Вестник РУДН. Серия: Инженерные исследования
RUDN Journal of Engineering Research

2023;24(2):166–176

ISSN 2312-8143 (Print); ISSN 2312-8151 (Online)
journals.rudn.ru/engineering-researches

DOI: 10.22363/2312-8143-2023-24-2-166-176
EDN: BNFZFA
UDC 629.5.083.5

Research article / Научная статья

Kinematic surfaces with congruent generatrix curves

Sergey N. Krivoshapko

RUDN University, *Moscow, Russian Federation*
sn_krivoshapko@mail.ru

Article history
Received: December 21, 2022
Revised: February 22, 2023
Accepted: February 26, 2023

Keywords:
rotative surface, spiroidal surface, translation surface, surface of congruent sections

Abstract. Kinematic surfaces of general type are considered to include rotational, spiroidal and translation surfaces. The author indicates that a class of kinematic surfaces under consideration also comprises instances of eleven other classes. A classification of general kinematic surfaces with congruent formations is compiled, with the most well-known specific kinematic surfaces indicated in addition to classes and groups containing surfaces formed by a rigid curve as it moves in space. The classification is based on the methods of forming the kinematic surfaces: (1) a stationary and mobile axoid with a generatrix curve rigidly connected to it; (2) a stationary directrix curve and a mobile rigid generatrix curve sliding along the directrix curve with the curves not necessarily having a common point; (3) the translation surfaces of one plane curve along another, with the curves sharing a common sliding point. The suggestion of organising a class of kinematic surfaces of general type does not imply their exception from the other classes of surfaces. The term "kinematic surfaces of general type" is used when it is necessary to show the wider group of surfaces but not to enumerate all classes with examined surfaces. The application of kinematic surfaces in construction, mechanical engineering is described, the explanation of some natural phenomena and processes in electrodynamics, fluid dynamics and astrophysics for the simulation of spiral objects is given.

For citation
Krivoshapko SN. Kinematic surfaces with congruent generatrix curves. *RUDN Journal of Engineering Research*. 2023;24(2):166–176. http://doi.org/10.22363/2312-8143-2023-24-2-166-176

Кинематические поверхности с конгруэнтными образующими кривыми

С.Н. Кривошапко

Российский университет дружбы народов, *Москва, Российская Федерация*
sn_krivoshapko@mail.ru

История статьи
Поступила в редакцию: 21 декабря 2022 г.
Доработана: 22 февраля 2023 г.
Принята к публикации: 26 февраля 2023 г.

Аннотация. Общепризнано, что кинематические поверхности общего вида полностью включают в себя ротативные и спироидальные поверхности, а также поверхности переноса. Показано, что класс рассматриваемых кинематических поверхностей включает в себя также

Ключевые слова:
ротативная поверхность, спироидальная поверхность, поверхность переноса, поверхности конгруэнтных сечений

представителей одиннадцати других классов. Составлена классификация кинематических поверхностей общего вида с конгруэнтными образующими, где помимо классов и групп, содержащих поверхности, образованные жесткой кривой при ее движении в пространстве, указаны наиболее известные конкретные кинематические поверхности. При этом учитывались способы образования кинематических поверхностей: 1) наличие неподвижного и подвижного аксоида с жестко связанной с ним образующей кривой; 2) неподвижной направляющей кривой и образующей подвижной жесткой кривой, скользящей вдоль направляющей кривой, причем кривым необязательно иметь общую точку; 3) поверхности переноса одной плоской кривой вдоль другой, причем кривые имеют одну общую точку скольжения. Предложение по организации класса кинематических поверхностей общего вида не подразумевает их исключения из других классов поверхностей. Термин «кинематические поверхности общего вида» используется, когда нужно показать более широкую группу поверхностей, а не перечислять все классы поверхностей, куда входят исследуемые поверхности. Описано применение кинематических поверхностей в строительстве, машиностроении, дано объяснение некоторых природных явлений и процессов в электродинамике, динамике жидкости и астрофизике для моделирования спиральных объектов.

Для цитирования
Krivoshapko S.N. Kinematic surfaces with congruent generatrix curves // Вестник Российского университета дружбы народов. Серия: Инженерные исследования. 2023. Т. 24. № 2. С. 166–176. http://doi.org/10.22363/2312-8143-2023-24-2-166-176

Introduction

The paper [1] allocates all analytical surfaces into 38 classes. The same surface can be included in several classes. For example, a spherical surface is in the class of rotational surfaces, in the class of constant Gaussian curvature surfaces, or constant mean curvature surfaces, or in the sub-class of cyclic rotational surfaces, or Weingarten surfaces, etc.

Kinematic surfaces of general type are identified as a separate class of surfaces. The generating curve of a *kinematic surface of general type,* moving to each subsequent position, may retain a certain character of movement, but the motion parameters, positions of axes and directions of infinitesimally small summands of motions of the generating line are continuously changing. These motions may be of the following types:

(1) translational motion of alternating direction;

(2) rotational motion with continuously alternating position in space and direction of the axis of rotation;

(3) screw motion with continuously alternating position and direction of the screw axis, and continuously alternating screw motion parameter.

Depending on the motion type of the generating curve, kinematic surfaces of general type may be classified into (1) translation surfaces; (2) rotational surfaces; (3) spiroidal surfaces (Figure 1).

Sometimes kinematic surfaces of a general type are called congruent section surfaces. A *congruent section surface* refers to a surface that contains a continuous uniparametric family of plane lines. The surface is derived from the motion of a given rigid plane line (generatrix) [2]. For the first time, the distinguishing of congruent section surfaces into a separate class was suggested by famous scientist, Doctor of Physical and Mathematical Sciences, Ivan I. Kotov (MAI, Moscow) in 1973.

The distinction of surfaces into separate classes simplified the presentation of construction methods of these surfaces by means of computer graphics and descriptive geometry of surfaces [3].

Kinetic surfaces and structures constitute a special group of structures that are closer to transforming structures. In recent years, kinetical facade surfaces have appeared: architectural facades which dynamically change, transforming buildings from static monoliths into permanently moving surfaces [4]. They are not included in the class of surfaces under consideration. The group of kinetic surfaces can comprise torse surfaces obtained by parabolic bending of a plane workpiece [5].

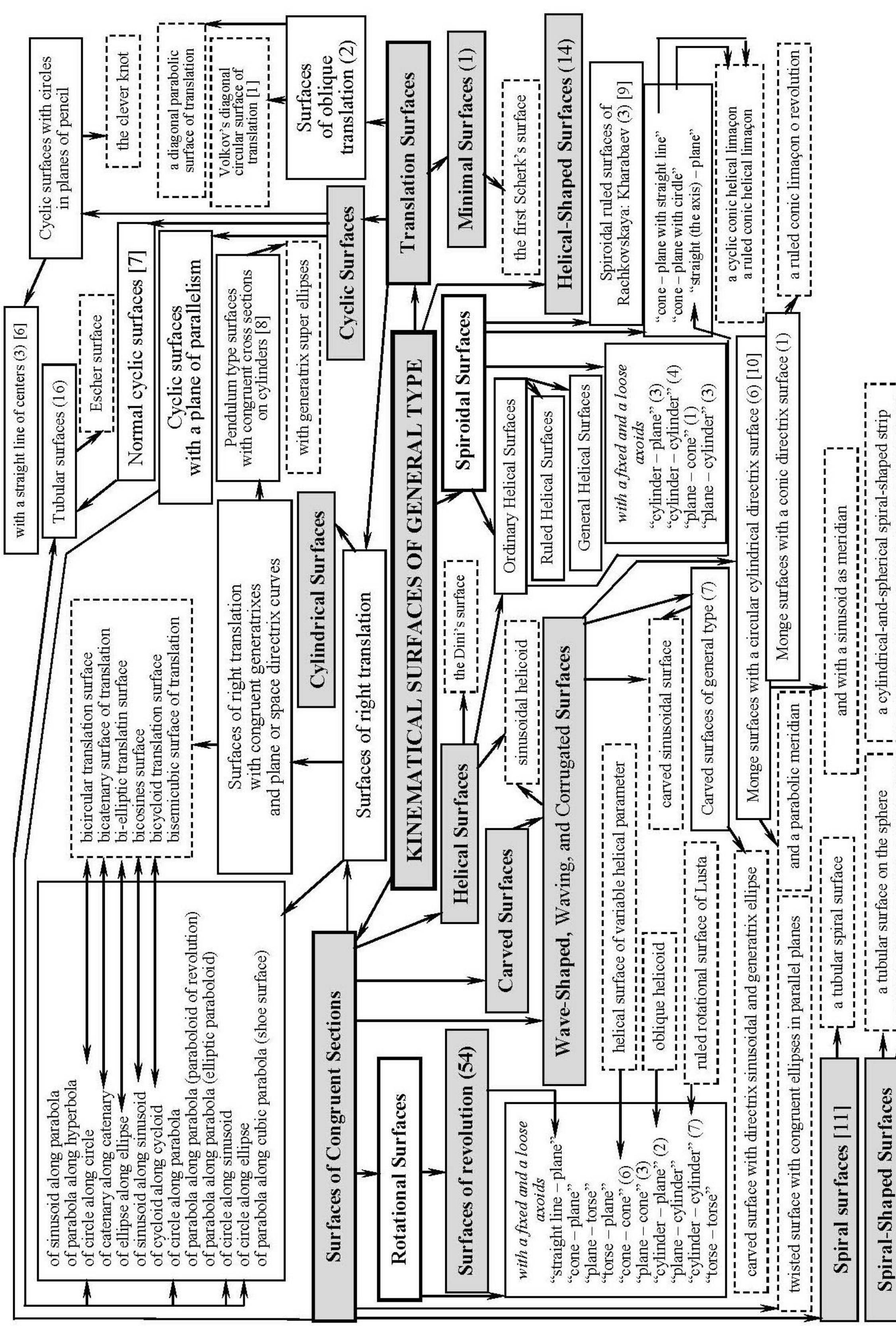

Figure 1. Distribution of kinematical surfaces of general type among the classes, the sub-classes, the groups, and the sub-groups

1. Purpose of the study

The purpose of the study is to search for the main published papers where kinematic surfaces are studied. Then, for ease of reference, to try and divide them into subclasses, groups and sub-groups and, finally, to classify them graphically, as it was done, for example, for linear surfaces [12], or rotational and translation surfaces [13].

The proposal for organizing a class of kinematic surfaces of general type does not imply their exclusion from other classes of surfaces. The term "kinematic surfaces of general type" is used when it is necessary to indicate a wider group of surfaces rather than to list all the surface classes which include the surfaces under study.

Tatiana Yu. Alaeva [14] believes that "the difficulty of classifying surfaces consists not only in their infinite diversity and complexity of geometrical characteristics, but primarily in the multiplicity and variety of ways of forming and defining surfaces, between which it is sometimes complicated to find something in common. Therefore, it is not possible to classify surfaces based on how they are defined. It is incorrect to divide surfaces into kinematic, wireframe, graphical, topographic, etc. These features describe the way the surface is defined, not the surface itself. Any surface can be defined in an unlimited number of ways. The basis for classifying surfaces can only be their geometric features."

However, despite the different opinions on classification issues, the method, based on the surface formation of a rigid (congruent) plane curve moving in space, will be considered as a basis for the classification of kinematic surfaces.

2. Explanation of the symbols

The darkened frames indicate the surface classes commonly accepted in practice. The non-darkened frames with an outline in the form of a solid line indicate subclasses, groups and sub-groups of surfaces. In the dotted outline frames, the names of the specific surfaces are given. The frames with bold outline specify the "Translation surfaces" class and two subclasses of "Rotational surfaces" and "Spiroidal surfaces", which are fully included in the "Kinematic surfaces of general type" class. The instances of other classes represented in the darkened frames in Figure 1 are only partially included (Figure 1).

The numbers in parentheses indicate how many specific surfaces described in the scientific literature belong to a given surface group. The numbers in square brackets identify the reference number in the "Reference list" section, in which the corresponding surfaces are examined most thoroughly.

3. Summary of studies on kinematic surfaces

According to Figure 1, the "Kinematic surfaces of general type" class has instances of 14 surface classes that have the same property – the surfaces are formed by the motion of congruent curves in space. For example, if we consider that *rotational surfaces* are formed by the rotation of a meridian around the surface axis, then these surfaces can be attributed to the considered class of surfaces.

It is generally accepted that kinematic surfaces are sub-divided into translation, rotational and spiroidal surfaces. However, the following classification demonstrates that some more surfaces of 11 classes can be included in this classification.

Translation surfaces refer to surfaces formed by a parallel (progressive) translation of a curve of one direction so that a certain point of the curve slides on another curve. The directrix and generatrix curves of the *direct translation* surface lie in mutually perpendicular planes. One translation surface, the *Scherck surface* (the first) is even in the "Minimal surfaces" class. *Diagonal translation surfaces* are formed by a parallel translation of a plane curve so that its two symmetric points continuously touch the plane outline. In some works, velaroidal surfaces are considered as translation surfaces. A *velaroidal* surface refers to a translation surface with a generatrix curve that changes its curvature in motion that results in a surface on a plane rectangular plane. Hence velaroidal surfaces cannot be considered a class of kinematic surfaces formed by congruent sections. An extended classification of the translation surfaces is given in the paper [13].

A *rotational surface* is formed by an arbitrary curve in the case of rolling without slipping of the moving torse, with which the generating curve is rigidly connected, over the stationary torse [15]. Thus, the whole class of rotational surfaces can be included in the class of kinematic surfaces of general type. Plane, cylinder, cone and straight line are private torse forms. The torse can only roll without slipping along its curvature. Figure 1 contains ten possible combinations of stationary and mobile axoids. A cone and a cylinder, a cylinder and a torse cannot form pairs of axoids for rotational motion. A *skew helicoid* can also be viewed as a rotational surface.

In this case, the mobile axoid must be assumed to be the plane that will roll along the stationary circular cylinder. The generatrix straight line must be parallel to the movable plane, cross the axis of the straight circular cylinder at a given acute angle and be rigidly connected to the movable plane. A Lust ruled rotational surface is formed by rolling a cylinder of $R/2$ radius on a cylinder of R radius with the generatrix straight line lying in the meridional plane of the movable cylinder [16].

A *spiroidal surface* can be defined by two stationary and mobile axoids adjacent to each other along a common generatrix (Figure 1), and a generatrix line rigidly connected to the mobile axoid in its initial position. The mobile axoid is due to the helical motion [15; 17]. A spiroidal surface with "cone-plane" axoids and a generating straight line lying in a movable plane is called a *ruled conical screw shell.* If, however, a circle is considered to be the generatrix curve, then it is called a *cyclic conical screw shell.* An *ordinary screw surface* can be called a degenerate cylindrical screw shell if we assume that the stationary axoid-cylinder has degenerated into its axis – a straight line. The *Rachkovskaya – Harabayev spiroidal ruled surfaces* are formed by the generatrix a selected straight generatrix of a given arbitrary cone, taken as a mobile axoid, which rolls with slipping along another torse surface, i.e. a stationary axoid. The apex of the cone is always on the edge of regression and the two ruled surfaces are in contact with each other at every moment along their common straight generatrix [9].

A large number of kinematic surfaces can be identified in the *"Cyclic surfaces"* class. For example, the group of tubular surfaces with a generating circle of constant radius contains interesting surfaces such as a circular torus, a tubular spiral, a tubular screw surface, a tubular surface on the sphere (Figure 2), a tubular loxodroma, etc. All these surfaces are described in the encyclopaedia [1]. Tubular surfaces refer to the "Normal cyclic surfaces" subclass [7]. There are also kinematic surfaces in the subclasses of *"Cyclic surfaces with parallelism plane"* and *"Cyclic surfaces with circles in beam planes".* The most familiar cyclic surfaces with circles of constant radius in the beam planes are the *St. Elijah surface, the clover knot, the circular screw and the circular spiral surfaces.* In principle, all cyclic surfaces with a generating circle of constant radius can also be included in the class of congruent section surfaces.

Recently a series of articles concerning study and visualization of congruent section surfaces on pendulum-type cylinders has appeared [8; 18]. Figure 3 shows a surface which can be simultaneously attributed to cyclic surfaces with parallelism plane or to congruent section surfaces of pendulum type on cylinders.

Figure 2. The tubular surface on the sphere

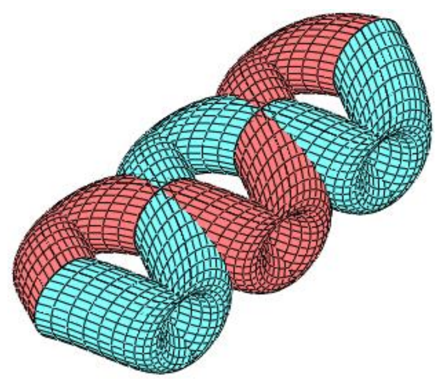

Figure 3. The computer model of the two joined cyclic surfaces of congruent circular sections of pendulum type

There is no consensus among geometricians on which surfaces should be included in the "Spiral surfaces" class. In [19], this class comprises spherical, conical and cylindrical spiral surfaces. However, the [1] divides these surfaces into 4 classes: "Spiral surfaces", "Screwed surfaces", "Screw-shaped surfaces" and "Spiral-shaped surfaces".

In this study, the classification is based on the following definitions of the 4 classes listed below, whose instances can be included in the kinematic surfaces of general type.

A *screw surface* is formed by a congruent curve in screw motion, i.e. the generating curve rotates evenly around the axis of rotation and simultaneously makes a translation motion in the direction of the same axis. If the ratio of straight-line velocity to angular velocity is constant, the screw motion is called *ordinary*, and the surface is called an *ordinary screw surface* (Figure 4). A *sinusoidal helicoid* is a *twisted surface with congruent sinusoids in parallel planes.* The surface under consideration is formed by rotating a sinusoid located in a plane perpendicular to the axis of rotation. The sinusoidal helicoid is presented only in [1]. The *Dini helicoid*, which is formed by the ordinary screw motion of a tractrix, is very well-known. *Pseudosphere* is a particular case of the Dini helicoid. If in screw motion the ratio of translation velocity to angular velocity is variable, then the trajectories of points of the generating curve will be cylindrical screw lines with variable pitch, and the screw sur-face will be termed a screw surface with variable pitch.

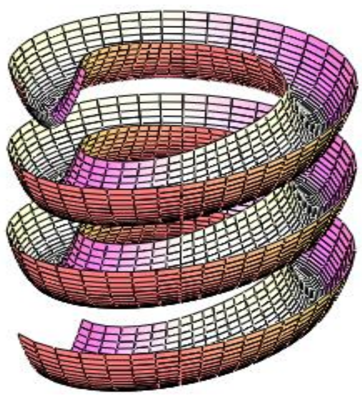

Figure 4. A variety of a screw surface of the congruent parabolic sections with the axes parallel to the axis of the guiding cylinder

The generatrix rigid curve forming a *spiral surface* makes a screw motion and simultaneously undergoes a similar transformation with a similarity coefficient, proportional to the rotation angle and with a constant similarity centre located on the rotation axis (Figure 5). The trajectories of curve points in the specified motion will be located on circular cones.

Figure 5. The spiral surface with a generatrix ellipse

Any spiral on any surface can be considered as the directrix curve of a *spiral surface*. However, the generatrix curve cannot change its shape as it moves along the directrix curve. Any spiral on any surface can be considered as the directrix curve of a spiral surface, but the forming curve cannot change its shape as it moves along the guiding curve. A large set of spirals on rotational surfaces is given in [20]. A tubular surface on the sphere (Figure 2) can also be called a spiral surface.

Screw-shaped surfaces refer to surfaces constructed by generatrix rigid curves which, in addition to a simple screw motion about the screw axis, make some additional motion (Figure 6). Screw-shaped surfaces can degenerate into screw surfaces under certain selection of geometric parameters. Screw-shaped surfaces with congruent generatrix curves are uncommon in the scientific literature.

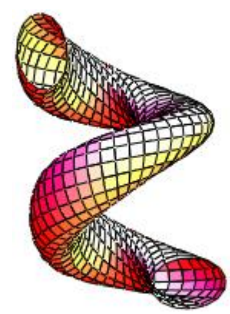

Figure 6. The screw-shaped twisted surface of the elliptical cross section in the beam planes

Carved surfaces are the surfaces in which the planes of one family of curvature plane lines are orthogonal to the surface.

A *carved surface of general type* may be considered a surface formed by a plane generatrix curve with one point moving along an arbitrary directrix curve. The curve must be in the normal plane of the directrix curve all the time. In this case, all coordinate lines of one family are congruent curves and the other family of coordinate lines is orthogonal to them. A *carved sinusoidal surface* is formed by a plane sine generatrix as one of its points moves along a plane sine directrix in the normal plane of the sine directrix (Figure 7). The carved sinusoidal surface can also be attributed to the class of undulating surfaces. Six more carved surfaces of general type are described in the encyclopaedia [1].

A *Monge carved surface* can be constructed by the kinematic method of rolling without slipping a plane with a plane line (meridian) along a developable surface. The simplest example of a Monge surface is a *rotational surface.* The method of constructing carved surfaces makes it possible to divide them into three groups depending on the type of directrix surface (*stationary axoid*): carved surfaces with cylindrical, conical and torse directrix surfaces. A *ruled conical shell of rotation* is a Monge carved surface with a conical directrix surface and a generatrix straight curve lying in a plane that rolls without slipping along a circular cone.

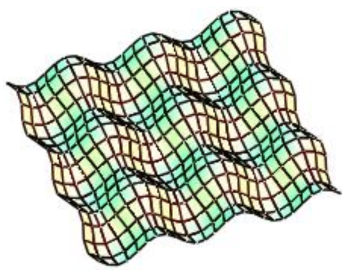

Figure 7. The carved sinusoidal surface

An example of a congruent section surface is a *twisted surface with congruent ellipses in parallel planes*, which is formed by rotation of ellipse $X = X(v) = b\cos v$; $Y = Y(v) = c\sin v$, located in the plane perpendicular to the rotation axis. The origin of the movable coordinate system oXY is located on the stationary coordinate axis Ox (Figure 8).

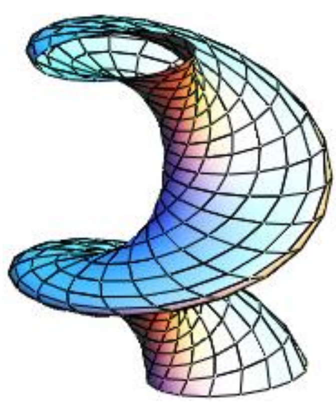

Figure 8. The twisted surface with congruent ellipses in parallel planes

Sometimes it is difficult to classify some surfaces into a familiar surface class, so it was necessary to organise the “Undulating, waving and corrugated surfaces” class.

Undulating surfaces are formed by the reciprocating and oscillating motion of rigid generatrix curves fluctuating relative to pre-selected basic surfaces, planes or lines. According to the definition of these surfaces, they can also include many familiar kinematic surfaces of a general type, as shown in Figure 1.

Waving surfaces are formed by the reciprocating and oscillating motion of generatrix curves which not only fluctuate relative to selected basic surfaces, planes or lines, but also deform themselves (Figure 9). Consequently, this group of surfaces cannot be included in the “Congruent section surfaces” class.

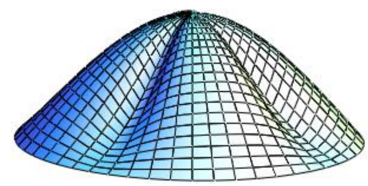

Figure 9. The waving surface with the pseudo Agnesi curls on a circular plan

Corrugated products are made by bending sheet metal and non-metal materials to give their surfaces a wave-shaped form with different sections in order to increase their strength.

4. Applications of kinematic surfaces

The monograph [21] addresses some issues of dynamic shaping in architecture. Some ideas from this research can be used for the surfaces under consideration.

Vyacheslav N. Ivanov [22] suggests several sketches of single and multi-wave vaulted coverings in the form of tubular shell fragments. Three real structures in the form of congruent section surfaces are indicated in [8]. In [23], among hundreds of structures illustrations, one can find a dozen of public buildings in the form of congruent section surfaces. Alexey V. Efremenko in his PhD thesis [24] reveals that the choice of the optimal coverage of a construction site can be made by selecting paired combinations of axoids and varying their parameters and parameters of generatrix curves, which allows obtaining multivariant solutions for the model based on rotational space transformations. The method developed by the author was used in the design of coverings for a water sports complex in Rostov-on-Don and a shopping mall in Kamensk.

Screw surface of parabolic congruent sections with axis parallel to vertical Oz axis in beam planes passing through Oz axis is used as chute for descending of bulk or liquid cargo (Figure 3). There is a description of a spiral screw conveyor surface [25].

In [26] it is proposed to use spiral minimum surfaces in astrophysics for simulation of spiral objects or in fluid dynamics. The technical literature gives examples of the use of spiral surfaces on circular cones in waste-heat boilers [27], vibrating conveyors, loading devices of a barrel, extraction apparatus, etc.

Vladimir V. Torshin [28] describes many natural objects and phenomena by means of spiral (spiral-shaped) surfaces. In addition, the researcher found an application of spiral surfaces to represent phenomena in electrodynamics.

Corrugated surfaces are the most widely used in mechanical engineering.

The possibility of practical application of engineering and computer graphics methods and computational geometry to visualise frames, required for machining of parts on metal-cutting equipment, is studied in [29]. Here, methods of system mathematical modelling of congruent section surfaces are considered.

Many examples of screw-shaped building and mechanical engineering constructions are given in the overview [30]. These include screw-shaped fragments of constructions and buildings, screw supports and anchors. Screw-shaped surfaces can be seen in constructions of building machines and mechanisms, screw lifts, in gear shapes and gear transmission grooves, and in heat-exchange units [31]. Screw and screw-shaped surfaces are common in blade shapes in the marine, aircraft and other mechanical engineering industries. Some examples of the cyclic surfaces, indicated in Figure 1, used in practice are given in [32].

There are hundreds of actually constructed facilities in the form of rotation and translation surfaces. They have been described in thousands of publications.

In view of the extensive use of kinematic surfaces in practice, mechanical engineers provide analytical [33] and numerical [34] methods for the calculating the strength of products and constructions in the form of various types of kinematic surfaces [35] under static and dynamic loads [31]. Geometricians also continued geometric researches [36].

5. Results

The surfaces of 38 classes were analysed. Thus, it was found that kinematic surfaces of general type with congruent generatrix curves exist in 14 classes. A classification of kinematic surfaces was compiled. It is necessary to note that the first attempt to set up a classification of kinematic surfaces was undertaken by Zolotuhin [37]. The methods of kinematic surface formation were used while making the classification. The classification will simplify the presentation of methods of construction of these surfaces by means of computer graphics. In [38] examples of surface construction with the system of plane coordinate lines using MathCad and AUTOCad software packages are considered. In particular, drawings of normal surfaces with constant generatrix curve, Monge carved surfaces, translation surfaces, cyclic surfaces, screw surfaces with generatrix curves in beam planes and other kinematic surfaces are shown in Figure 1.

The material in the “Applications of kinematic surfaces” section provides an insight into how geometric studies influence the development of certain branches of mechanical engineering, construction and the modeling of natural phenomena.

Conclusion

In this study, a classification of kinematic surfaces with a rigid mobile generatrix curve is first proposed. The inclusion of individual surface groups into the classification is explained. The references mentioned are focused only on the geometrical problems of the considered surfaces, their application in practice and visualization of the surfaces by means of computer graphics.

References

1. Krivoshapko SN, Ivanov VN. *Encyclopedia of analytical surfaces*. Springer; 2015. https://doi.org/10.1007/978-3-319-11773-7

2. Krivoshapko SN, Ivanov VN. Surfaces of congruent sections on cylinder. *Vestnik MGSU*. 2020;15(12): 1620–1631. (In Russ.) https://doi.org/10.22227/1997-0935.2020.12.1620-1631

3. Kheyfets AL, Galimov D, Shleykov I. Kinematic and analytical surfaces programming for solution of architectural designing tasks. *GraphiCon’ 2001, September*. Nizhny Novgorod; 2001. p. 283–286.

4. Bi M, He Y, Li Z, Lee T-U, Min Xie Y. Design and construction of kinetic structures based on elastic strips. *Automation in Construction*. 2023;146:104659. https://doi.org/10.1016/j.autcon.2022.104659

5. Krivoshapko SN. On parabolic bending of plane metal sheet into torse structure. *Tekhnologiya Mashinostroeniya*. 2020;11(221):14–24. (In Russ.)

6. Krivoshapko SN. Cyclic surfaces with the circles in the planes of pencil and with the straight directrixes. *Structural Mechanics of Engineering Constructions and Buildings*. 2004;(13):8–13. (In Russ.)

7. Ivanov VN, Shmelyova AA. Geometry and forming of thin-walled space structures on the basis of normal cyclic surfaces. *Structural Mechanics of Engineering Constructions and Buildings*. 2016;(6):3–8. (In Russ.)

8. Alborova LA, Strashnov SV. Surfaces of congruent sections of pendulum type on cylinders with generatrix superellipses. *Structural Mechanics of Engineering Constructions and Buildings*. 2022;18(1):64–72. http://doi.org/10.22363/1815-5235-2022-18-1-64-72

9. Rachkovskaya GS, Kharabaev YuN, Rachkovskaya NS. The computer modelling of kinematic linear surfaces (based on the complex moving a cone along a torse). *Proceedings of the International Conference on Computing, Communication and Control Technologies (CCCT 2004)*. Austin, Texas; 2004. p. 107–111.

10. Romanova VA. Forming of Monge's surfaces by kinematical method with the help of AUTOCAD. *Structural Mechanics of Engineering Constructions and Buildings*. 2019;15(2):106–116. (In Russ.) http://doi.org/10.22363/1815-5235-2019-15-2-106-116

11. Virich SO. Design of spiral surfaces with the help of congruences of helical lines. *Geometrichne i Kompyutrne Modelyuvannya*. 2005;(9):28–31.

12. Krivoshapko SN. Analytical ruled surfaces and their complete classification. *Structural Mechanics of Engineering Constructions and Buildings*. 2020;16(2):131–138. (In Russ.) http://doi.org/10.22363/1815-5235-2020-16-2-131-138

13. Strashnov SV, Rynkovskaya MI. On question of classification of analytical surfaces. *Geometry and Graphics*. 2022;10(1):36–43. https://doi.org/10.12737/2308-4898-2022-10-1-36-43

14. Alaeva TYu. On question of classification of surfaces. *Actual Problems of Science in the Agro-Industrial Complex: Proceedings of the 71st International Scientific-Practical Conference* (vol. 2). Karavaevo; 2020. p. 6–11. (In Russ.)

15. Krivoshapko SN, Shambina SL. Researches and visualization of rotative and spiroidal surfaces. *Applied Geometry and Engineering Graphics. Proceedings of TDATU*. 2011;49(4):33–41.

16. Lusta GI. A review on rotational surfaces. *Trudy Moskovskogo Nauchno-Metodicheskogo Seminara po Nachertatelnoy Geometrii i Ingenernoy Grafike*. 1963;(2): 120–124. (In Russ.)

17. Yadgarov DYa, Sholomov IH. Analytical method of design of spiroidal surfaces with axoids "torse – torse". *Prikladnaya Geometriya ta Inzhenernaya Grafika*. 1983;(35):102–105. (In Russ.)

18. Krivoshapko SN, Shambina SL. The pendulum type surfaces with congruential cross sections. *Structural Mechanics of Engineering Constructions and Buildings*. 2021;17(2):165–174. https://doi.org/10.22363/1815-5235-2021-17-2-165-174

19. Lebedev VA, Solovjov VP, Webb BW. View factors of spherical, conic, and cylindrical spiral surfaces. *Journal of Quantitative Spectroscopy and Radiative Transfer*. 2021; 274(1):107866. https://doi.org/10.1016/j.jqsrt.2021.107866

20. Lazureanu C. Spirals on surfaces of revolution. *VisMath*. 2014;16(2):1–10.

21. Saprykina NA. *The bases of dynamic forming in architecture*. Moscow: Architecture-C Publ.; 2005. (In Russ.)

22. Ivanov VN. Geometry and design of tubular shells. *RUDN Journal of Engineering Research*. 2005; (1):109–114. (In Russ.)

23. Krasic S. *Geometrijske površi u arhitekturi*. Gradevinsko-arhitektonski fakultet Univerzitet u Nišu; 2012.

24. Efremenko AV. *Research of ruled and non-ruled surfaces on the basis of new types of transformation of space* (Thesis of PhD diss.). Nizhniy Novgorod; 2000. (In Russ.)

25. Isaev YuM, Grishin OP, Nastin AA, Semashkin NM, Shurekov AV. Rolling of grains from spiral-and-helical surface of the conveyer. *Modern High Technologies*. 2008;(7):86–87.

26. Kiselev AV, Varlamov VI. *On spiral minimal surfaces*. Cornell University; 2006. p. 1–18.

27. Tzetzerin YuA, Serogodskiy VN. *Repair of reclamation kettle in chemical industry*. Moscow: Khimiya Publ.; 1984.

28. Torshin VV. *Spiral formation in nature and electrodynamics*. Moscow: TzP VASIZDAST Publ.; 2008.

29. Nartya VI. *Unitized-and-matrix method of mathematical modelling of surfaces*. Vologda: Infa-Ingeneriya Publ.; 2016. (In Russ.)

30. Krivoshapko SN. *Analysis and design of helix-shaped structures used in building and building machines* (review information) (issue 1). Moscow: VNIINTPI Publ.; 2006. (In Russ.)

31. Whiston GS. Use of screw translational symmetry for the vibration analysis of structures. *International Journal for Numerical Methods in Engineering*. 1982;18(3):435–444.

32. Bock Hyeng C.A., Yamb E.B. Application of cyclic shells in architecture, machine design, and bionics. *International Journal of Modern Engineering Researches*. 2012;2(3):799–806.

33. Juhanio Marulanda Ar. *Analysis of shells in the form of carved surfaces of Monge*. Ph diss. Moscow: UDN Publ.; 1970. (In Russ.)

34. Yakupov NM. Analysis of carved surfaces. *Actual Problems of Mechanics of Shells*. Kazan: KISI Publ.; 1988. (In Russ.)

35. Skidan IA. *Kinematical modelling of kinematical surfaces in special coordinates*. DS diss. Moscow: MADI Publ.; 1989. (In Russ.)

36. Efimov MI. Calculation of volume of compartments limited by some rotative surfaces. *Nachertatelnaya Geometriya i eyo Prilozheniya.* 1979;(3):103–105 (In Russ.)

37. Zolotuhin VF. Classification of surfaces. *Collection of Scientific and Methodological Articles on Descriptive Geometry and Engineering Graphics* (issue 10). Moscow: Vysshaya shkola Publ.; 1983. p. 3–12. (In Russ.)

38. Ivanov VN, Romanova VA. *Constructive forms of spatial structures (visualization of surfaces in MathCad, AutoCad).* Moscow: ASV Publ.; 2016. (In Russ.)

Список литературы

1. *Кривошапко С.Н., Иванов В.Н.* Энциклопедия аналитических поверхностей. М.: ЛИБРОКОМ, 2010. 560 с.

2. *Кривошапко С.Н., Иванов В.Н.* Поверхности конгруэнтных сечений на цилиндрах // Вестник МГСУ. 2020. Т. 15. Вып. 12. С. 1620–1631. https://doi.org/10.22227/1997-0935.2020.12.1620-1631

3. *Kheyfets A.L., Galimov D., Shleykov I.* Kinematic and analytical surfaces programming for solution of architectural designing tasks // GraphiCon' 2001, September. Nizhny Novgorod, 2001. Pp. 283–286.

4. *Bi M., He Y., Li Z., Lee T-U., Min Xie Y.* Design and construction of kinetic structures based on elastic strips // Automation in Construction. 2023. Vol. 146. https://doi.org/10.1016/j.autcon.2022.104659

5. *Кривошапко С.Н.* О параболическом изгибании плоского металлического листа в торсовую конструкцию // Технология машиностроения. 2020. № 11 (221). С. 14–24.

6. *Кривошапко С.Н.* Циклические поверхности с окружностями в плоскостях пучка и с прямыми направляющими // Строительная механика инженерных конструкций и сооружений. 2004. № 13. С. 8–13.

7. *Иванов В.Н., Шмелева А.А.* Геометрия и формообразование тонкостенных пространственных конструкций на основе нормальных циклических поверхностей // Строительная механика инженерных конструкций и сооружений. 2016. № 6. С. 3–8.

8. *Alborova L.A., Strashnov S.V.* Surfaces of congruent sections of pendulum type on cylinders with generatrix superellipses // Строительная механика инженерных конструкций и сооружений. 2022. Т. 18. № 1. С. 64–72. http://doi.org/10.22363/1815-5235-2022-18-1-64-72

9. *Rachkovskaya G.S., Kharabaev Yu.N., Rachkovskaya N.S.* The computer modelling of kinematic linear surfaces (based on the complex moving a cone along a torse) // Proceedings of the International Conference on Computing, Communication and Control Technologies (CCCT 2004). Austin, Texas, 2004. Pp. 107–111.

10. *Романова В.А.* Образование поверхностей Монжа кинематическим способом в среде AUTOCAD // Строительная механика инженерных конструкций и сооружений. 2019. Т. 15. № 2. С. 106–116. http://doi.org/10.22363/1815-5235-2019-15-2-106-116

11. *Вірич С.О.* Вилучення спіральної поверхні з конгруенції конічних гвинтових ліній// Геометричне та комп'ютерне моделювання. Харків: ХДУХТ, 2005. Вип. 9. С. 28–31.

12. *Кривошапко С.Н.* Аналитические линейчатые поверхности и их полная классификация // Строительная механика инженерных конструкций и сооружений. 2020. Т. 16. № 2. С. 131–138. https://doi.org/10.22363/1815-5235-2020-16-2-131-138

13. *Страшнов С.В., Рынковская М.И.* К вопросу о классификации аналитических поверхностей// Геометрия и графика. 2022. Т. 10. № 1. С. 36–43. https://doi.org/10.12737/2308-4898-2022-10-1-36-43

14. *Алаева Т.Ю.* К вопросу о классификации поверхностей // Актуальные проблемы науки в агропромышленном комплексе: сборник статей 71-й международной научно-практической конференции: в 3 томах. Том 2 / под ред. С.В. Цыбакина, М.А. Ивановой, А.В. Рожнова. Караваево, 2020. С. 6–11.

15. *Кривошапко С.Н., Шамбина С.Л.* Исследование и визуализация ротативных и спироидальных поверхностей // Прикладна геометрія та інженерна графіка. Праці Таврійський державний агротехнологічний університет. Вип. 4. Т. 49. Мелітополь: ТДАТУ, 2011. С. 33–41.

16. *Луста Г.И.* Обзор ротативных поверхностей // Труды Московского научно-методического семинара по начертательной геометрии и инженерной графике. М., 1963. Вып. 2. С. 120–124.

17. *Ядгаров Д.Я., Шоломов И.Х.* Аналитический способ конструирования спироидальных поверхностей с аксоидами торс – торс // Прикладная геометрия и инженерная графика. Киев, 1983. Вып. 35. С. 102–105.

18. *Krivoshapko S.N., Shambina S.L.* The pendulum type surfaces with congruential cross sections// Строительная механика инженерных конструкций и сооружений. 2021. Т. 17. № 2. С. 165–174. https://doi.org/10.22363/1815-5235-2021-17-2-165-174

19. *Lebedev V.A., Solovjov V.P., Webb B.W.* View factors of spherical, conic, and cylindrical spiral surfaces // Journal of Quantitative Spectroscopy and Radiative Transfer. 2021. Vol. 274. Issue 1. https://doi.org/10.1016/j.jqsrt.2021.107866

20. *Lazureanu C.* Spirals on surfaces of revolution // VisMath. 2014. Vol. 16. No. 2. Pp. 1–10.

21. *Сапрыкина Н.А.* Основы динамического формообразования в архитектуре. М.: Архитектура-С, 2005. 312 с.

22. *Иванов В.Н.* Геометрия и конструирование трубчатых оболочек // Вестник Российского университета дружбы народов. Серия: Инженерные исследования. 2005. № 1. С. 109–114.

23. *Krasic S.* Geometrijske površi u arhitekturi. Građevinsko-arhitektonski fakultet Univerzitet u Nišu, 2012. 238 p.

24. *Ефременко А.В.* Исследование линейчатых и нелинейчатых поверхностей на основе новых видов преобразования пространства: автореф. дис. … канд. техн. наук. Н. Новгород, 2000. 26 с.

25. *Исаев Ю.М., Гришин О.П., Настин А.А., Семашкин Н.М., Шуреков А.В.* Скатывание зерна со спирально-винтовой поверхности транспортера // Современные наукоемкие технологии. 2008. № 7. С. 86–87.

26. *Kiselev A.V., Varlamov V.I.* On spiral minimal surfaces. Cornell University, 2006. Pp. 1–18.

27. *Цсрерин Ю.А., Серогодский В.Н.* Ремонт котлов-утилизаторов в химической промышленности. М.: Химия, 1984. 88 с.

28. *Торшин В.В.* Спиральные образования в природе и электродинамике. М.: ЦП ВАСИЗДАСТ, 2008. 251 с.

29. *Нартя В.И.* Блочно-матричный метод математического моделирования поверхностей. Вологда: Инфра-Инженерия, 2016. 236 с.

30. *Кривошапко С.Н.* Расчет и проектирование винтообразных конструкций, применяемых в строительстве и строительных машинах (обзорная информация). М.: ВНИИНТПИ, 2006. Вып. 1. 60 с.

31. *Whiston G.S.* Use of screw translational symmetry for the vibration analysis of structures // International Journal for Numerical Methods in Engineering. 1982. Vol. 18. No 3. Pp. 435–444.

32. *Bock Hyeng C.A., Yamb E.B.* Application of cyclic shells in architecture, machine design, and bionics // International Journal of Modern Engineering Researches. 2012. Vol. 2. Issue 3. Pp. 799–806.

33. *Юханио Маруланда Ар.* Расчет оболочек в форме резных поверхностей Монжа: дис. … канд. техн. наук. М.: УДН, 1970. 154 с

34. *Якупов Н.М.* Расчет оболочек типа резных // Актуальные проблемы механики оболочек: тезисы докладов. Казань: КИСИ, 1988. С. 242

35. *Скидан И.А.* Кинематическое моделирование кинематических поверхностей в специальных координатах: дис. … д-р техн. наук. М.: МАДИ, 1989.

36. *Ефимов М.И.* Об определении объемов отсеков, ограниченных некоторыми ротативными поверхностями // Начертательная геометрия и ее приложения. 1979. № 3. С. 103–105.

37. *Золотухин В.Ф.* Классификация поверхностей // Сборник научно-методических статей по начертательной геометрии и инженерной графике. М.: Высшая школа, 1983. Вып. 10. С. 3–12

38. *Иванов В.Н., Романова В.А.* Конструкционные формы пространственных конструкций. Визуализация поверхностей в системах MathCad, AutoCad. М.: Изд-во АСВ, 2016. 412 с.

About the author

Sergey N. Krivoshapko, DSc, Professor of the Department of Civil Engineering, Academy of Engineering, RUDN University, 6 Miklukho-Maklaya St, Moscow, 117198, Russian Federation; ORCID: 0000-0002-9385-3699, Scopus Author ID: 6507572305, eLIBRARY SPIN-code: 2021-6966; sn_krivoshapko@mail.ru

Сведения об авторе

Кривошапко Сергей Николаевич, доктор технических наук, профессор департамента строительства, инженерная академия, Российский университет дружбы народов, Российская Федерация, 117198, Москва, ул. Миклухо-Маклая, д. 6; ORCID: 0000-0002-9385-3699, Scopus Author ID: 6507572305, eLIBRARY SPIN-код: 2021-6966; sn_krivoshapko@mail.ru