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Coplanar multi-turn rendezvous in near-circular orbit using a low-thrust engine

Andrey A. Baranov[a], **Adilson P. Olivio**[b]

[a]Keldysh Institute of Applied Mathematics, Russian Academy of Sciences, *Moscow, Russian Federation*
[b]Peoples' Friendship University of Russia (RUDN University), *Moscow, Russian Federation*
pedrokekule@mail.ru

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Abstract. The authors describe an algorithm that allows calculating the parameters of maneuvers performed on several turns by the low-thrust engine, which ensure the flight of the active spacecraft to the specified vicinity of the target space object. The movement takes place in the vicinity of a circular orbit. Linearized equations of motion are used in solving the problem. The influence of the non-centrality of the gravitational field and the atmosphere is not taken into account. The determination of maneuver parameters takes place in three stages. At the first and third stages, the parameters of the pulse transition and the transition performed by the low-thrust engine, are determined analytically. At the second stage, the distribution of maneuvering between turns, which provides a solution to the meeting problem, is carried out by iterating over one variable. This method of solving the problem provides simplicity and high reliability of determining the parameters of maneuvers, which allows it to be used on board the spacecraft. The paper investigates the dependence of the total characteristic speed of solving the meeting problem on the number of turns of the flight and the magnitude of the engine thrust.

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Компланарная многовитковая встреча на околокруговой орбите с помощью двигателей малой тяги

А.А. Баранов[a], **А.П. Оливио**[b]

[a]Институт прикладной математики имени М.В. Келдыша РАН, *Москва, Российская Федерация*
[b]Российский университет дружбы народов, *Москва, Российская Федерация*
pedrokekule@mail.ru

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Аннотация. Описан алгоритм, позволяющий рассчитать параметры маневров, исполняемых на нескольких витках двигателем малой тяги, обеспечивающих перелет активного космического аппарата в заданную окрестность целевого космического объекта. Движение происходит в

Ключевые слова:
космический аппарат, расчет параметров маневров, космический объект, малая тяга, круговая орбита, импульс скорости

окрестности круговой орбиты. При решении задачи используются линеаризованные уравнения движения. Влияние нецентральности гравитационного поля и атмосферы не учитываются. Определение параметров маневров происходит в три этапа. На первом и третьем этапах параметры импульсного перехода и перехода, выполняемого двигателем малой тяги, определяются аналитически. На втором этапе распределение маневрирования между витками, обеспечивающее решение задачи встречи, осуществляется перебором по одной переменной. Данный метод решения задачи прост и гарантирует высокую надежность определения параметров маневров, что позволяет использовать его на борту космического аппарата. Исследуется зависимость суммарной характеристической скорости решения задачи встречи от числа витков перелета и величины тяги двигателя.

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Introduction

The problem of the meeting in a near-circular orbit using a low-thrust engines is of great importance in the practice of spacecraft (SC) flights. This problem is solved during the rendezvous and docking of spacecraft, the implementation of a group flight of several spacecraft, the formation of a given configuration of satellite systems, the removal of space debris, and maintenance of the spacecraft.

Over the past sixty years, the problem of the meeting has been considered in the papers of many authors. Particular attention was paid to the problem of rendezvous in near-circular orbits, when maneuvers are performed by high-thrust engines. A problem of this type was encountered most often in practical paper.

As the first notable papers in this area, we can note the papers of J.E. Prussing [1], who studied the problem of a meeting with a duration of no more than three turns for the case of two circular coplanar orbits, and J.-P. Marec [2], who solved the classical problem of a meeting of average duration in near-circular orbits.

Currently, three main approaches to solving complex multi-impulse problems of spacecraft maneuvering are widely used. In the first case, the problem of maneuvering in the plane of the orbit and the problem of turning the plane of the orbit are solved independently of each other. A similar scheme was used, in particular, to implement the Shuttle rendezvous with the orbital station[1], to control the movement of geostationary satellites [3], satellites included in satellite systems [4], etc. The advantage of such a scheme is its simplicity and reliability, the disadvantage is the excessive costs of the total characteristic speed for maneuvering.

In the second case, numerical methods are used to find the optimal solution to the most complex multi-impulse problems, taking into account a wide range of restrictions [5; 6]. Most often, the simplex method is used to calculate the parameters of maneuvers [7; 8].

In the third method, at the first stage, using the solution of the Lambert problem, the parameters of the two-impulse solution of the meeting problem are determined. Then the behavior of the hodograph of the basis vector corresponding to the found solution is analyzed, and, if necessary, additional velocity impulses are added to obtain the optimal solution. This approach was first applied in the papers of Lion and Handelsman [9], Jezewski and Roosendaal [10].

There are also methods that are at the junction of different approaches. For example, in papers [11; 12] proposed numerical-analytical methods for solving the multi-impulse meeting problem, combining the advantages of the first and second of the previously listed approaches. They allow using the results obtained in the early papers of T. Edelbaum [13], J.-P. Marec [2], when solving modern practical problems.

Since the 1960s, the process of using electric rocket engines (ERE) on spacecraft has begun. Due to the high specific impulse, EREs can significantly reduce fuel costs for orbital maneuvering. However, the small (compared to traditional liquid rocket engines) thrust of the ERE leads to the need to take into account their long-term operation.

[1] Shuttle Press Kit: STS-92. Available from: https://historycollection.jsc.nasa.gov/JSCHistoryPortal/history/shuttle_pk/pk/Flight_100_STS-092_Press_Kit.pdf (accessed: 17.08.2022).

Problems of this type occupy a special place among the problems of optimal spacecraft maneuvering. A significant number of papers have been devoted to them, several very interesting monographs have been published [14; 15]. Particularly noteworthy are the papers of V.G. Petukhov [16], Petukhov and Olivio [17] and [18; 19]. Due to the complexity of the problems in which it is assumed that maneuvering is carried out using low-thrust propulsion systems, they have traditionally been solved by numerical methods using the Pontryagin maximum principle or the continuation method. In recent years, to solve problems with a large extent of maneuvers, Yu.P. Ulybyshev successfully uses the interior point method [20].

In the method considered in this paper, the meeting problem is solved both in an impulse setting and taking into account the long-term operation of a low-thrust engine.

To analyze the relative motion of a spacecraft in the vicinity of circular orbits, it is necessary to use special mathematical models of motion. The most popular mathematical model of the relative motion of spacecraft in the vicinity of circular orbits is the Hill – Clohessy – Wiltshire (HCW) model. Linearized differential equations for the relative motion of a spacecraft in the vicinity of a circular orbit for the rendezvous and docking problem were obtained by Clohessy – Wiltshire in 1960 [21], but as early as the 19th century, similar equations were used by Hill in his theory of the motion of the Moon [22]. In this mathematical model, to obtain the equations of relative motion, a rotating (orbital) coordinate system and linearization of differential equations of relative motion are used, based on the assumption that the distance between the considered spacecraft is small compared to the average radius of the orbit. In this paper, we use linearized equations obtained by P.E. Elyasberg [23].

Due to the increase in the number of maneuvering SC, the increase in the efficiency of solving problems, at present there is a tendency to transfer the process of calculating maneuvers on board the spacecraft. This leads to the need to simplify the process of calculating the parameters of maneuvers and increase.

1. Formulation of the meeting problem

The problem of calculating the parameters of flight maneuvers between close near – circular orbits is solved in an approximate impulse formulation, within the framework of the unperturbed Keplerian motion.

The flight conditions using N velocity impulse in a fixed time from the initial orbit to a given point of the final orbit (meeting problem) in linear approximation can be written as [12]:

$$\sum_{i=1}^{N}(\Delta V_{ri}\sin\varphi_i+2\Delta V_{ti}\cos\varphi_i)=\Delta e_x; \quad (1)$$

$$\sum_{i=1}^{N}(-\Delta V_{ri}\cos\varphi_i+2\Delta V_{ti}\sin\varphi_i)=\Delta e_y; \quad (2)$$

$$\sum_{i=1}^{N}2\Delta V_{ti}=\Delta a; \quad (3)$$

$$\sum_{i=1}^{N}(2\Delta V_{ri}(1-\cos\varphi_i)+\Delta V_{ti}\times \times(-3\varphi_i+4\sin\varphi_i))=\Delta t, \quad (4)$$

where $\Delta e_x = e_f\cos\omega_f - e_0\cos\omega_0$; $\Delta e_y = e_f\sin\omega_f - e_0\sin\omega_0$; $\Delta a = (a_f - a_0) / r_0$; $\Delta t = \lambda_0(t_f - t_0)$; $\Delta V_{ti} = \Delta V^*_{ti} / V_0$; $\Delta V_{ri} = \Delta V^*_{ri} / V_0$; f, 0 – indices corresponding to the final and initial orbits; e_f, e_0 – eccentricities of the orbits; a_f, a_0 – semi-major axes of the orbits; ω_f, ω_0 – angles between the direction to the pericenter of the corresponding orbit and the direction to the point specified on the final orbit (the axis Ox is directed to this point); t_f – the necessary arriving time to this point; t_0 – time at which, when moving in the initial orbit, the projection of the radius-vector in the plane of the final orbit falls on a beam passing through a given meeting point; V_0, λ_0 – orbital and angular velocities along the reference circular orbit of radius r_0 ($r_0 = a_f$); N – number of velocity impulse; φ_i – angle of application of the i-th velocity impulse, calculated from the direction to the given meeting point towards the motion of the SC; ΔV^*_{ti}, ΔV^*_{ri} – transversal and radial components of the i-th velocity impulse respectively.

It must be taken into consideration that the angles φ_i – are negative, i.e., because it was assumed that $\varphi_f = 0$ at the given point.

The problem of finding the parameters of optimal maneuvers can be formulated as follows: it's necessary to determine ΔV_{ri}, ΔV_{ti}, φ_i (i = 1, …, N), at which the total characteristic velocity of maneuvers ΔV is minimal

$$\Delta V = \sum_{i=1}^{N} \Delta V_i = \sum_{i=1}^{N} \sqrt{\Delta V_{ri}^2 + \Delta V_{ti}^2}$$

при ограничениях (1)–(4).

2. Algorithm for solving the transfer problem

When solving the problem of transfer between coplanar orbits, the first three equations of system (1)–(4) are used.

There are three types of solutions for which the necessary optimality conditions are satisfied:

a) on the μ, λ plane, the hodograph of the basis vector is an *ellipse*, the center of which is located on the μ axis, but is shifted from the origin of the coordinate system; the ellipse is tangent to a circle of unit radius at a point on the μ axis;

b) the hodograph of the basis vector degenerates into a *point* coinciding with the point of intersection of a circle of unit radius with the μ axis;

c) the hodograph of the basis vector – an *ellipse* centered at the origin of the coordinate system, touching the circle at *two* points on the μ axis.

Since all possible optimal solutions have решений $\lambda = 0$, and $\mu \neq 0$, then the velocity impulses of these solutions are purely transversal.

Assuming that the velocity impulses are applied at the optimal points for correcting the eccentricity vector,

$$\operatorname{tg}\varphi_e = \frac{\Delta e_y}{\Delta e_x}; \quad \varphi_1 = \varphi_e; \ \ \varphi_2 = \varphi_1 + \pi,$$

we find the magnitude of the velocity impulses of the optimal solution:

$$\Delta V_{t1} = \frac{1}{4}(\Delta a + \Delta e). \tag{5}$$

$$\Delta V_{t2} = \frac{1}{4}(\Delta a - \Delta e). \tag{6}$$

3. Algorithm for solving the meeting problem

When solving the meeting problem, the values of the velocity impulses ΔV_{t1}, ΔV_{t2}, determined when solving the transition problem, are distributed among N turns allowed for maneuvering:

$$\Delta V_{1t} = \sum_{i=1}^{N} \Delta V_{1ti}; \tag{7}$$

$$\Delta V_{2t} = \sum_{i=1}^{N} \Delta V_{2ti}, \tag{8}$$

where N is the number of turns on which maneuvering is allowed.

The further goal is to choose such a distribution of the magnitudes of the velocity impulses over the coils so that equation (4) is satisfied.

To significantly simplify the solution of the problem, we assume that the magnitudes of the velocity impulses along the turns change linearly:

$$\Delta V_{1ti} = \Delta V_{1t1} + (\Delta V_{1tN} - \Delta V_{1t1})(i-1)/(N-1); \tag{9}$$

$$\Delta V_{2ti} = \Delta V_{2t1} + (\Delta V_{2tN} - \Delta V_{2t1})(i-1)/(N-1), \tag{10}$$

where ΔV_{1t1}, ΔV_{1tN} and ΔV_{2t1}, ΔV_{2tN} are the value of the velocity of impulses on the first and last allowed turns of maneuvering, which is part of the first and second velocity of solving the transfer problem.

Substituting the values of the velocity impulses calculated by formulas (9), (10) into (7) and (8) we get:

$$\Delta V_{1t} = \sum_{i=1}^{N} \Delta V_{1ti} = 0.5N(\Delta V_{1t1} + \Delta V_{1tN}); \tag{11}$$

$$\Delta V_{2t} = \sum_{i=1}^{N} \Delta V_{2ti} = 0.5N(\Delta V_{2t1} + \Delta V_{2tN}). \tag{12}$$

Using (11) and (12), we obtain formulas for determining ΔV_{1tN}, ΔV_{2tN}:

$$\Delta V_{1tN} = \frac{\Delta V_{1t}}{0.5N} - \Delta V_{1t1}; \tag{13}$$

$$\Delta V_{2tN} = \frac{\Delta V_{1t1}}{0.5N} - \Delta V_{2t1}. \tag{14}$$

Substituting the found values ΔV_{1tN}, ΔV_{2tN} into formulas (9) and (10), we obtain:

$$\Delta V_{1ti} = 2\Delta V_1(i-1)/ /N(N-1) + \Delta V_{1t1}\left[1 - \frac{2(i-1)}{N-1}\right]; \quad (15)$$

$$\Delta V_{2ti} = 2\Delta V_2(i-1)/ /N(N-1) + \Delta V_{2t1}\left[1 - \frac{2(i-1)}{N-1}\right]. \quad (16)$$

Thus, we found the values of all impulses of velocity, expressed only through ΔV_{1t1} and ΔV_{2t1}. Substituting them into equation (3), we obtain a linear equation with two unknowns ΔV_{1t1}, ΔV_{2t1}. The coefficients for velocity impulses are known, since their application angles are known:

$$\varphi_{1i} = \varphi_e + 2\pi(N_i - N); \quad (17)$$

$$\varphi_{2i} = \varphi_e + \pi + 2\pi(N_i - N). \quad (18)$$

Going through the value of the variable ΔV_{1t1}, within the given limits, for each value from equation (3) we find the value of the variable ΔV_{2t1}.

Then, using (15) and (16), we find the values of all velocity impulses. Adding the modules of all velocity impulses, we find the total characteristic velocity of the next solution. The solution for which the total characteristic velocity is minimal is taken as a solution to the meeting problem. If the total characteristic velocity of the found solution coincides with the total characteristic velocity of the solution to the transition problem, then a solution was found with the minimum possible total characteristic velocity.

At the next stage, we estimate the duration of each of the found maneuvers.

The duration of each of the maneuvers is estimated using the ratio:

$$\Delta\varphi_i = \frac{w_c}{w}\Delta V_i, \quad (19)$$

where w_c – the centripetal acceleration of the reference circular orbit $\left(w_c = \frac{V_0^2}{r_0}\right)$; w – the acceleration created by the propulsion system $\left(w = \frac{P}{m}\right)$; m – the mass of the active SC; P – thrust of its engine.

If the duration of the largest velocity impulse doesn't exceed 20°, then the solution is close to the impulse one and we consider that the problem has been solved. If the duration of the maneuvers is significant, then we proceed to the solution with low thrust.

4. Solving the problem with "low thrust"

For each turn, we find what changes in the eccentricity and the semi-major axis produce velocity impulses determined on this turn

$$\Delta e_i = 2\Delta V_{1ti} - 2\Delta V_{2ti}; \quad (20)$$

$$\Delta a_i = 2\Delta V_{1ti} + 2\Delta V_{2ti}. \quad (21)$$

Then we determine the required duration of low-thrust maneuvers, which will produce the same change in these elements:

$$4\sin\frac{\Delta\varphi_{1i}}{2} - 4\sin\frac{\Delta\varphi_{2i}}{2} = \frac{w_c\Delta e_i}{w};$$

$$2\Delta\varphi_{1i} + 2\Delta\varphi_{2i} = \frac{w_c\Delta a_i}{w}. \quad (22)$$

From system (22) one can find the quantities $\Delta\varphi_{1i}$, $\Delta\varphi_{2i}$ [24]:

$$\Delta\varphi_1 = \frac{w_c\Delta a}{4wn} + 2\arcsin\frac{w_c\Delta e}{8wn\cos\frac{w_c\Delta a}{8wn}};$$

$$\Delta\varphi_2 = \frac{w_c\Delta a}{4wn} - 2\arcsin\frac{w_c\Delta e}{8wn\cos\frac{w_c\Delta a}{8wn}}. \quad (23)$$

Thus, turn by turn, we find the duration of all maneuvers. The problem with low thrust is correctly solved. If the argument of the arcsine is greater than 1, then there is no solution (with the available thrust and mass of the spacecraft, it is impossible to solve the rendezvous problem for a given number of turns).

The found solution with "low thrust" gives the same change in the semi-major axis and the eccentricity vector as the original impulse solution. Equation (4) is also quite accurate, since the middle of long maneuvers coincides with the moments of application of velocity impulses, and the same change in the major axis on the turn is made, therefore, the required time of arrival at the meeting point is provided.

5. Iterative procedure

In the formulated meeting problem, linearized equations of motion are used, the non-centrality of the gravitational field, the influence of the atmosphere, etc. are not taken into account. This leads to the fact that the actual accuracy of the fulfillment of the terminal conditions in the system (1)–(6) will be insufficient. To solve the problem with a given accuracy, you can use the iterative scheme [5; 6], which consists of the following steps:

1. At the beginning of the next iteration, the "approximate" problem is solved: under the previously accepted simplifying assumptions, the parameters of the maneuvers that ensure the formation of the "target" orbit are determined (at the first iteration, the "target" orbit coincides with the final orbit).
2. Then, taking into account the calculated velocity impulses, using the models of all necessary disturbances, an "accurate" prediction of the spacecraft motion is carried out and the parameters of the formed orbit are found.
3. The deviations of the parameters of the formed orbit from the corresponding parameters of the final orbit are calculated.
4. If the deviations exceed the allowable values, then the parameters of the "target" orbit are changed by the value of the calculated deviations, and the next iteration is carried out.
5. The procedure ends when the terminal conditions are met with the specified accuracy.
6. For "accurate" forecasting, as a rule, numerical and/or high-precision numerical-analytical integration is used. It is possible to use different forecasting methods at different iterations, but the accuracy of the forecast should increase with the number of the current iteration.
7. Numerical integration takes into account the influence of non-centrality of the gravitational field, atmosphere, light pressure, etc., the operation of the spacecraft engines is accurately modeled, therefore, in spite the fact that the maneuver parameters and are found at each iteration using the simplest motion model, but as a result of the iterative procedure, they provide access to the final orbit with the required accuracy.

6. Problem solving examples

Considering the motion of a spacecraft (SC) relative to the point O, moving along a non-perturbed near-circular orbit with a radius of 6871 km. Let's take the gravitational parameter of the Earth equal to $3.9860044 \cdot 10^{14}$ m^3/s^2. Consider the problem of a flight with the help of N impulses of velocity in a fixed time from the initial orbit to a given point of the final orbit from a point in phase space $\boldsymbol{r}_0 = (10, 100, -5)$ km, $\boldsymbol{v}_0 = (1, -10, 3)$ m/s to the origin, i.e, to the point $\boldsymbol{r}_f = (0, 0, 0)$ km, with a velocity $\boldsymbol{v}_f = (0, 0, 0)$ m/s. For the problem, we'll take the initial mass of the SC equal to 1000 kg, the specific impulse of the SC propulsion system is 220 seconds (2157.463 m/s), and the thrust (T) will be varied in the range from 0.362 to 100 N. The flight is carried out in $N = 4$ and 13 turns.

Double impulse transfer. In the Table 1 shows the results of the calculation the parameters of a two-impulse transfer between coplanar orbits, that is, the magnitude of the transversal components of the velocity impulses, the angles of application of the first and second impulses, as can be seen, the minimum value of the characteristic velocity that a SC must have for the transfer maneuver, is 4.485 m/s.

Multi-impulse solution. The value of the first velocity impulse is moved within the limits from –2.785 to 0.5 m/s with a step of 0.024 m/s.

Parameters of the optimal solutions. In the Tables 2–4 show the parameters of the optimal solution, that is, the values of the velocity impulses on the turns for the cases when $N = 4$ and 13.

Solution of the problem with low thrust. In the Tables 5, 6 and 7 shown the results of calculating the problem with low thrusts "Thrust = 1 and Thrust = 0.362", that is, the values of the velocity impulses and the duration of maneuvers on turns for cases when N=4 and 13. In some cases, there're no solutions because the value of the arcsine argument is out of range (–1; 1). With increased thrust, the duration of the maneuvers is reduced, and the costs of the total characteristic velocity (CXC) of the low thrust solution during thrust increment, coincides with costs of the CXC of the impulse solution.

Table 1

Results of the calculation the parameters of the coplanar transfer problem

ΔV_1, m/s	ΔV_2, m/s	$\lvert\Delta V\rvert$, m/s	φ_e °	φ_1°	φ_2 °
–2.785	1. 7	4.485	6.4	186.4	366.4

Table 2

Parameters of the optimal solution for *N* = 4

N	ΔV_{1i}, m/s	ΔV_{2i}, m/s	$\lvert\Delta V_{1i}\rvert + \lvert\Delta V_{2i}\rvert$, m/s
1	–0.024	0.848	0.872
2	–0.472	0.566	1.038
3	–0.92	0.284	1.204
4	–1.369	0.002	1.371
∑	–2.785	1.7	4.485

Table 3

Parameters of the optimal solution for *N* = 13

N	ΔV_{1i}, m/s	ΔV_{2i}, m/s	$\lvert\Delta V_{1i}\rvert + \lvert\Delta V_{2i}\rvert$, m/s
1	–0.001	0.199	0.2
2	–0.037	0.187	0.224
…	…	…	…
12	–0.392	0.074	0.466
13	–0.427	0.063	0.49
∑	–2.785	1.7	4.485

Table 4

Results of the calculation of the problem with low thrust, for *N* = 4

T, N	0.362	0.37	0.4	0.5	1	2	5	10	100
$\Delta\varphi$°	There is not solution	There is not solution	There is not solution	There is not solution	300.137	144.199	57.082	28.499	2.849
ΔV, m/s	There is not solution	There is not solution	There is not solution	There is not solution	4.726	4.541	4.494	4.487	4.485

Table 5

Results of the calculation of the problem with low thrust “Thrust = 1”, for *N* = 4

N	ΔV_{1i}, m/s	ΔV_{2i} , m/s	$\lvert\Delta V_{1i}\rvert + \lvert\Delta V_{2i}\rvert$, m/s	$\Delta\varphi_{1i}$°	$\Delta\varphi_{2i}$°	$(\lvert\Delta\varphi_{1i}\rvert + \lvert\Delta\varphi_{2i}\rvert)$°
1	–0.04	0.864	0.904	–2.561	54.87	57.431
2	–0.48	0.573	1.053	–30.473	36.424	66.897
3	–0.942	0.306	1.248	–59.841	19.434	79.275
4	–1.443	0.077	1.52	–91.65	4.884	96.534
∑	–2.905	1.82	4.725	–184.525	115.612	300.137

Table 6

Results of the calculation of the problem with low thrust, for *N* = 13

T, N	0.362	0.37	0.4	0.5	1	2	5	10	100
$\Delta\varphi$°	809.865	791.329	728.952	578.15	285.884	142.555	56.979	28.487	2.849
ΔV, m/s	4.616	4.61	4.591	4.551	4.501	4.489	4.486	4.485	4.485

Table 7

Results of the calculation of the problem with low thrust “Thrust = 0.362”, for *N* = 13

N	ΔV_{1i}, m/s	ΔV_{2i}, m/s	$\lvert\Delta V_{1i}\rvert + \lvert\Delta V_{2i}\rvert$, m/s	$\Delta\varphi_{1i}$°	$\Delta\varphi_{2i}$°	$(\lvert\Delta\varphi_{1i}\rvert + \lvert\Delta\varphi_{2i}\rvert)$°
1	–0.003	0.2	0.203	–0.45	35.149	35.599
2	–0.038	0.189	0.227	–6.642	33.117	39.759
…	…	…	…	…	…	…
12	–0.405	0.087	0.492	–71.015	15.253	86.268
13	–0.444	0.08	0.524	–77.953	13.967	91.92
∑	–2.85	1.765	4.615	–500.114	309. 751	809.865

Conclusion

In the paper describes an algorithm for calculating the parameters of a multi-turn multi-impulse encounter. The main advantage of the proposed algorithm is its simplicity and reliability, which makes it possible to use it not only in ground control centers, but also on board the spacecraft. Simultaneously, this algorithm makes it possible to obtain the optimal solution of the problem in the case when the initial phase belongs to the optimal phase range and the total characteristic velocity of the solution of the meeting problem coincides with the total characteristic velocity of the optimal solution of the transfer problem. The algorithm makes it possible to obtain a solution even when the maneuvers are performed by low thrust engines. The examples given in the paper confirm the operability of this algorithm and the high quality of the resulting solution.

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About the authors

Andrey A. Baranov, Candidate of Physical and Mathematical Sciences, leading researcher, Institute for Applied Mathematics, Russian Academy of Sciences, 4 Miusskaya Ploshchad', Moscow, 125047, Russian Federation; ORCID: 0000-0003-1823-9354, Scopus Author ID: 14059618600, eLIBRARY SPIN-code: 6606-3690; andrey_baranov@list.ru

Adilson Pedro Olivio, postgraduate, Department of Mechanics and Control Processes, Academy of Engineering, Peoples' Friendship University of Russia (RUDN University), 6 Miklukho-Maklaya St, Moscow, 117198, Russian Federation; ORCID: 0000-0001-5632-3747, Scopus Author ID: 57223139496, eLIBRARY SPIN-code: 7628-6084; pedrokekule@mail.ru

Сведения об авторах

Баранов Андрей Анатольевич, кандидат физико-математических наук, ведущий научный сотрудник, Институт прикладной математики имени М.В. Келдыша, Российская академия наук, Российская Федерация, 125047, Москва, Миусская пл., д. 4; ORCID: 0000-0003-1823-9354, Scopus Author ID: 14059618600, eLIBRARY SPIN-код: 6606-3690; andrey_baranov@list.ru

Оливио Адильсон Педро, аспирант, департамент механики и процессов управления, Инженерная академия, Российский университет дружбы народов, Российская Федерация, 117198, Москва, ул. Миклухо-Маклая, д. 6; ORCID: 0000-0001-5632-3747, Scopus Author ID: 57223139496, eLIBRARY SPIN-код: 7628-6084; pedrokekule@mail.ru