *Proceedings of the Royal Society A* 333 (1973) 403–418.
13. O. V. Babourova, B. N. Frolov, V. C. Zhukovsky, Gauge Field Theory for Poincaré–Weyl Group, *Physical Review D* 74 (2006) 064012–1–125.
14. O. V. Babourova, B. N. Frolov, V. C. Zhukovsky, Theory of Gravitation on the Basis of the Poincaré–Weyl Gauge Group, *Gravitation and Cosmology* 15(1) (2009) 13–15.
15. O. V. Babourova, B. N. Frolov, Dark Energy, Dirac’s Scalar Field and the Cosmological Constant Problem, *ArXiv: 1112.4449 [gr-qc]* (2011).
16. O. V. Babourova, B. N. Frolov, R. S. Kostkin, Dirac’s Scalar Field as Dark Energy with the Frameworks of Conformal Theory of Gravitation in Weyl–Cartan Space, *ArXiv: 1006.4761[gr-qc]* (2010).
17. O. V. Babourova, R. S. Kostkin, B. N. Frolov, The Problem of a Cosmological Constant within the Conformal Gravitation Theory in the Weyl–Cartan Space, *Russian Physics Journal* 54(1) (2011) 121–123.
18. O. V. Babourova, B. N. Frolov, *Mathematical Foundations of the Modern Theory of Gravitation*, MPGU, Moscow, 2012, (in Russian).
19. E. B. Gliner, Inflationary Universe and the Vacuumlike State of Physical Medium Space, *Physics-Uspekhi (Advances in Physical Sciences)* 45(2) (2002) 213–220.
20. O. V. Babourova, K. N. Lipkin, B. N. Frolov, Theory of Gravity With the Dirac Scalar Field and the Problem of Cosmological Constant, *Russian Physics Journal* 55(7) (2012) 855–857.
21. O. V. Babourova, B. N. Frolov, K. N. Lipkin, Theory of Gravity With a Dirac Scalar Field in the Exterior Form Formalism and Cosmological Constant Problem, *Gravitation and Cosmology* 18(4) (2012) 225–231.
22. O. V. Babourova, B. N. Frolov, Dark Energy as a Cosmological Consequence of Existence of the Dirac Scalar Field in Nature, *Physical Research International (ID 952181)* (2015) 1–6.
23. S. Weinberg, The Cosmological Constant Problem, *Reviews of Modern Physics* 61(1) (1989) 1–23.
24. M. Li, X.-D. Li, S. Wang, Y. Wang, Dark Energy, *Communications in Theoretical Physics* 56 (2011) 525–560.
25. T. Matos, L. A. Urena-Lopez, On the Nature of Dark Matter, *International Journal of Modern Physics D* 13 (2004) 2287–2292.
26. D. F. Mota, V. Salzano, S. Capozziello, Unifying Static Analysis of Gravitational Structures With a Scale-Dependent Scalar Field Gravity as an Alternative to Dark Matter, *Physical Review D* 83 (2011) 084038.
27. O. V. Babourova, B. N. Frolov, E. V. Febres, Spherically Symmetric Solution of Gravitation Theory With a Dirac Scalar Field in the Cartan–Weyl Space, *Russian Physics Journal* 57(9) (2015) 1297–1299.
28. O. V. Babourova, B. N. Frolov, P. E. Kudlaev, E. V. Romanova, Spherically Symmetric Solution in Cartan–Weyl Space with Dirac Scalar Field, in: *Proceedings of the Twelfth Asia-Pacific International Conference on Gravitation, Astrophysics, and Cosmology dedicated to the Centenary of Einstein’s General Relativity*, Moscow, 28 Jun–5 July 2015, World Scientific Publishing Co. Pte. Ltd., Singapore, 2016, pp. 191–195.
29. O. V. Babourova, B. N. Frolov, E. A. Klimova, Plane Torsion Waves in Quadratic Gravitational Theories in Riemann–Cartan Space, *Classical and Quantum Gravity* 16 (1999) 1149–1162.

30. H. Yilmaz, New Approach to General Relativity, *Physical Review* 111 (1958) 1417–1420.
31. N. Rosen, A Bi-Metric Theory of Gravitation, *Annals of Physics (New York)* 84 (1974) 455–473.
32. Y. Itin, A Class of Quasi-Linear Equations in Coframe Gravity, *General Relativity and Gravitation* 31 (1999) 1891–1911.
33. S. D. Majumdar, A Class of Exact Solutions of Einstein's Field Equations, *Physical Review* 72(5) (1947) 390–398.
34. A. Papapetrou, Solution of the Equations of the Gravitational Field for an Arbitrary Charge-Distribution, *Proceedings of Royal Irish Academy A* 51 (1947) 191–204.
35. L. Iorio, Gravitational Anomalies in the Solar System?, *International Journal of Modern Physics D* 24(6) (2015) 1530015 (37 p.).

УДК 530.122:531.266.3

### Сферически-симметричное решение теории гравитации Вейля–Дирака и её следствия

О. В. Бабурова, Б. Н. Фролов, П. Э. Кудлаев, Е. В. Романова

*Московский педагогический государственный университет, Москва, Россия*

В историческом аспекте обсуждаются Пуанкаре- и Пуанкаре-Вейль-калибровочные теории гравитации в постримановых пространствах со скалярным полем Дирака с лагранжианами, квадратичными по кривизне и кручению. Рассматриваются различные гипотезы о возможном построении моделей тёмной материи с помощью скалярного поля. Развивается новая конформная теория гравитации Вейля–Дирака, представляющая собой теорию гравитации в пространстве-времени Картана–Вейля со скалярным полем Дирака, которое рассматривается как модель тёмной материи. Найдено статическое сферически-симметричное решение уравнений поля в вакууме для центральной компактной массы в виде метрики, конформной метрике Илмаза–Розена. На основе этого решения рассмотрено радиальное движение космического аппарата, стартующего с Земли. В ньютоновом приближении показано, что асимптотическое значение скорости аппарата на значительном удалении от Земли зависит от параметра решения. Тем самым возникает возможность при сравнении с наблюдательными данными определить значение этого параметра, что позволит оценить величину массы покоя кванта скалярного поля Дирака.

**Ключевые слова:** тёмная материя, скалярное поле Дирака, теория гравитации Вейля–Дирака, пространство-время Картана–Вейля, метрика Илмаза–Розена, облёт Земли космическим аппаратом

### Литература

1. *Kibble T. W. B.* Lorentz Invariance and the Gravitational Field // *Journal of Mathematical Physics*. — 1961. — Vol. 2. — Pp. 212–221.
2. *Фролов Б. Н.* Принцип локальной инвариантности и теорема Нетер // *Вестник Московского университета, серия физики, астрономии*. — 1963. — Т. 6. — С. 48–58.
3. *Фролов Б. Н.* Принцип локальной инвариантности и теорема Нетер // *Современные проблемы гравитации / Труды 2-й Советской гравитационной конференции*. — Тбилиси: Издательство Тбилисского университета, 1967. — С. 270–278.
4. *Frolov B. N.* Tetrad Palatini Formalism and Quadratic Lagrangians in the Gravitational Field Theory // *Acta Physica Polonica B*. — 1978. — Vol. 9. — Pp. 823–829.
5. *Frolov B. N.* On Foundations of Poincaré-gauged Theory of Gravity // *Gravitation and Cosmology*. — 2004. — Vol. 6. — Pp. 116–120.
6. *Hayashi K.* Gauge Theories of Massive and Massless Tensor Fields // *Progress of Theoretical Physics*. — 1968. — Vol. 39. — Pp. 494–515.

7. Metric-Affine Gauge Theory of Gravity: Field Equations, Noether Identities, World Spinors, and Breaking of Dilaton Invariance / F. W. Hehl, J. L. McCrea, E. W. Mielke, Y. Ne'eman // *Physics Reports*. — 1995. — Vol. 258. — Pp. 1–171.
8. *Фролов Б. Н.* Пуанкаре калибровочная теория гравитации. — Москва: МПГУ, 2003.
9. *Blagojević M., Hehl F. W.* Gauge Theory of Gravitation. — London: WSPC, Imperial College Press, 2013.
10. *Фролов Б. Н.* Гравитация и электромагнетизм. — Минск.: Университетское, 1992. — С. 174–178.
11. *Frolov B. N.* Gravity, Particles and Spacetime / Ed. by P. Pronin, G. Sardanashvily. — Singapore: World Scientific, 1996. — Pp. 113–144.
12. *Dirac P. A. M.* Long Range Forces and Broken Symmetries // *Proceedings of the Royal Society A*. — 1973. — Vol. 333. — Pp. 403–418.
13. *Babourova O. V., Frolov B. N., Zhukovsky V. C.* Gauge Field Theory for Poincaré–Weyl Group // *Physical Review D*. — 2006. — Vol. 74. — Pp. 064012–1–125.
14. *Babourova O. V., Frolov B. N., Zhukovsky V. C.* Theory of Gravitation on the Basis of the Poincaré–Weyl Gauge Group // *Gravitation and Cosmology*. — 2009. — Vol. 15(1). — Pp. 13–15.
15. *Babourova O. V., Frolov B. N.* Dark Energy, Dirac's Scalar Field and the Cosmological Constant Problem. — ArXive: 1112.4449 [gr-qc]. — 2011.
16. *Babourova O. V., Frolov B. N., Kostkin R. S.* Dirac's Scalar Field as Dark Energy with the Frameworks of Conformal Theory of Gravitation in Weyl–Cartan Space. — ArXive: 1006.4761[gr-qc]. — 2010.
17. *Babourova O. V., Kostkin R. S., Frolov B. N.* The Problem of a Cosmological Constant within the Conformal Gravitation Theory in the Weyl–Cartan Space // *Russian Physics Journal*. — 2011. — Vol. 54(1). — Pp. 121–123.
18. *Бабурова О. В., Фролов Б. Н.* Математические основы современной теории гравитации. — Москва: МПГУ, 2012.
19. *Gliner E. B.* Inflationary Universe and the Vacuumlike State of Physical Medium Space // *Physics-Uspekhi (Advances in Physical Sciences)*. — 2002. — Vol. 45(2). — Pp. 213–220.
20. *Babourova O. V., Lipkin K. N., Frolov B. N.* Theory of Gravity With the Dirac Scalar Field and the Problem of Cosmological Constant // *Russian Physics Journal*. — 2012. — Vol. 55(7). — Pp. 855–857.
21. *Babourova O. V., Frolov B. N., Lipkin K. N.* Theory of Gravity With a Dirac Scalar Field in the Exterior Form Formalism and Cosmological Constant Problem // *Gravitation and Cosmology*. — 2012. — Vol. 18(4). — Pp. 225–231.
22. *Babourova O. V., Frolov B. N.* Dark Energy as a Cosmological Consequence of Existence of the Dirac Scalar Field in Nature // *Physical Research International*. — 2015. — Vol. (ID 952181). — Pp. 1–6.
23. *Weinberg S.* The Cosmological Constant Problem // *Reviews of Modern Physics*. — 1989. — Vol. 61(1). — Pp. 1–23.
24. Dark Energy / M. Li, X.-D. Li, S. Wang, Y. Wang // *Communications in Theoretical Physics*. — 2011. — Vol. 56. — Pp. 525–560.
25. *Matos T., Urena-Lopez L. A.* On the Nature of Dark Matter // *International Journal of Modern Physics D*. — 2004. — Vol. 13. — Pp. 2287–2292.
26. *Mota D. F., Salzano V., Capozziello S.* Unifying Static Analysis of Gravitational Structures With a Scale-Dependent Scalar Field Gravity as an Alternative to Dark Matter // *Physical Review D*. — 2011. — Vol. 83. — P. 084038.
27. *Babourova O. V., Frolov B. N., Febres E. V.* Spherically Symmetric Solution of Gravitation Theory With a Dirac Scalar Field in the Cartan–Weyl Space // *Russian Physics Journal*. — 2015. — Vol. 57(9). — Pp. 1297–1299.
28. Spherically Symmetric Solution in Cartan–Weyl Space with Dirac Scalar Field / O. V. Babourova, B. N. Frolov, P. E. Kudlaev, E. V. Romanova // *Proceedings of*



- the Twelfth Asia-Pacific International Conference on Gravitation, Astrophysics, and Cosmology dedicated to the Centenary of Einstein's General Relativity, Moscow, 28 Jun–5 July 2015. — Singapore: World Scientific Publishing Co. Pte. Ltd., 2016. — Pp. 191–195.
29. Babourova O. V., Frolov B. N., Klimova E. A. Plane Torsion Waves in Quadratic Gravitational Theories in Riemann–Cartan Space // *Classical and Quantum Gravity*. — 1999. — Vol. 16. — Pp. 1149–1162.
  30. Yilmaz H. New Approach to General Relativity // *Physical Review*. — 1958. — Vol. 111. — Pp. 1417–1420.
  31. Rosen N. A Bi-Metric Theory of Gravitation // *Annals of Physics (New York)*. — 1974. — Vol. 84. — Pp. 455–473.
  32. Itin Y. A Class of Quasi-Linear Equations in Coframe Gravity // *General Relativity and Gravitation*. — 1999. — Vol. 31. — Pp. 1891–1911.
  33. Majumdar S. D. A Class of Exact Solutions of Einstein's Field Equations // *Physical Review*. — 1947. — Vol. 72(5). — Pp. 390–398.
  34. Papapetrou A. Solution of the Equations of the Gravitational Field for an Arbitrary Charge-Distribution // *Proceedings of Royal Irish Academy A*. — 1947. — Vol. 51. — Pp. 191–204.
  35. Iorio L. Gravitational Anomalies in the Solar System? // *International Journal of Modern Physics D*. — 2015. — Vol. 24(6). — P. 1530015 (37 p.).

© Babourova O. V., Frolov B. N., Kudlaev P. E., Romanova E. V., 2016