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Non-Coplanar Rendezvous in Near-Circular Orbit with the Use a Low Thrust Engine

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Abstract. Presented method allows one to calculate the of maneuvers performed on several turns using a low-thrust engine. These maneuvers ensure the flight of an active spacecraft within a given area of the target space object. The flight is carried out in the vicinity of a circular orbit. Simplified mathematical models of motion are used to solve this problem. The influence of the non-centrality of the gravitational field and atmosphere is not taken into account in the calculations. The process of determining the parameters of the maneuvers is divided into several stages: in the first and third stages, the parameters of the impulse transfer and the transfer carried out by the low-thrust engine are calculated using analytical methods. In the second stage, the distribution of maneuvering between turns, ensuring a successful solution to the meeting problem, is determined by changing one variable. This method is characterized by its simplicity and high reliability in determining the parameters of maneuvers, which makes it applicable on board a spacecraft. As part of the study, an analysis of the dependence of the total characteristic velocity of solving the meeting problem on the amount of engine thrust was also carried out. The maneuver parameters can be refined using an iterative procedure to take into account the main disturbances.

Keywords: spacecraft, near-circular orbit, velocity impulse, calculation of maneuver parameters, space object, low thrust engine

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Некомпланарная встреча на околокруговой орбите с помощью двигателя малой тяги

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Нераздельное соавторство.

Аннотация. Представлен метод, позволяющий вычислить параметры маневров, выполняемых на нескольких витках с применением двигателя малой тяги. Эти маневры обеспечивают перелет активного космического аппарата в пределы заданной области целевого космического объекта. Перелет осуществляется в окрестности круговой орбиты. Для решения данной задачи применяются упрощенные математические модели движения. Влияние нецентральности гравитационного поля и атмосферы в расчетах не учитывается. Процесс определения параметров маневров разбит на несколько этапов: на первом и третьем этапах параметры импульсного перехода и перехода, осуществляемого двигателем малой тяги, вычисляются с использованием аналитических методов. На втором этапе распределение маневрирования между витками, обеспечивающее успешное решение задачи встречи, определяется путем изменения одной переменной. Данный метод отличается простотой и высокой надежностью в определении параметров маневров, что делает его применимым на борту космических аппаратов. В рамках исследования также проведен анализ зависимости суммарной характеристической скорости решения задачи встречи от величины тяги двигателя. Параметры маневров могут быть уточнены с помощью итерационной процедуры, чтобы учесть основные возмущения.

Ключевые слова: космический аппарат, околокруговая орбита, импульс скорости, расчет параметров маневров, космический объект, малая тяга

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Introduction

The problem of meeting in a near-circular orbit using low-thrust engines is important in the practice of spacecraft (SC) flights. Typical examples are the problem of rendezvous and docking of spacecraft, the implementation of a group flight of several spacecraft, the formation of a given configuration of satellite systems, during removal of space debris, during servicing of spacecraft and other astronautical missions involving more than one spacecraft.

Due to the great complexity of solving problems of spacecraft meeting with greater accuracy, over the past few years many authors have developed algorithms for solving the problem of spacecraft meeting [1–2].

Currently, three main approaches are widely used in solving complex problems of multi-impulse maneuvering of spacecraft. In the first case, the problems of maneuvering in the orbital plane and the problems of rotating the orbital plane are solved independently. This scheme was used, for example, when approaching the Shuttle spacecraft with an orbital station, to control the movement of geostationary satellites [3], satellites within satellite systems [4–6], and so on. The advantage of this scheme lies in its simplicity and reliability, and the disadvantage is the excessive cost of characteristic velocity for maneuvering.

In the second case, numerical methods are used to find optimal solutions for the most complex multi-impulse problems, taking into account a wide range of restrictions [7–8]. The simplex method is most often used [9–10].

In the third method, at the initial stage, the solution to the Lambert problem is used to determine the parameters of the two-impulse solutions to the meeting problem. Then the behavior of the hodograph of the basis vector corresponding to the found solution is analyzed, and, if necessary, additional velocity impulses are added to obtain the optimal solution. This approach was first used in the works of Lion and Handelsman [11], Jezewski and Rozendaal [12].

There are also methods that are at the intersection of different approaches. For example, in [13–14] numerical and analytical methods for solving the multi-impulse meeting problem are proposed, combining the advantages of the first and second of the previously listed approaches. They make it possible to use the results obtained in the early papers of T. Edelbaum [15], J.P. Marec [2], when solving modern practical problems.

Since the 1960s, the process of using electric rocket engines (ERM) on spacecraft began. Thanks to their high specific impulse, electric propulsion engines can significantly reduce fuel costs for orbital maneuvering. However, the low (compared to traditional liquid rocket engines) thrust of electric propulsion engines leads to the need to take into account their long-term operation.

Problems of this type take a special place a special place among the problems of optimal maneuvering of a spacecraft. A significant number of articles have been devoted to them, and several very interesting monographs have been published [16; 17]. Particularly noteworthy are the papers of V.G. Petukhov [18–20]. Due to the complexity of the problems in which it is assumed that maneuvering is carried out using a propulsion system (PS), they have traditionally been solved numerically and by methods using the Pontryagin maximum principle or the continuation method. In recent years, Yu.P. Ulybyshev has successfully used the interior point method to solve problems with long maneuvers [21].

In the method considered in this paper, the non-coplanar meeting problem is solved both in the impulse formulation and taking into account the long-term operation of the low-thrust engine [22–24].

To analyze the relative motion of a spacecraft in the vicinity of circular orbits, it is necessary to use special mathematical models of motion. The most popular mathematical model of the relative motion of a spacecraft in the vicinity of circular orbits is the Hill–Clohessy–Wiltshire (HCW) model. Linearized differential equations for the relative motion of a spacecraft in the vicinity of a circular orbit for the problem of rendezvous and docking were obtained by Clohessy–Wiltshire in 1960 [25], but back in the 19th century similar equations were used by Hill in his theory of lunar motion [26].

In this mathematical model, to obtain the equations of relative motion, a rotating (orbital) coordinate system and linearization of the differential equations of relative motion are used, based on the assumption that the distance between the considered spacecraft is small compared to the average orbital radius. This work uses linearized equations obtained by P.E. Elyasberg [27]. They were obtained using a cylindrical coordinate system and are significantly more convenient for solving the problem of long-duration encounters, in which there are significant deviations along the orbit.

Due to the increase in the number of maneuvering spacecraft and the increase in the efficiency of solving problems, there is currently a tendency to transfer the process of calculating maneuvers on board the spacecraft. This leads to the need to simplify the process of calculating maneuver parameters and increase the reliability of this process. The algorithm considered in this paper has precisely these properties.

1. Formulation of the meeting problem

The problem of calculating the parameters of transfer maneuvers between close near-circular orbits is solved in an approximate impulse formulation, within the framework of unperturbed Keplerian motion.

The conditions for transferring with the help of N velocity impulses in a fixed time from the initial orbit to a given point of the final non-coplanar orbit (meeting problem) in a linear approximation can be written in the form Ilyin and Kuzmak [22]:

$$\sum_{i=1}^{N}(\Delta V_{ri}\sin\varphi_i+2\Delta V_{ti}\cos\varphi_i)=\Delta e_x, \quad (1)$$

$$\sum_{i=1}^{N}(-\Delta V_{ri}\cos\varphi_i+2\Delta V_{ti}\sin\varphi_i)=\Delta e_y, \quad (2)$$

$$\sum_{i=1}^{N}2\Delta V_{ti}=\Delta a, \quad (3)$$

$$\sum_{i=1}^{N}(2\Delta V_{ri}(1-\cos\varphi_i)+\Delta V_{ti}(-3\varphi_i+4\sin\varphi_i))=\Delta t, \quad (4)$$

$$\sum_{i=1}^{N}-\Delta V_{zi}\sin\varphi_i=\Delta z, \quad (5)$$

$$\sum_{i=1}^{N}\Delta V_{zi}\cos\varphi_i=\Delta V_z, \quad (6)$$

where

$\Delta e_x = e_f\cos\omega_f - e_0\cos\omega_0$,

$\Delta e_y = e_f\sin\omega_f - e_0\sin\omega_0$,

$\Delta a=(a_f-a_0)/r_0$, $\Delta t=\lambda_0(t_f-t_0)$,

$\Delta z= z_0/r_0$, $\Delta V_z=\Delta V_{z0}/V_0$,

$\Delta V_{ti}=\Delta V_{ti}^*/V_0$, $\Delta V_{ri}=\Delta V_{ri}^*/V_0$, $\Delta V_{zi}=\Delta V_{zi}^*/V_0$.

Here «f», «0» — the indices corresponding to the final and initial orbits, e_f, e_0 — the eccentricities of the orbits; a_f, a_0 — semi-axes major of orbits; ω_f, ω_0 — angles between the direction to the pericenter of the corresponding orbit and the direction to a point specified on the final orbit (the Ox axis is directed to this point); t_f — the required time of arrival at a given point, t_0 — the time at which, when moving along the initial orbit, the projection of the radius vector onto the plane of the final orbit hits the ray passing through the given meeting point; z_0 — the deviation of the spacecraft in the initial orbit from the plane of the final orbit at time t_0; Vz_0 — lateral relative velocity at this moment; V_0, λ_0 — orbital and angular velocities of movement along the reference circular orbit of radius r_0 ($r_0 = a_f$); N — number of velocity impulses; φ_i — the angle of application of the i-th velocity impulse, measured from the direction to a given meeting point in the direction of the SC motion; ΔV_{ti}^*, ΔV_{ri}^*, ΔV_{zi}^* — transversal, radial and lateral components of the i-th velocity impulse, respectively. It is necessary to take into account that the angles φ_i — negative, because it was assumed that at a given point $\varphi_f = 0$.

The problem of searching for parameters of optimal maneuvers can be formulated as follows: it is necessary to determine ΔV_{ri}, ΔV_{ti}, ΔV_{zi}, φ_i ($i = 1, \ldots, N$), at which the total characteristic velocity of maneuvers ΔV is minimal.

$$\Delta V=\sum_{i=1}^{N}\Delta V_i=\sum_{i=1}^{N}\sqrt{\Delta V_{ri}^2+\Delta V_{ti}^2+\Delta V_{zi}^2},$$

under restrictions (1)–(6).

In this paper problem of the meeting is solved in several stages. At the first stage, the problem of impulses transfer between non-coplanar orbits is solved (Section 2). The velocity impulses for solving the transfer problem are then distributed among the turns allowed for maneuvering to ensure that equation (4) is satisfied (Section 3). In sections (4) and (5), a solution to the low-thrust transfer problem is sought.

The maneuver parameters can be refined using an iterative procedure to take into account all disturbances (the influence of compression of the Earth, atmosphere, etc.).

2. Algorithm for solving the transfer problem

When solving the problem of transfer between non-coplanar orbits, five equations of the system (1)–(6) are used.

The angle φ_1 (the angle of application of the first velocity impulse) is searched and for each successive value of the angle the values of the velocity impulses and the angle φ_2 are found:

$$\Delta V_{t1}=\frac{\Delta e^2-\Delta a^2}{4(\Delta e_y\sin\varphi_{1f}+\Delta e_x\cos\varphi_{1f}-\Delta a)}, \quad (7)$$

$$\Delta V_{t2}=\frac{\Delta a}{2}-\Delta V_{t1}, \quad (8)$$

$$\tan\varphi_2=\frac{\frac{\Delta e_y}{2}-\Delta V_{t1}\sin\varphi_{1f}}{\frac{\Delta e_x}{2}-\Delta V_{t1}\cos\varphi_{1f}}, \quad (9)$$

and then from equations (5)–(6) the values of the lateral components of the velocity impulses are determined:

$$\Delta V_{z1}=-\left(\frac{\Delta z\cos\varphi_2+\Delta V_z\sin\varphi_2}{\sin(\varphi_1-\varphi_2)}\right), \quad (10)$$

$$\Delta V_{z2} = -\left(\frac{\Delta z \cos \varphi_1 + \Delta V_z \sin \varphi_1}{\sin(\varphi_1 - \varphi_2)}\right). \quad (11)$$

From the entire set of solutions found, the one that provides the minimum total characteristic velocity is selected. Further, the parameters of this solution are indicated by the index «*m*» ΔV_{t1m}, ΔV_{z1m}, φ_{1m}, ΔV_{t2m}, ΔV_{z2m}, φ_{2m}.

3. Algorithm for solving the meeting problem

When solving the meeting problem, the values of the velocity impulses ΔV_{t1}, ΔV_{t2}, determined when solving the transfer problem, are distributed among N turns allowed for maneuvering:

$$\Delta V_{1tm} = \sum_{i=1}^{N} \Delta V_{1ti}\,; \quad (12)$$

$$\Delta V_{2tm} = \sum_{i=1}^{N} \Delta V_{2ti}\,. \quad (13)$$

Here N is the number of turns on which maneuvering is allowed.

The lateral components are distributed in proportion to the transversal

$$\Delta V_{1zi} = \frac{|\Delta V_{1ti}|}{|\Delta V_{1t}|} \Delta V_{1zm}\,,$$

and

$$\Delta V_{2zi} = \frac{|\Delta V_{2ti}|}{|\Delta V_{2t}|} \Delta V_{2zm}\,. \quad (14)$$

The further goal is to select such a distribution of the magnitudes of the velocity impulses along the turns so that equation (4) is satisfied.

To significantly simplify the solution of the problem, we assume that the magnitudes of the velocity impulses along the turns change linearly:

$$\Delta V_{1ti} = \Delta V_{1t1} + \\ +(\Delta V_{1tN} - \Delta V_{1t1})(i-1)/(N-1), \quad (15)$$

$$\Delta V_{2ti} = \Delta V_{2t1} + \\ +(\Delta V_{2tN} - \Delta V_{2t1})(i-1)/(N-1). \quad (16)$$

Here ΔV_{1t1}, ΔV_{1tN} и ΔV_{2t1}, ΔV_{2tN} are the magnitudes of the velocity impulses on the first and last permitted turns of maneuvering, which are a part of the first and second velocity impulses of solving the transfer problem.

Substituting the values of velocity impulses calculated using formulas (15), (16) into (12) and (13) we obtain:

$$\Delta V_{1tm} = \sum_{i=1}^{N} \Delta V_{1ti} = 0.5N(\Delta V_{1t1} + \Delta V_{1tN}); \quad (17)$$

$$\Delta V_{2tm} = \sum_{i=1}^{N} \Delta V_{2ti} = 0.5N(\Delta V_{2t1} + \Delta V_{2tN}). \quad (18)$$

Using (17) and (18), we obtain formulas for determining $\Delta V_{1tN}, \Delta V_{2tN}$:

$$\Delta V_{1tN} = \frac{\Delta V_{1t}}{0.5N} - \Delta V_{1t1}\,; \quad (19)$$

$$\Delta V_{2tN} = \frac{\Delta V_{2t}}{0.5N} - \Delta V_{2t1}. \quad (20)$$

Substituting the found values $\Delta V_{1tN}, \Delta V_{2tN}$ into formulas (15) and (16), we obtain:

$$\Delta V_{1ti} = \\ = 2\Delta V_1 (i-1)/N(N-1) + \Delta V_{1t1}\left[1 - \frac{2(i-1)}{N-1}\right], \quad (21)$$

$$\Delta V_{2ti} = \\ = 2\Delta V_2 (i-1)/N(N-1) + \Delta V_{2t1}\left[1 - \frac{2(i-1)}{N-1}\right]. \quad (22)$$

Thus, we found the values of all velocity impulses, expressed only in terms of ΔV_{1t1} and ΔV_{2t1}. Substituting them into equation (3), we obtain a linear equation with two unknowns $\Delta V_{1t1}, \Delta V_{2t1}$. The coefficients for velocity impulses are known, since their angles of application are known:

$$\varphi_{1i} = \varphi_{1m} + 2\pi(N_i - N)\,, \quad (23)$$

$$\varphi_{2i} = \varphi_{2m} + 2\pi(N_i - N)\,. \quad (24)$$

By sorting through the value of the variable ΔV_{1t1}, within the specified limits, for each value from equation (3) we find the value of the variable ΔV_{2t1}.

Then, using (23) and (24), we find the values of all velocity impulses. Adding the modules of all velocity impulses, we find the total characteristic velocity of the next solution. The solution whose total characteristic velocity is minimal is accepted as a solution to the meeting problem. If the total characteristic velocity of the found solution coincides with the total characteristic velocity of the solution to the transfer problem, then a solution with the minimum possible total characteristic velocity was found.

If the duration of the largest velocity impulse does not exceed 20°, then the solution is close to an impulse one and we consider that the problem has already been solved. Taking into account all disturbances (non-centrality of the gravitational field, the influence of the atmosphere, etc.), the operation of a real propulsion system can be carried out using the iterative procedure described in Section 5. If the duration of the maneuvers is significant, then we proceed to solving the problem taking into account low thrust PS.

4. Solving the problem with «low thrust»

It is assumed that the orientation of the propulsion system during the execution of the maneuver is fixed in the orbital coordinate system.

For each turn, we find what changes in eccentricity and semi-major axis produce the found velocity impulses determined at this turn

$$\Delta e_{1ix}=2\Delta V_{1ti}\cos\varphi_{1i}+2\Delta V_{2ti}\cos\varphi_{2i}, \quad (25)$$

$$\Delta e_{1iy}=2\Delta V_{1ti}\sin\varphi_{1i}+2\Delta V_{2ti}\sin\varphi_{2i}, \quad (26)$$

$$\Delta a_i=2\Delta V_{1ti}+2\Delta V_{2ti}. \quad (27)$$

Similarly for changing the lateral parameters on a turn

$$\Delta V_{iz}=\Delta V_{1zi}\cos\varphi_{1i}+\Delta V_{2zi}\cos\varphi_{2i}, \quad (28)$$

$$\Delta z_i=\Delta V_{1zi}\sin\varphi_{1i}+\Delta V_{2ti}\sin\varphi_{2i}. \quad (29)$$

Then we determine the required duration of low-thrust maneuvers that will produce the same change in these elements [20]:

$$\Delta\varphi_1=2\arcsin\frac{w_c\,\Delta V_1}{2w}, \quad (30)$$

$$\Delta\varphi_2=2\arcsin\frac{w_c\,\Delta V_2}{2w}. \quad (31)$$

Thus, turn by turn we find the duration of all maneuvers. The low thrust problem has been correctly solved. If the arcsine argument is greater than 1, then there is no solution (with the existing thrust and mass of the spacecraft, it is impossible to solve the meeting problem for a given number of turns).

The found solution with “low thrust” gives the same change in the eccentricity vector and orientation of the orbital plane as the original impulse solution, because the midpoints of long maneuvers coincide with the moments of application of velocity impulses.

However, the difficulty is that the change in the semi-major axis becomes larger than necessary, since it changes with orbital orientation more effectively than eccentricity. Therefore, as a result of the maneuvers, an error remains in the formation of the required value of the semi-major axis, and to eliminate this error, you can use the iterative procedure described in [20].

Let us assume that the initial deviation of the semi-major axis was $\Delta a_0=a_f-a_0$ (for example, $\Delta a_0>0$), and the deviations Δa_0, Δe_{x0}, Δe_{y0}, Δi_0, $\Delta\vartheta_0$ (the angle between the line of intersection of the orbital planes and the line of apses relative orbits) were used in determining the parameters of the maneuvers.

As a result of performing the calculation maneuvers, the semi-major axis a_1 will be formed ($a_1>a_f$). In the next iteration, deviations $\Delta a_1=\Delta a_0+a_f-a_1, \Delta e_{x0}, \Delta e_{y0}, \Delta i_0, \Delta\vartheta_0$ will be used, at the next iteration $\Delta a_2=\Delta a_1+a_f-a_2$, etc., until the semi-major axis is formed with the given accuracy.

Since at each turn the same change in the semi-major axis will be made as in the impulse solution, the meeting problem will be solved with the same accuracy.

5. Iterative procedure

In the formulated meeting problem, linearized equations of motion are used, the non-centrality of the gravitational field, the influence of the atmosphere, etc. are not taken into account. This leads to the fact that the actual accuracy of fulfilling the terminal conditions in system (1)–(6) will be insufficient. To solve a problem with a given accuracy, you can use an iterative scheme [7–8], which consists of the following stages:

1. In the beginning of the next iteration, an "approximate" problem is solved: under the previously accepted simplifying assumptions, the parameters of maneuvers that ensure the formation of a "target" orbit are determined (at the first iteration, the "target" orbit coincides with the final orbit).

2. Then, taking into account the calculated maneuvers, using models of all necessary disturbances, an "accurate" prediction of the spacecraft motion is carried out and the parameters of the formed orbit are found.

3. The deviations of the parameters of the formed orbit from the corresponding parameters of the final orbit are calculated.

4. If the deviations exceed the permissible ones, then the parameters of the "target" orbit are changed by the value of the calculated deviations, and the next iteration is carried out.

5. The procedure ends when the terminal conditions are met with the specified accuracy.

6. For "accurate" forecasting, as a rule, numerical and/or high-precision numerical-analytical integration are used. It is possible to use different forecast methods at different iterations, but the accuracy of the forecast should increase with the number of the current iteration.

7. During numerical integration, the influence of the non-centrality of the gravitational field, atmosphere, light pressure, etc. is taken into account, the operation of the spacecraft engines is carefully modeled, therefore, despite the fact that the maneuver parameters are found at each iteration using the simplest motion model, but as a result of an iterative procedure, they ensure access to the final orbit with the required accuracy.

6. Example of solving the non-coplanar meeting problem

Let us consider the motion of a spacecraft (SC) relative to point O, moving in an undisturbed near circular orbit with a radius of 6871 km. Let us take the gravitational parameter of the Earth equal to 3.9860044·1014 m^3/s^2. Let us consider the flight problem using N velocity impulses in a fixed time from the initial orbit to a given point in the final orbit from a point in phase space $\mathbf{r}_0 = (10, 100, -5)$ km, $\mathbf{v}_0 = (1, -10, 3)$ m /c to the origin, that is, to the point $\mathbf{r}_f = (0, 0, 0)$ km, with a velocity $\mathbf{v}_f = (0, 0, 0)$ m/s. For the problem, we will take the initial mass of the spacecraft equal to 1000 kg, the specific impulse of the spacecraft propulsion system is 220 seconds (2157.463 m/s), and the thrust (T) will be varied in the range from 1 to 100 N. The flight is carried out in $N = 15$ turns.

Solution of the two-impulse transfer problem

Table 1 shows the results of calculations of the parameters of the optimal two-impulse transfer between non-coplanar orbits, that is, the values of the transversal and lateral components of the velocity impulses, the angles of application of the first and second impulses are given as well. The angle of application of the first velocity impulse was varied from 0 to 360° with a step of 0.75°. It can be seen that the minimum value of the characteristic velocity that a spacecraft (SC) must have for the transfer maneuver is 10.308 m/s.

Multi-impulse solution to the meeting problem

To obtain an impulse solution to the meeting problem, the velocity impulses of the two-impulse solution are distributed between 15 turns so that condition (4) is satisfied. For this purpose, the algorithm described in Section 3. The value of the first velocity impulse is varied within the range from -3.452 m/s to 0.5 m/s with a step of 0.023 m/s.

Table 2 shows parameters of the distributed impulse solution

Table 3 shows the deviations of orbital elements for each turn corresponding to the influence of distributed velocity impulses.

This impulse solution can be transformed to take into account the real thrust of the engine.

The process of obtaining a solution for 1N thrust is shown below.

At the first stage, the durations of the maneuvers are calculated, which for a real low thrust (1N) provide the changes in the orbital elements shown in Table 3 at each orbit (except for the semi-major axis). These durations are shown in Table 4.

Then the change in the semi-major axis produced for a given duration of maneuvers is calculated and a new target value of the semi-major axis is formed for the next iteration. These data are shown in Table 5.

The next iteration is performed and the parameters of the new impulse solution, the duration of the maneuver and changes made of the semi-major axis under the influence of low thrust and errors in the correction of the semi-major axis are shown in Tables 6, 7 and 8.

Table 1

Results of the calculation the parameters of the optimal non-coplanar impulse transfer problem

φ_1°	φ_2 °	ΔV_{t1}, m/s	ΔV_{t2}, m/s	ΔV_{z1}, m/s	ΔV_{z2}, m/s	ΔV_t, m/s	ΔV_z, m/s	ΔV_1, m/s	ΔV_2, m/s	ΔV, m/s
155	55.851	-3.452	2.367	-0.637	-6.372	5.819	7.01	3.51	6.798	10.308

Table 2

Distribution of the two-impulse solution by turns

N	ΔV_{t1}, m/s	ΔV_{t2}, m/s	ΔV_{z1}, m/s	ΔV_{z2}, m/s	ΔV_t, m/s	ΔV_z, m/s	ΔV_1, m/s	ΔV_2, m/s	ΔV, m/s
1	-0.022	0.314	-0.004	-0.844	0.336	0.848	0.023	0.9	0.923
2	-0.052	0.291	-0.01	-0.784	0.343	0.794	0.053	0.837	0.89
3	-0.082	0.269	-0.015	-0.724	0.351	0.739	0.083	0.773	0.856
4	-0.111	0.247	-0.021	-0.664	0.358	0.685	0.113	0.709	0.822
5	-0.141	0.224	-0.026	-0.604	0.366	0.63	0.143	0.645	0.788
6	-0.171	0.202	-0.031	-0.545	0.373	0.576	0.174	0.581	0.755
7	-0.2	0.18	-0.037	-0.485	0.38	0.522	0.204	0.517	0.721
8	-0.23	0.158	-0.043	-0.425	0.388	0.467	0.234	0.453	0.687
9	-0.26	0.136	-0.048	-0.365	0.395	0.413	0.264	0.389	0.653
10	-0.29	0.113	-0.053	-0.305	0.403	0.358	0.294	0.325	0.619
11	-0.319	0.091	-0.059	-0.245	0.41	0.304	0.325	0.261	0.586
12	-0.349	0.069	-0.064	-0.185	0.418	0.249	0.349	0.198	0.547
13	-0.379	0.047	-0.07	-0.125	0.425	0.195	0.385	0.134	0.519
14	-0.408	0.024	-0.075	-0.066	0.433	0.141	0.415	0.07	0.485
15	-0.438	0.002	-0.081	-0.006	0.44	0.087	0.446	0.006	0.452
$\sum$	-3.452	2.367	-0.637	-6.372	5.819	7.01	3.51	6.798	10.308

Table 3

Results of deviations of orbital elements by turns

N	$\Delta e_{1ix} \times 10^{-4}$	$\Delta e_{1iy} \times 10^{-4}$	$\Delta e_i \times 10^{-4}$	φ_{ei} °	$\Delta a_{0i} \times 10^{-4}$	$\Delta V_{zi} \times 10^{-4}$	$\Delta z_i \times 10^{-4}$	φ_{zi} °
1	-0.41	-0.71	0.816	59.877	0.765	0.627	-0.915	55.578
2	-0.306	-0.691	0.755	66.096	0.629	0.589	-0.847	55.163
3	-0.203	-0.675	0.705	73.302	0.492	0.552	-0.779	54.68
4	-0.099	-0.66	0.667	81.466	0.356	0.514	-0.711	54.112
5	0.005	-0.644	0.644	-89.598	0.219	0.476	-0.642	53.435
6	0.108	-0.629	0.638	-80.254	0.083	0.439	-0.574	52.612
7	0.212	-0.614	0.649	-70.979	-0.053	0.401	-0.506	51.594
8	0.315	-0.598	0.676	-62.228	-0.19	0.364	-0.438	50.302
9	0.419	-0.583	0.718	-54.318	-0.326	0.326	-0.37	48.609
10	0.522	-0.568	0.771	-47.388	-0.463	0.288	-0.302	46.299
11	0.626	-0.552	0.835	-41.432	-0.599	0.251	-0.234	42.976
12	0.729	-0.537	0.906	-36.362	-0.736	0.213	-0.166	37.832
13	0.833	-0.521	0.983	-32.057	-0.872	0.176	-0.097	29.028
14	0.936	-0.506	1.064	-28.395	-1.009	0.138	-0.029	11.991
15	1.04	-0.491	1.15	-25.267	-1.145	0.1	0.039	-21.159

Table 4

Duration of the maneuver for *N* = 15

N	$\Delta\varphi_{1i}$ °	$\Delta\varphi_{2i}$ °	$\Delta\varphi_i$ °
1	1.427	59.874	61.301
2	3.347	55.244	58.591
3	5.268	50.71	55.978
4	7.19	46.259	53.449
5	9.115	41.881	50.996
6	11.041	37.567	48.608
7	12.971	33.306	46.277
8	14.905	29.093	43.998
9	16.843	24.92	41.763
10	18.786	20.78	39.566
11	20.734	16.667	37.401
12	22.689	12.575	35.264
13	24.65	8.5	33.15
14	26.618	4.436	31.054
15	28.595	0.377	28.972

Table 5

Changes made of the semi-major axis under the influence of low thrust and errors in the correction of the semi-major axis

N	$\Delta a_{0i} \times 10^{-4}$	$\Delta a_i \times 10^{-4}$	$\delta a_i \times 10^{-4}$	$\Delta a_{1i} \times 10^{-4}$
1	0.765	0.804	-0.039	0.727
2	0.629	0.659	-0.0304	0.598
3	0.492	0.516	-0.0235	0.469
4	0.356	0.374	-0.018	0.338
5	0.219	0.232	-0.013	0.2065
6	0.083	0.092	-0.0089	0.074
7	-0.053	-0.048	-0.0056	-0.059
8	-0.19	-0.187	-0.00278	-0.193
9	-0.326	-0.326	-0.00036	-0.327
10	-0.463	-0.464	-0.0018	-0.461
11	-0.599	-0.603	0.00375	-0.596
12	-0.736	-0.741	0.00565	-0.73
13	-0.872	-0.88	0.0076	-0.865
14	-1.009	-1.018	0.0097	-0.999
15	-1.145	-1.157	0.01203	-1.133

Table 6

Parameters of the new impulse solution for *N* = 15

N	$\Delta V_{t1}, m/s$	$\Delta V_{t2}, m/s$	$\Delta V_{z1}, m/s$	$\Delta V_{z2}, m/s$	$\Delta V_t, m/s$	$\Delta V_z, m/s$	$\Delta V_1, m/s$	$\Delta V_2, m/s$	$\Delta V, m/s$
1	-0.04	0.317	0.045	0.843	0.357	0.888	0.060	0.901	0.961
2	-0.066	0.294	0.029	0.791	0.36	0.82	0.072	0.844	0.916
3	-0.092	0.271	0.014	0.73	0.363	0.744	0.093	0.779	0.872
…	…	…	…	…	…	…	…	…	…
13	-0.375	0.046	-0.079	0.124	0.421	0.203	0.383	0.132	0.515
14	-0.404	0.024	-0.086	0.065	0.428	0.151	0.413	0.069	0.482
15	-0.435	0.003	-0.089	-0.009	0.438	0.098	0.444	0.009	0.453
$\sum$	-3.501	2.378	-0.501	5.402	5.879	7.073	3.585	6.82	10.405

It can be seen that the accuracy of the semi-major axis formation has increased.

It took four iterations to solve the problem. The information about the fourth iteration is given below (in Table 9).

Fourth iteration. In the next iteration, an impulse solution is first sought for the deviations of the orbital elements at each turn.

Then, the duration of the maneuvers is determined and shown in Table 10.

The change made in the semi-major axis is determined and shown in Table 11.

The good accuracy of the semi-major axis formation was obtained, so the iterative procedure is completed.

The duration of maneuvers is converted into impulse values. These results are shown in Table 12.

Maneuvers are calculated in a similar way for various thrust values from a given range.

The results are shown in the summary Table 13.

Table 7

Duration of the maneuver for *N* = 15

N	$\Delta\varphi_{1i}$ °	$\Delta\varphi_{2i}$ °	$\Delta\varphi_{i}$ °
1	3.82	60.588	64.408
2	4.57	55.777	60.347
3	5.944	51.103	57.047
...	...	...	...
13	24.551	8.425	32.976
14	26.499	4.379	30.878
15	28.498	0.636	29.134

Table 8

Changes made of the semi-major axis under the influence of low thrust and errors in the correction of the semi-major axis

N	$\Delta a_{0i} \times 10^{-4}$	$\Delta a_{i} \times 10^{-4}$	$\delta a_{i} \times 10^{-4}$	$\Delta a_{1i} \times 10^{-4}$
1	0.765	0.717	0.00483	0.775
2	0.629	0.617	0.0116	0.61
3	0.492	0.494	-0.0017	0.467
4	0.356	0.3614	-0.0055	0.333
5	0.219	0.225	-0.00569	0.201
6	0.083	0.0876	-0.00461	0.0694
7	-0.053	-0.0503	-0.00314	-0.0622
8	-0.19	-0.188	-0.00164	-0.194
9	-0.326	-0.326	-0.00022	-0.327
10	-0.463	-0.454	0.0011	-0.46
11	-0.599	-0.602	0.00233	-0.593
12	-0.736	-0.723	-0.0124	-0.7425
13	-0.872	-0.869	0.00466	-0.867
14	-1.009	-1.014	0.00566	-0.993
15	-1.145	-1.15	0.00437	-1.129

Table 9

Parameters of the next impulse solution for *N* = 15

N	$\Delta V_{t1}, m/s$	$\Delta V_{t2}, m/s$	$\Delta V_{z1}, m/s$	$\Delta V_{z2}, m/s$	$\Delta V_{t}, m/s$	$\Delta V_{z}, m/s$	$\Delta V_{1}, m/s$	$\Delta V_{2}, m/s$	$\Delta V, m/s$
1	-0.031	0.315	0.019	0.848	0.346	0.867	0.036	0.905	0.941
2	-0.061	0.293	0.015	0.788	0.354	0.803	0.063	0.841	0.904
3	-0.093	0.271	0.016	0.73	0.364	0.746	0.094	0.779	0.873
...	...	...	...	...	...	...	...	...	...
13	-0.373	0.046	-0.084	0.124	0.419	0.208	0.382	0.132	0.514
14	-0.401	0.024	-0.096	0.065	0.425	0.161	0.412	0.069	0.481
15	-0.434	0.005	-0.093	-0.012	0.439	0.105	0.444	0.013	0.457
Σ	-3.496	2.381	-0.517	6.382	5.877	7.039	3.562	6.834	10.396

Table 10

Duration of the maneuver for *N* = 15

N	$\Delta\boldsymbol{\varphi}_{1i}$ °	$\Delta\boldsymbol{\varphi}_{2i}$ °	$\Delta\boldsymbol{\varphi}_{i}$ °
1	2.273	60.18	62.453
2	3.991	55.573	59.564
3	5.999	51.128	57.127
…	…	…	…
13	24.499	8.401	32.9
14	26.406	4.437	30.843
15	28.463	0.832	29.295

Table 11

Changes made of the semi-major axis under the influence of low thrust and errors in the correction of the semi-major axis

N	$\Delta a_{0i} \times 10^{-4}$	$\Delta a_i \times 10^{-4}$	$\delta a_i \times 10^{-4}$	$\Delta a_{1i} \times 10^{-4}$
1	0.765	0.774	-0.00872	0.738
2	0.629	0.638	-0.009	0.6
3	0.492	0.492	0.000198	0.468
4	0.356	0.356	0.0000162	0.333
5	0.219	0.219	-0.0000054	0.2
6	0.083	0.0831	-0.000121	0.0672
7	-0.053	-0.0532	-0.000263	-0.0646
8	-0.19	-0.19	-0.0002505	-0.385
9	-0.326	-0.326	-0.0000469	-0.327
10	-0.463	-0.463	0.000289	-0.458
11	-0.599	-0.6	0.000481	-0.589
12	-0.736	-0.734	-0.00196	-0.745
13	-0.872	-0.875	0.0029	-0.857
14	-1.009	-1.01	0.001031	-0.987
15	-1.145	-1.145	0.000124	-1.127

Table 12

Parameters of the solution with low thrust for *N* = 15

N	ΔV_{t1}, m/s	ΔV_{t2}, m/s	ΔV_{z1}, m/s	ΔV_{z2}, m/s	ΔV_t, m/s	ΔV_z, m/s	ΔV_1, m/s	ΔV_2, m/s	ΔV, m/s
1	0.035	0.33	0.006	-0.888	0.365	0.894	0.036	0.947	0.983
2	0.062	0.305	0.011	-0.82	0.367	0.831	0.063	0.875	0.938
3	0.093	0.28	0.017	-0.755	0.373	0.772	0.095	0.805	0.900
4	0.12	0.256	0.022	-0.688	0.376	0.71	0.122	0.734	0.856
5	0.148	0.231	0.027	-0.623	0.379	0.650	0.150	0.664	0.815
6	0.176	0.207	0.032	-0.558	0.383	0.59	0.179	0.595	0.774
7	0.204	0.184	0.038	-0.494	0.388	0.532	0.208	0.527	0.735
8	0.232	0.16	0.043	-0.431	0.392	0.474	0.236	0.460	0.696
9	0.261	0.137	0.048	-0.368	0.398	0.416	0.265	0.393	0.658
10	0.29	0.114	0.054	-0.306	0.404	0.36	0.295	0.327	0.622
11	0.319	0.091	0.059	-0.245	0.41	0.304	0.324	0.261	0.586
12	0.349	0.07	0.064	-0.187	0.419	0.251	0.355	0.200	0.554
13	0.379	0.046	0.07	-0.124	0.425	0.194	0.385	0.132	0.518
14	0.409	0.024	0.075	-0.065	0.433	0.14	0.416	0.069	0.485
15	0.441	0.005	0.081	-0.012	0.446	0.093	0.448	0.013	0.461
$\sum$	3.518	2.44	0.647	-6.564	5.958	7.211	3.577	7.003	10.580

Table 13

Parameters of the solution with respect to maximal thrust magnitude

T, N	ΔV, m/s	M, kg
1	10.580	4.892
2	10.377	4.798
5	10.32	4.772
10	10.318	4.771
100	10.308	4.766

Conclusion

The paper describes an algorithm for calculating the parameters of the multi-turn, multi-impulse meeting. The main advantage of the proposed algorithm is its simplicity and reliability, which allows it to be used not only in ground control centers, but also on board a spacecraft. In the same time, this algorithm makes it possible to obtain an optimal solution to the problem in the case when the initial phase belongs to the optimal phase range and the total characteristic velocity of solving the meeting problem coincides with the total characterristic velocity of the optimal solution to the transfer problem. The algorithm makes it possible to obtain a solution even in the case when maneuvers are performed by low-thrust engines. Each stage of the algorithm is transparent for understanding and control. The examples given in the article confirm the performance of this algorithm and the high quality of the resulting solution.

References

1. Prussing JE. Optimal two- and three-impulse fixed-time rendezvous in the vicinity of a circular orbit. *AIAA Journal.* 1970;8(7):46–56. https://doi.org/10.2514/3.5876

2. Marec J.P. *Optimal space trajectories* (vol. 1). Amsterdam, Oxford, New York: Elsevier Sci. Publ. Co.; 1979.

3. Bulynin YuL. Ballistic support for orbital motion control of geostationary spacecraft at various stages of operation. *System Analysis, Control and Navigation: Abstracts of Reports*. Crimea, Yevpatoria; 2008. P. 73–74. ISBN 978-5-4465-3279-7. (In Russ.)

4. Rylov YuP. Control of a spacecraft entering the satellite system using electric rocket engines. *Kosmicheskie issledovaniya.* 1985;23(5):691–700. (In Russ.)

5. Kulakov AYu. *Model and algorithms of reconfiguration of the spacecraft motion control system (dissertation of the candidate of Technical Sciences).* St. Petersburg; 2017. (In Russ.)

6. Tkachenko IS. Analysis of key technologies for creating multisatellite orbital constellations of small spacecraft. *Ontology of Designing.* 2021;11(4):478–499. https://doi.org/10.18287/2223-9537-2021-11-4-478-499

7. Bazhinov IK, Gavrilov VP, Yastrebov VD, et al. *Navigation support for the flight of the Salyut — 6-Soyuz-Progress orbital complex.* Moscow: Nauka Publ.; 1985. (In Russ.)

8. Baranov A.A. Algorithm for calculating the parameters of four-impulse transitions between close almost-circular orbits. *Cosmic Research.* 1986;24(3):324–327.

9. Lidov ML. Mathematical analogy between some optimal problems of trajectory corrections and selection of measurements and algorithms of their solution. *Kosmicheskie Issledovaniya.* 1971;9(5):687–706. (In Russ.)

10. Gavrilov V, Obukhov E. Correction problem with fixed number of impulses. *Kosmicheskie Issledovaniya.*1980;18(2):163–172. (In Russ.)

11. Lion PM, Handelsman M. Basis-vector for pulse trajectories with a given flight time. *Rocket Technology and Cosmonautics.* 1968;6(1):153–160. (In Russ.)

12. Jezewski DJ, Rozendaal HL. An efficient method for calculating optimal free-space n-impulse trajectories. *AIAA Journal.* 1968;6(11):2160–2165. (In Russ.)

13. Baranov AA. Geometric solution of the problem of a rendezvous on close nearly circular coplanar orbits. *Cosmic Research.* 1989;27(6):689–697.

14. Baranov AA, Roldugin DS. Six-impulse maneuvers for rendezvous of spacecraft in near-circular non-coplanar orbits. *Cosmic Research.* 2012;50(6):441–448.

15. Edelbaum TN. Minimum Impulse Transfer in the Vicinity of a Circular Orbit. *Journal of the Astronautical Sciences.* 1967;XIV(2):66–73.

16. Lebedev VN. *Calculation of the motion of a spacecraft with low thrust*. Moscow: Publishing House of the USSR Academy of Sciences, 1968. (In Russ.)

17. Grodzovsky GL, Ivanov YuN, Tokarev VV. *Mechanics of low-thrust space flight*. Moscow: Nauka Publ.; 1966. (In Russ.)

18. Petukhov VG. Continuation method for optimization of low-thrust interplanetary trajectories. *Cosmic Research.* 2012;50(3):258–270. (In Russ.) EDN: OXXIVF

19. Petukhov VG, Olívio AP. Optimization of the finite-thrust trajectory in the vicinity of a circular orbit. *Advances in the Astronautical Sciences.* 2021;174:5–15.

20. Baranov AA. *Maneuvering in the vicinity of a circular orbit.* Moscow: Sputnik+ Publ.; 2016. (In Russ.)

21. Ulybyshev YuP. Optimization of multi-mode rendezvous trajectories with constraints. *Cosmic Research.* 2008;46(2):133–145. (In Russ.)

22. Ilyin VA, Kuzmak GE. *Optimal flights of spacecraft.* Moscow: Nauka Publ.; 1976. (In Russ.)

23. Baranov AA, Olivio AP. Coplanar multi-turn rendezvous in near-circular orbit using a low-thrust engine. *RUDN Journal of Engineering Research.* 2022; 23(4):283–292. http://doi.org/10.22363/2312-8143-2022-23-4-283-29

24. Baranov A.A, de Prado AFB, Razumny VY., Baranov Jr.AA. Optimal low-thrust transfers between close near-circular coplanar orbits. *Cosmic Research.* 2011;49(3):269–279. https://doi.org/10.1134/S0010952511030014

25. Clohessy WH, Wiltshire RS. Terminal Guidance System for Satellite Rendezvous. *Journal of the Aerospace Sciences.* 1960;27(9):653–678. https://doi.org/10.2514/8.8704

26. Hill GW. Researches in Lunar Theory. *American Journal of Mathematics.* 1878;1:5–26.

27. Elyasberg PE. *Introduction to the theory of flight of artificial Earth satellites.* Moscow: Nauka Publ.; 1965. (In Russ.)

Список литературы

1. *Прассинг Ж.Е.* Оптимальные двух- и трехимпульсные встречи в окрестности круговой орбиты при фиксированном времени перехода // Ракетная техника и космонавтика. 1970. Т. 8. № 7. С. 46–56. https://doi.org/10.2514/3.5876

2. *Marec J.P.* Optimal space trajectories // Studies in Astronautics. Vol. 1. Amsterdam, Oxford, New York: Elsevier Sci. Publ. Co.; 1979. 329 p.

3. *Булынин Ю.Л.* Баллистическое обеспечение управления орбитальным движением геостационарных КА на различных этапах эксплуатации // 13-я Международная научная конференция «Системный анализ, управление и навигация»: тезисы докладов. Крым, Евпатория, 2008. С. 73–74. ISBN 978-5-4465-3279-7.

4. *Рылов Ю.П.* Управление космическим аппаратом, входящим в спутниковую систему при помощи электроракетных двигателей // Космические исследования. 1985. Т. 23. № 5. С. 691–700.

5. *Кулаков А.Ю.* Модель и алгоритмы реконфигурации системы управления движением космического аппарата: дис. … канд. техн. наук. СПб., 2017. 156 с.

6. *Ткаченко И.С.* Анализ ключевых технологий создания многоспутниковых орбитальных группировок малых космических аппаратов // Онтология проектирования. 2021. Т. 11. № 4 (42). С. 478–499. https://doi.org/10.18287/2223-9537-2021-11-4-478-499

7. *Бажинов И.К., Гаврилов В.П., Ястребов В.Д. и др.* Навигационное обеспечение полета орбитального комплекса «Салют — 6» — «Союз» — «Прогресс». М.: Наука, 1985.

8. *Баранов А.А.* Алгоритм расчета параметров четырехимпульсных переходов между близкими околокруговыми орбитам // Космические исследования. 1986. Т. 24. № 3. С. 400–403

9. *Лидов М.Л.* Математическая аналогия между некоторыми оптимальными задачами коррекции траекторий и выбора состава измерений и алгоритмы их решения // Космические исследования. 1971. Т. 9. № 5. С. 687–706.

10. *Гаврилов В.П.*, *Обухов Е.В.* Задача коррекции с ограничением на число импульсов // Космические исследования. 1980. Т. 18. № 2. С. 163–172.

11. *Лайон П.М., Хенделсмен М.* Базис-вектор для импульсных траекторий с заданным временем перелёта // Ракетная техника и космонавтика. 1968. Т. 6. № 1. С. 153–160.

12. *Ежевски Д.Дж., Розендал Х.Л.* Эффективный метод расчета оптимальных *N*–импульсных траекторий полета в космическом пространстве // Ракетная техника и космонавтика. 1968. Т. 6. № 11. С. 138–145.

13. *Баранов А.А.* О геометрическом решении задачи встречи на близких почти круговых компланарных орбитах // Космические исследования. 1989. Т. 27. № 6. С. 808–816.

14. *Баранов А.А., Ролдугин Д.С.* Шестиимпульсные маневры встречи КА на околокруговых некомпланарных орбитах // Космические исследования. 2012. Т. 50. № 6. С. 472–479.

15. *Edelbaum T.N.* Minimum Impulse Transfer in the Vicinity of a Circular Orbit // Journal of the Astronautical Sciences. 1967. Vol. XIV. No. 2. P. 66–73.

16. *Лебедев В.Н.* Расчет движения космического аппарата с малой тягой. М.: Изд. ВЦ АН СССР, 1968. 108 с.

17. *Гродзовский Г.Л.*, *Иванов Ю.Н.*, *Токарев В.В.* Механика космического полета с малой тягой. М.: Наука, 1966. 680 с.

18. *Петухов В.Г.* Метод продолжения для оптимизации межпланетных траекторий с малой тягой // Космические исследования. 2012. Т. 50. № 3. С. 258–270.

19. *Petukhov V.G., Olívio A.P.* Optimization of the Finite-Thrust Trajectory in the Vicinity of a Circular Orbit // Advances in the Astronautical Sciences. 2021. Vol. 174. P. 5–15.

20. *Баранов А.А.* Маневрирование в окрестности круговой орбиты. М.: Спутник, Москва. 2016. 512 с.

21. *Улыбышев Ю.П.* Оптимизация межорбитальных перелетов с малой тягой при ограничениях // Космические исследования. 2012. Т. 50. № 5. С. 403–418.

22. *Ильин В.А., Кузмак Г.Е.* Оптимальные перелеты космических аппаратов. М.: Наука, 1976. 744 с.

23. *Baranov A.A., Olivio A.P.* Coplanar multi-turn rendezvous in near-circular orbit using a low-thrust engine // Вестник Российского университета дружбы

народов. Серия: Инженерные исследования. 2022. Т. 23. № 4. С. 283–292. http://doi.org/10.22363/2312-8143-2022-23-4-283-292

24. *Баранов А.А., Прадо А.Ф.Б., Разумный В.Ю., Баранов А.А.* Оптимальные переходы с малой тягой между близкими околокруговыми компланарными орбитами // Космические исследования. 2011. Т. 49. № 3. С. 278–288.

25. *Clohessy W.H., Wiltshire R.S.* Terminal Guidance System for Satellite Rendezvous // Journal of the Aerospace Sciences. 1960. Vol. 27. No. 9. P. 653–678. https://doi.org/10.2514/8.8704

26. *Hill G.W.* Researches in Lunar Theory // American Journal of Mathematics. 1878. Vol. 1. P. 5–26.

27. *Эльясберг П.Е.* Введение в теорию полета искусственных спутников Земли. М.: Наука, 1965. 540 с.

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