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Designing the low-energy lunar transfers trajectories which pass in the vicinity of the libration points of the Earth – Moon system. Part 1. Theory and method

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Abstract. A method for designing low-energy trajectory of transfer to the Moon with the insertion of a spacecraft into a low circumlunar orbit is proposed. The analysis of this trajectory is based on the solution of a boundary value problem for the system of differential equations of the restricted four-body problem. The trajectory of the low-energy flight passes through a region of space where the gravitational attraction of the Earth, the Moon, and the Sun tend to cancel. The trajectory turns out to be very sensitive to the initial conditions of the spacecraft motion. Difficulties arise in solving the boundary value problem. Weak stability boundary issue appears. An additional difficulty in designing the trajectory of a low-energy transfer of a spacecraft is related to the multi-extremality of the optimization problem under consideration. The authors assume that the transfer trajectory passes in the vicinity of the libration point L1 or L2 of the Earth – Moon system and introduces some restrictions on the velocity vector of the spacecraft at the moment the spacecraft passes the vicinity of the libration point. This assumption and the use of enumeration in space of the two main parameters of the transfer pattern allows to find an initial approximation for the low-energy transfer trajectory.

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Проектирование низкоэнергетических лунных перелетов, траектория которых проходит в окрестности точек либрации системы Земля – Луна. Часть 1. Теория и метод

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Аннотация. Предложен метод проектирования низкоэнергетических перелетов к Луне с выведением космического аппарата на низкую окололунную орбиту. Анализ траектории низкоэнергетического лунного перелета основывается на решении краевой задачи для системы дифференциальных уравнений ограниченной задачи четырех тел. Траектория низкоэнергетического перелета проходит через область пространства, где гравитационное притяжение Земли, Луны и Солнца очень близки. Поэтому траектория оказывается крайне чувствительной к начальным условиям движения космического аппарата и возникает проблема при решении краевой задачи. Дополнительная трудность проектирования траектории низкоэнергетического лунного перелета связана с многоэкстремальностью рассматриваемой оптимизационной проблемы. В исследовании выдвигается предположение, что перелетная траектория проходит в окрестности точки либрации L1 или L2 системы Земля – Луна и вводятся некоторые ограничения на вектор скорости космического аппарата в момент прохождения им окрестности точки либрации. Данное предположение с использованием перебора в пространстве двух основных параметров схемы перелета позволяет найти начальное приближение для траектории низкоэнергетического перелета.

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Introduction

The aim of the paper is to develop the method for designing low-energy trajectories for the flight to the Moon with the spacecraft (SC) insertion into the low Moon orbit (LMO, low circumlunar orbit). Traditional two impulses pattern for flights to the LMO require a relatively large braking impulse while the transition from the trajectory approaching the Moon to the circumlunar orbit takes place. This velocity impulse turns out to be greater than 800 m/s if the height of the LMO is 100 km. There is a possibility (primarily due to the use of solar gravitational perturbations) to reduce this velocity impulse. That is why; this type of trajectory is called as low-energy transfer trajectory. In literature, weak stability boundary (WSB-trajectories) is often used for low-energy trajectories of lunar flights. In this paper, a method for designing low-energy flight trajectories is proposed.

Finding low-energy lunar flights can be considered as an important part of the theory of lunar flights. The development of this theory, the development of methods for designing low-energy trajectories of lunar flights, and finding the flight patterns and trajectories of such flights are the subject of research by many teams and many researchers.

It is supposed that on low-energy flight trajectories:

– there must be a section where the perturbing action of the Sun provides the approach of the SC to the vicinity of the Moon with a small value of the selenocentric velocity,

– on the trajectory of the SC approach to the circumlunar orbit, gravitational perturbations from the Earth should provide the decrease in the energy of the selenocentric motion of the SC and the "temporary capture" of the SC by the Moon (the energy constant of the selenocentric osculating orbit becomes negative over a long time interval).

At present, low-energy lunar trajectories are realities and they have been proven not only theoretically, but also practically. These projects were successfully implemented using the considered flight trajectories: the Hiten project (MUSES-A), the ARTEMIS project and the GRAIL project. The problem of analyzing low-energy trajectories to the Moon is considered in many works [1–11]. Some of these (in particular, in [1]) propose methods for finding low-energy lunar trajectories. However, despite the fact that some of the methodological ideas used in these works are very interesting, the problem cannot be considered that it had been solved. In the present work, first of all, due to the narrowing of the area of the analyzed flight trajectories, it is possible to propose an algorithm for designing the trajectories of low-energy lunar trajectories.

1. Statement of the problem for designing the low-energy lunar transfer trajectory

The problem of finding a rational pattern for the flight from the low Earth's orbit (LEO) to the LMO is considered. The propulsion system of the SC is supposed to be chemical. In order not to tie the study to the characteristics of the used chemical propulsion system, the impulse approximation of active sites is used.

Many characteristics of LEO are considered to be known and it is considered as low circular. The orbital inclination is considered to be given. The longitude of the ascending node of the orbit (Ω) is a chosen parameter of the flight pattern. The starting point from the described orbit is also considered as a chosen parameter. It is determined by the latitude argument of the starting point (u_0) from LEO.

The target orbit of the artificial satellite of the Moon is assumed to be low circular orbit. The height of this orbit is assumed to be given. Note that the developed technique makes it possible to design launch trajectories to high circular circumlunar orbits, but the energy gain from the analyzed lunar flight pattern may turn out to be less significant than for flights to a low orbit. The fact is that when launching the SC into high circumlunar orbits, it is possible to use a three-impulse-maneuvering pattern in the vicinity of the Moon. Such a flight pattern may turn out to be more profitable than the traditional single-pulse pattern when launching the SC into high circumlunar orbits.

1.1. The flight pattern of low-energy flight to the circumlunar orbit

There is no strict definition of the concept of low-energy lunar flight. These flights are based on the fundamental possibility of using a ballistic flight to the Moon, when the SC is temporarily captured by the Moon without any rocket-dynamic maneuver (without turning on the SC engine). Temporal capture is characterized by the negative energy of the osculating selenocentric orbit of the SC. That is, the eccentricity of the osculating selenocentric trajectory of the SC becomes less than one. The selenocentric osculating orbit of the SC is a highly elongated elliptical orbit.

From the point of view of the practice of lunar flight, the temporary capture trajectory itself is to be unlikely interested. For practice, the SC must be inserted into some given target orbit. It is impossible using of the SC propulsion system. Therefore, it is supposed to use a chemical propulsion system when approaching the Moon, ensures the transfer of the SC to the given circumlunar orbit. To do this, it is necessary that the height of the circumlunar orbit of the mentioned selenocentric trajectory be no more than the height of the final circumlunar orbit. Then there is a possibility to realize the deceleration impulse of the speed, which ensures the flight to the final circumlunar orbit.

The following pattern of the flight to the Moon is analyzed. The SC on the LEO is given the velocity impulse that increases the velocity of the SC without changing the direction of the velocity vector. This velocity impulse provides a transition to the highly elongated osculating geocentric orbit, the apogee radius of which is greater than the radius of the Earth's gravity sphere. Three celestial bodies actively influence the formation of the further flight trajectory: the Earth, the Sun and the Moon. Due to the strong solar gravitational perturbation, the SC enters the vicinity of the Moon with a relatively low selenocentric velocity. Subsequently, the SC approaches the Moon and its height above the lunar surface becomes equal to the given height of the circumlunar orbit. At this moment, the SC is given by velocity impulse, which ensures the movement of the SC along the target circumlunar orbit.

1.2. Statement of the problem for designing the flight trajectory to the circumlunar orbit

In the general case, the problem of finding a rational trajectory for the flight to the circumlunar orbit can be formulated as follows. Find the following parameters of the flight pattern: the date of start (T_{st}) at the analyzed given epoch, the longitude of the ascending node of the LEO (Ω), the latitude argument of the starting point (u_0), the magnitude of the accelerating velocity impulse at the start (ΔV_1), the flight time to the target orbit of the artificial satellite of the moon (t_p), the magnitude and direction of the braking velocity impulse at the end point of the flight trajectory to the moon (ΔV_{br}) in order to:

a) the SC perform the transport task (the SC ended up on a circumlunar trajectory of a given height) and

b) the flight required minimal energy input.

Instead of the magnitude of the accelerated velocity impulse ΔV_1, we will consider the apogee radius of the osculating orbit, to which the SC is transferred by this velocity impulse r_a. We will call this orbit the intermediate one. The entire flight trajectory in the formulation under consideration is completely determined by the initial conditions of the SC motion when starting from LEO, that is, by the values of four characteristics: T_{st}, Ω, u_o, r_a. The conditions of motion at the end-point of the known flight trajectory depend on the flight time tp. With this formulation of the problem, the execution of the transport task can be reduced to satisfying the following two conditions of the equality type.

1. At the end point of the flight trajectory, the SC distance from the Moon's surface must be equal to the height of the LMO H_f. That is, the magnitude of the radius vector of the SC relative to the Moon was equal to the sum of the radius of the Moon (R_{Moon}) and the height of the orbit (H_f)

$$r_{SC\_Moon} = R_{Moon} + H_f. \qquad (1)$$

2. The radius vector of the SC relatively to the Moon and its velocity vector to the Moon must provide a given inclination of the target circumlunar orbit. The condition can be written using the expression for the unit vector of the angular momentum vector of the selenocentric orbit in the form:

$$\frac{[\mathbf{r}_{SC\_Moon}\mathbf{V}_{SC\_Moon}]_z}{\left|[\mathbf{r}_{SC\_Moon}\mathbf{V}_{SC\_Moon}]\right|} = \cos(i). \qquad (2)$$

In the last equality, the expression in square brackets is the cross product of the selenocentric radius of the SC and its selenocentric velocity (angular momentum vector). The subscript **z** denotes the projection of the angular momentum vector onto the *z*-axis of the selenocentric equatorial coordinate system (the Moon's axis of rotation). The denominator of the left side of the equality uses the modulus of the angular momentum vector. **i** on the right side of the equation is the given inclination of the plane of the target circumlunar orbit. For the often-analyzed case of a polar circumlunar orbit, the last condition takes the form $[\mathbf{r}_{SC\_Moon}\mathbf{V}_{SC\_Moon}]_z = 0$.

The listed two conditions must be satisfied by the choice of five parameters of the flight pattern: T_{st}, Ω, u_o, r_a, and t_p. Thus, the execution of the transport task is reduced to finding the flight pattern parameters that satisfy satisfy the two listed conditions of the equality type. It is clear that there are many solutions to a problem in which the number of unknowns is greater than the number of conditions of the equality type in the general case. We may be interested only in those solutions that require minimal energy for the flight. Two variants of the flight pattern optimization criterion are considered. In one of them, the summary velocity impulse (ΔV_Σ, the sum of the magnitudes of the velocity impulse departing from the Earth ΔV_1 and the breaking impulse of velocity carried out in the vicinity of the Moon when the SC is inserted into the LMO ΔV_{br}): $\Delta V_\Sigma = \Delta V_1 + \Delta V_{br}$ is considered as the optimization criterion. In the second variant, only the velocity impulse, which is carried out in the vicinity of the Moon during the insertion of the SC into the LMO (ΔV_{br}) is considered as optimization criterion.

From the point of view of the practical implementation of the considered maneuver, the second criterion is interesting because of practical implementation of the considered maneuver better perfom. The fact is that the first impulse of velocity is imparted to the SC in the LEO by the upper stage, and the margin of the characteristic velocity is quite large. In this case, a relatively small increase in the fueling of this block during the considered maneuver (a small increase in the magnitude of the first velocity impulse) leads to a strong increase in the apogee of the geocentric trajectory of the SC, which is required to implement the trajectory of a low-energy flight to the Moon. The braking impulse of velocity in the vicinity of the Moon is performed using the SC engine itself, using the fuel of the propulsion system of the SC. That is why the expediency of using a low-energy lunar flight pattern is often proved by analyzing the possibility of reducing the decelerated impulse of velocity in the vicinity of the Moon, without paying attention to the magnitude of the velocity impulse departing from the Earth.

From the point of view of the developed methodology for designing low-energy lunar trajectories, the choice of one of the two listed criteria is unprincipled. In the process of numerical analysis, the authors used both the first and second performance indicators. For practice – example, apparently, it is natural to formulate the problem in the following way: consider the magnitude of the decelerated velocity impulse as an optimization criterion, but introduce an upper limit on the magnitude of the velocity impulse of departing from the Earth. When analyzing a specific lunar mission, it is necessary to move from the impulsive formulation to the formulation of the problem with finite thrust, while the optimization criterion is to use the inserted mass into the circumlunar orbit.

Thus, the mathematical formulation of the problem of finding patterns of low-energy flight trajectories when launching the SC into circumlunar orbits can be as follows.

Find such five parameters of the flight pattern: T_{st}, Ω, u_o, r_a, t_p, which ensure the satisfaction of the two conditions listed above (1), (2) and provide a minimum indicator of energy costs or in the form

$$\Delta V_\Sigma = \Delta V_\Sigma\,(T_{\text{st}}, \Omega, u_o, r_a, t_p) \to \min, \qquad (3)$$

or in the form

$$\Delta V_{\text{br}} = \Delta V_{\text{br}}\,(T_{\text{st}}, \Omega, u_o, r_a, t_p) \to \min. \qquad (4)$$

The mathematical formulation of the problem involves finding the minimum of a function of five variables when two conditions of the equality type are satisfied (constrained optimization problem).

It is possible to reformulate the problem so that the number of equality-type conditions is reduced to one condition. This is due to the possibility of choosing the velocity impulse during the transition to a LMO not as purely braking, but in an arbitrary direction. In this case, the only condition of the equality type will be the condition for the SC to reach a point in the vicinity of the Moon with a given height (condition (1)). By choosing the velocity impulse vector $\mathbf{\Delta V_{br}}$, it is always possible to ensure the subsequent movement of the SC along a circular orbit with a given inclination, if the declination of the SC selenocentric radius vector relative to the lunar equator at the end point of the flight trajectory is less than the given inclination of the circumlunar orbit. For a typical variant of a polar circumlunar orbit, this constraint is always satisfied.

Fundamentally, a decrease in the number of satisfied equality constraints can have a favorable effect on the convergence of the iterative process of searching for a rational flight pattern and is considered as an important methodological technique.

2. Mathematical model describing the trajectory of the lunar flight

To describe the motion of the SC during its flight to the Moon, the system of differential equations of the restricted four-body problem is used. On the entire flight trajectory, the gravitational effects of the Earth, the Moon and the Sun are taken into account as material points. The position of celestial bodies is determined using the DE-406 ephemeris software. The entire trajectory is divided into geocentric and selenocentric sections. Since all gravitational forces are taken into account in both sections, the choice of the trajectory split point has practically no effect on the accuracy of the calculation. This choice is related to the technique used for finding the low-energy flight trajectory and will be explained below.

The analysis of the trajectory sections was carried out in geocentric and selenocentric coordinate systems, the main plane of which was chosen to be the plane of the ecliptic of the J2000 epoch. The axes of these coordinate systems are parallel. The trajectory sections used their own dimensionless variables.

The following system of differential equations was used to analyze the geocentric section of SC trajectory:

$$\left.\begin{aligned}
&\frac{dV_x}{dt} = -\frac{1}{r^3}x + \Phi_{\text{Moon}\,x} + \Phi_{\text{Sun}\,x};\\
&\frac{dV_y}{dt} = -\frac{1}{r^3}y + \Phi_{\text{Moon}\,y} + \Phi_{\text{Sun}\,y};\\
&\frac{dV_z}{dt} = -\frac{1}{r^3}z + \Phi_{\text{Moon}\,z} + \Phi_{\text{Sun}\,z};\\
&\frac{dx}{dt} = V_x;\ \frac{dy}{dt} = V_y;\ \frac{dz}{dt} = V_z.
\end{aligned}\right\} \qquad (5)$$

The above differential equations use dimensionless characteristics: x, y, z are the components of the geocentric radius-vector of the spacecraft in the ecliptic coordinate system; $r = \sqrt{x^2 + y^2 + z^2}$ is the value of this radius vector; V_x, V_y, V_z are the components of the spacecraft geocentric inertial velocity; $\Phi_{Moon x}$, $\Phi_{Moon y}$, Φ_{Moon} are the components of the perturbing acceleration caused by Moon; $\Phi_{Sun x}$, $\Phi_{Sun y}$, $\Phi_{Sun z}$ are the components of the perturbing acceleration caused by the Sun; t is the time.

The components of the perturbing acceleration from the Moon are:

$$\Phi_{\text{Moon}\,x} = \mu_{\text{Moon}\,b}\left(\frac{x_{\text{Moon}} - x}{r^3_{\text{SC\_Moon}}} - \frac{x_{\text{Moon}}}{r^3_{\text{Moon}}}\right);$$

$$\Phi_{\text{Moon}\,y} = \mu_{\text{Moon}\,b}\left(\frac{y_{\text{Moon}} - y}{r^3_{\text{SC\_Moon}}} - \frac{y_{\text{Moon}}}{r^3_{\text{Moon}}}\right);$$

$$\Phi_{\text{Moon}\,z} = \mu_{\text{Moon}\,b}\left(\frac{z_{\text{Moon}} - z}{r^3_{\text{SC\_Moon}}} - \frac{z_{\text{Moon}}}{r^3_{\text{Moon}}}\right), \tag{6}$$

where $\mu_{Moon b}$ is the dimensionless gravitational constant of the Moon (ratio of the gravitational constant of the Moon to the gravitational constant of the Earth) $\mu_{\text{Earth}\,b} = \frac{\mu_{\text{Moon}}}{\mu_{\text{Earth}}}$; x_{Moon}, y_{Moon}, z_{Moon} are components of the radius-vector of the Moon relative to the Earth; $r_{\text{Moon}} = \sqrt{x^2_{\text{Moon}} + y^2_{\text{Moon}} + z^2_{\text{Moon}}}$ is the value of this radius vector; $r_{\text{SC\_Moon}} = \sqrt{(x - x_{\text{Moon}})^2 + (y - y_{\text{Moon}})^2 + (z - z_{\text{Moon}})^2}$ – the value of the radius-vector of the spacecraft relative to the Moon.

The components of the perturbing acceleration from the Sun are:

$$\Phi_{\text{Sun}\,x} = \mu_{\text{Sun}\,b}\left(\frac{x_{\text{Sun}} - x}{r^3_{\text{SC\_Sun}}} - \frac{x_{\text{Sun}}}{r^3_{\text{Sun}}}\right);$$

$$\Phi_{\text{Sun}\,y} = \mu_{\text{Sun}\,b}\left(\frac{y_{\text{Sun}} - y}{r^3_{\text{SC\_Sun}}} - \frac{y_{\text{Sun}}}{r^3_{\text{Sun}}}\right);$$

$$\Phi_{\text{Sun}\,z} = \mu_{\text{Sun}\,b}\left(\frac{z_{\text{Sun}} - z}{r^3_{\text{SC\_Sun}}} - \frac{z_{\text{Sun}}}{r^3_{\text{Sun}}}\right), \tag{7}$$

where $\mu_{Sun b}$ is the dimensionless gravitational constant of the Sun (ratio of the gravitational constant of the Sun to the gravitational constant of the Earth $\mu_{\text{Sun}\,b} = \frac{\mu_{\text{Sun}}}{\mu_{\text{Earth}}}$); x_{Sun}, y_{Sun}, z_{Sun} are the components of the radius-vector of the Sun relative to the Earth; $r_{\text{Sun}} = \sqrt{x^2_{\text{Sun}} + y^2_{\text{Sun}} + z^2_{\text{Sun}}}$ is the value of this radius vector; $r_{\text{SC\_Sun}} = \sqrt{(x - x_{\text{Sun}})^2 + (y - y_{\text{Sun}})^2 + (z - z_{\text{Sun}})^2}$ is the value of the SC radius-vector relative to the Sun.

The motion on the selenocentric section of the trajectory is analyzed using a system similar to the shown system of differential equations (5). In this case, the Moon is considered as the central body, and the Earth and the Sun are the perturbing ones.

3. Methodical ideas of the developed method for designing low-energy lunar flights

The general idea of reducing the energy costs for the flight is associated with the possibility of using gravitational disturbances in the limited four-body problem (Earth-Moon-Sun-SC) during the flight to the Moon. First of all, the flight trajectory must be chosen so that solar gravitational perturbations ensure the SC approach to the vicinity of the Moon with a small value of selenocentric velocity. Therefore, like many researchers, the authors of this article tried to estimate gravitational solar perturbations as a function of the relative position of the Sun at the time of launch of the SC and the elements of the geocentric osculating orbit, to which the SC is transferred when starting from the LEO. Unfortunately, our attempts did not give a positive result. We believe there are two reasons for this:

– the fact is that the solar perturbations are very large. Therefore, the use of a technique that allows estimating perturbations as quadrature of functions depending on the elements of the unperturbed orbit turns out to be incorrect;

– simultaneously with the gravitational solar perturbations, the geocentric trajectory is also strongly perturbed by the Moon. There is a superposition (interference) of solar and lunar disturbances. This makes it difficult to estimate solar disturbances.

An attempt to consider the formulated problem as a mathematical programming problem and use local methods to find its solution is doomed to failure. The reason is a very large number of local extremums. The number of local extremums is so large that it is difficult to expect that search methods focused on finding a global extremum (for example, genetic algorithms) will cope with solving the problem without choosing a good initial approximation. In addition, the main idea of the proposed method is to find this initial approximation, significantly narrowing the range of possible solutions.

3.1. Constriction the class of considered transfer trajectories. Introduction of the conditions for the SC to fly a vicinity of the libration point of the Earth-Moon system

The paper proposes to consider only such patterns of flight to the Moon, the trajectories of which pass through the vicinity of the libration points L1 or L2 of the Earth-Moon system. More precisely, we analyze such flight trajectories that can be obtained using as an initial approximation the trajectories passing through the vicinity of these libration points. The authors do not claim that low-energy trajectories necessarily pass through the neighborhood of libration points, but it is precisely such flights that are unsearcning in the present work.

In this paper, it is assumed that the trajectory of the SC does not just pass in the vicinity of the libration point, but some restrictions are introduced on the magnitude and direction of the SC velocity vector at this moment of time. It is considered that the SC geocentric velocity vector at the moment of passage of the libration point is such that the following two conditions are satisfied: the perigee radius and the apogee radius of the osculating geocentric orbit of the SC are close to the perigee and apogee radius of the geocentric osculating orbit of the libration point.

The introduction of such restrictions can be explained as follows. It follows from the Jacobi integral of the restricted three-body problem (Earth – Moon – SC) that, within the framework of this problem, the SC can “penetrate” into the vicinity of the Moon through the vicinity of the libration point. In this case, the SC geocentric velocity vector should be close to the geocentric velocity vector of the libration point. In principle, it was possible to choose another an variant when restrictions were introduced both on the distance of the SC to the libration point and on the components of the velocity vector. The authors of the article were stopped by the fact that then (with the approach used by the authors) it would be necessary to compare values of different dimensions (distance and speed). Therefore, the described option was chosen. It analyzes (minimizes) the sum of three positive values of the same dimension (three distances):

$$J = \Delta r_L + \left| r_{\mathrm{SC}p} - r_{Lp} \right| + \left| r_{\mathrm{SC}a} - r_{La} \right|, \qquad (8)$$

where the first term Δr_L is the SC distance from the libration point (it is found as the difference between the geocentric vectors of the SC and the libration point); $r_{\mathrm{SC}p}$ and $r_{\mathrm{SC}a}$ – radius of perigee and apogee of the osculating geocentric orbit of the SC; r_{Lp} and r_{La} – perigee and apogee radius of the osculating geocentric orbit of the libration point.

Note that in some cases it may be appropriate not to use modules in the expression of the introduced functional J. To do this, we can change the functional using the squares of each of the terms.

The terms of the functional J depend on the four parameters of the flight pattern T_{st}, Ω, u_o, r_a, which determine the conditions for the motion of the SC after its launch from LEO, and the current time of motion of the SC t: $J(T_{\mathrm{st}}, \Omega, u_o, r_a, t)$. On each flight trajectory, there is a time t_1 when J is minimal. Let us denote this minimum value as I and call it the total miss of the libration point:

$$I(T_{\mathrm{st}}, \Omega, u_o, r_a) = \min_t J(T_{\mathrm{st}}, \Omega, u_o, r_a, t), \qquad (9)$$

The total miss I is a function of four arguments. Finding the initial approximation for these arguments is proposed to be performed as follows. In order to minimize the angle between the plane of the intermediate orbit and the plane of the Moon's orbit, the longitude of the ascending angle of the LEO is assumed to be equal to the longitude of the ascending angle of the Moon's orbit. The analysis shows that the longitude of the ascending node of the Moon's orbit (relative to the Earth's equator) varies in a relatively small range from –13.4° to +13.4° with a period of 18.6 years. For 2024, the longitude of the ascending node of the Moon's orbit changes from 3.77° to 0.137°. Therefore, it is acceptable to consider the initial approximation for the chosen parameter of the lunar flight pattern Ω for the launch dates in 2024 as zero. Let us also pay attention to the fact that with such a choice of the longitude of the ascending node of the LEO, the angle between the ecliptic plane and the plane of the intermediate orbit turns out to be minimal. This leads to an increase in the perturbation of the intermediate geocentric orbit of the SC by solar gravitational acceleration.

We suppose that the gravitational solar acceleration should deform the trajectory of the SC when it moves as far as possible from the Earth (at the apogee of the intermediate orbit). In order for this deformation to be significant, we considered it expedient to choose the initial approximation for the argument of the latitude of the starting point from LEO u_o so that the starting point is located near the ecliptic plane. Therefore, the argument of the latitude of the starting point from the LEO, counted from the line of nodes of the LEO relative to the plane of the ecliptic, must be equal to either zero or 180°. It is these values that are considered as initial approximations for this latitude argument. In this case, if the longitude of the ascending node of the intermediate orbit is zero, then the arguments of the latitude of the starting point, counted relative to the plane of the earth's equator, will be the same (0° and 180°). If the longitude of the ascending node of the intermediate orbit is chosen equal to 3.77°, then the value of the latitude argument will be greater than these values by 3.168°.

3.2. Use of direct enumeration of two parameters of the flight pattern to find the areas of their values that ensure the satisfaction of the conditions for passing the libration point

A serious problem in the design of lunar flight patterns is multi-extremality. Therefore, an attempt to use an approach that uses the methods of finding a local extremum, as a rule, is not successful. The solution does not provide a zero residual value of the boundary conditions. The search process in a local minimum that is significantly different from zero.

To overcome these difficulties, it is ideal to enumerate all the parameters of the flight pattern in space (with a small step for each parameter), to find such sets of parameters in which the transport problem is solved with relatively good accuracy (there will be a finite small number of such sets). Then refine the values of the parameters of each received set using local methods. A modern computer is not able to implement such an approach. To overcome the described difficulty, it is proposed to use following approach.

Of the four parameters that determine the transfer trajectory (start date T_{st}, radius of the intermediate orbit apogee r_a, longitude of the ascending node of the LEO Ω, argument of the latitude of the launch point u_o), two are selected, the rational values of which are difficult to foresee. These parameters are T_{st} and r_a. For these parameters, a complete enumeration of their values from the possible range is carried out. The start date T_{st} varies throughout the analyzed range of dates (for example, the annual range) with a fairly small step (for example, one hour). The apogee radius of the intermediate orbit r_a varies in the range of 1–1.5 million km with a step of 5 thousand km. For each point of the parameter plane $T_{st} - r_a$ with fixed values of the other two parameters chosen from rational considerations, the SC trajectory is analyzed and the value of the total miss of the libration point I is calculated: At the same time, the time t_1 of the maximum approach of the SC to the libration point is also found.

An analysis of the level lines of the total miss of the libration point on the plane $T_{st} - r_a$ makes it possible to find such launch dates and radii of the apogee of the intermediate geocentric orbit, at which the SC can reach the vicinity of the considered libration point and, at the same time, the SC velocity vector has such a magnitude and direction that the shape and size of the osculating geocentric orbit of the SC are close to the shape and size of the osculating geocentric orbit of the libration point. It is these trajectories that are considered as the initial approximation for low-energy transfer trajectories.

The date of reaching the minimum total miss of the libration point t_1 is considered as the boundary point of the geocentric and selenocentric sections of the trajectory.

At further stages of the analysis, a transition is made to the formulation, where the total miss of the libration point is not analyzed, the achieved height above the lunar surface is considered as an indicator of the solution of the transport problem. The required-height is ensured with the use of local search methods.

At the final stage of the analysis, when it is possible to achieve a given height of the SC above the lunar surface, the gradient projection method is used, which makes it possible to iteratively improve the parameters of the flight pattern (according to the criterion of fuel consumption), while remaining on the trajectory that transfers the SC to a LMO of a given height.

3.3. Introduction to the consideration of the intermediate velocity impulse – a methodological technique in the design of low-energy lunar trajectories

When designing low-energy lunar flights, a methodological technique was used related to the introduction of an intermediate velocity impulse on the flight trajectory (ΔV_c) into the flight pattern. The idea was to make it easier to solve the boundary value problem (satisfying the conditions of the transport problem). With the introduction of an intermediate velocity impulse, the dimension of the vector of the selected parameters of the flight pattern is increased by four units at once: (date of the velocity impulse, its magnitude and direction, characterized by two angles). In this case, the class of possible flight trajectories is greatly expanded, and the solution of the boundary value problem is simplified. When introducing an intermediate velocity impulse, it was assumed that in the final solution, its value would be reduced to very small (infinitely small) values, and this introduction was considered as a methodological technique. In the course of the analysis, it is possible to consider as a low-energy flight such flights on which the intermediate impulse of velocity is not infinitely small, but simply small (for example, equal to 20–30 m/s), and the sum of this impulse ΔV_c and the braking impulse at the Moon is less velocity impulse of traditional direct flights.

The initial approximation for the date of the intermediate velocity impulse is the date of reaching the trajectory point with the minimum total miss of the libration point.

Conclusion

The statement considered in this paper does not analyze the problem of optimizing the number of velocity pulses on the trajectory of a low-energy flight to the Moon. This problem is expected to be analyzed in the future. One of the possible research methods is to use the necessary optimality conditions of the maximum principle for the impulse flight trajectory.

It can also be noted that in further studies, we do not exclude the possibility of introducing an additional term into the *J*-function (8), will ensure, at the time of the passage of the vicinity of the libration point, the proximity of the plane of the geocentric osculating orbit of the SC to the plane of the geocentric orbit of the Moon.

In the second part of this work, the developed algorithm for designing a low-energy trajectory for a flight to the Moon will be described. The results of a numerical analysis of the obtained trajectories will be presented.

References

1. Parker JS, Anderson RL. *Low-energy lunar trajectory design*. Hoboken, New Jersey: John Wiley & Sons, Inc.; 2014. https://doi.org/10.1002/9781118855065

2. McCarthy BP, Howell KC. Cislunar transfer design exploiting periodic and quasi-periodic orbital structures in the four-body problem. *71st International Astronautical Congress, The CyberSpace Edition, October 12–14, 2020.* Paris; 2020.

3. Scheuerle ST, McCarthy BP, Howell KC. Construction of ballistic lunar transfers leveraging dynamical systems techniques. *AAS/AIAA Astrodynamics Specialist Conference, Lake Tahoe, California (Virtual), August 9–12, 2020.* South Lake Tahoe, California; 2021.

4. McCarthy BP, Howell KC. Trajectory design using quasi-periodic orbits in the multi-body problem. *Proceedings of the 29th AAS/AIAA Space Flight Mechanics Meeting, 2019.* Maui; 2019.

5. Ivashkin VV. On the Earth-to-Moon trajectories with temporary capture of a particle by the moon. *54th International Astronautical Congress, Bremen, Germany, September 29 – October 3, 2003.* Paper IAC-03-A.P.01. https://doi.org/10.2514/6.IAC-03-A.P.01

6. Ivashkin VV. Low energy trajectories for the Moon-to-Earth space flight. *Journal of Earth System Science.* 2005;114:613–618. https://doi.org/10.1007/BF02715945

7. Belbruno EA, Carrico JP. Calculation of weak stability boundary ballistic lunar transfer trajectories. *Proceedings of the AIAAJ'AAS Astrodynamics Specialist Conference, August 14–17, 2000, Denver, Colorado*. Paper AIAA 2000-4142. https://doi.org/10.2514/6.2000-4142

8. Belbruno EA, Miller JK. Sun-perturbed Earth-to-Moon transfers with ballistic capture. *Journal of Guidance, Control, and Dynamics*. 1993;16(4):770–774. https://doi.org/10.2514/3.21079

9. Koon WS, Lo MW, Marsden JE, Ross SD. Low energy transfers to the Moon. *Celestial Mechanics and Dynamical Astronomy*. 2001;81(1):63–73. https://doi.org/10.1023/A:1013359120468

10. Miller JK. Lunar transfer trajectory design and four body problem. *13th AAS/AIAA Space Flight Mechanics Meeting at Ponce, Puerto Rico, 2003.* American Astronomical Society, American Institute of Aeronautics and Astronautics; 2003.

11. Miller JK, Hintz GR. Weak stability boundary and trajectory design. *Spaceflight Mechanics*. Conference paper AAS 15-297. Williamsburgh, VA; 2015.

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