Вестник РУДН. Серия: Инженерные исследования
RUDN Journal of Engineering Research

2022;23(1):30–37

ISSN 2312-8143 (Print); ISSN 2312-8151 (Online)
journals.rudn.ru/engineering-researches

DOI 10.22363/2312-8143-2022-23-1-30-37
UDC 621.78.01:621.785-97

Research article / Научная статья

Generating hydrodynamic surfaces by families of Lame curves for modelling submarine hulls

Valery V. Karnevich

Peoples' Friendship University of Russia (RUDN University), *Moscow, Russian Federation*
valera.karnevich@gmail.com

Article history
Received: December 19, 2021
Revised: February 23, 2022
Accepted: February 28, 2022

Keywords:
hydrodynamic surface, algebraic surface, submarine, vessel, midsection, Lame curve

Abstract. This paper investigates the construction of hydrodynamic surfaces, which are defined by algebraic equations and describe the theoretical hull of a vessel. A technique for automation of generating hydrodynamic surfaces is proposed. This technique allows to create a vast variety of hull shapes, for which Lame curves with variable exponents are used as surface generator lines. The surface is constructed by a family of curves in one of the three mutually perpendicular planes, which permits to obtain three algebraically different, but geometrically identical surfaces. This paper introduces parametric equations for each of the three surfaces, generated by families of sections, buttocks and waterlines in the form of Lame curves. The algorithm of modelling a submarine hull with different fore and aft bodies and a parallel middle body by a closed surface is demonstrated and the modelling results are illustrated. The presented technique may be effectively applied at the early stages of ship design when choosing the optimal hull shape, for which a number of surfaces need to be considered.

For citation
Karnevich VV. Generating hydrodynamic surfaces by families of Lame curves for modelling submarine hulls. *RUDN Journal of Engineering Research*. 2022;23(1):30–37. http://doi.org/10.22363/2312-8143-2022-23-1-30-37

Построение гидродинамических поверхностей каркасами из кривых Ламе на примере корпуса подводной лодки

В.В. Карневич

Российский университет дружбы народов, *Москва, Российская Федерация*
valera.karnevich@gmail.com

История статьи
Поступила в редакцию: 19 декабря 2021 г.
Доработана: 23 февраля 2022 г.
Принята к публикации: 28 февраля 2022 г.

Аннотация. Исследуется построение гидродинамических поверхностей, которые описываются аналитическими уравнениями и формируют теоретический корпус судна. Предлагается методика автоматизации построения гидродинамических поверхностей с возможностью создания

Ключевые слова:
гидродинамическая поверхность, алгебраическая поверхность, субмарина, подводная лодка, судно, мидельшпангоут, кривые Ламе

широкого разнообразия форм корпусов, для чего используются кривые Ламе с произвольными степенями как образующие каркас поверхности. Поверхность образуется каркасом сечений в одной из трех взаимно перпендикулярных плоскостей, что позволяет получить три алгебраически отличающиеся, но геометрически идентичные поверхности. Впервые выводятся параметрические уравнения каждой из таких трех поверхностей, образованных каркасами теоретических шпангоутов, батоксов и ватерлиний в форме кривых Ламе. Продемонстрирован алгоритм моделирования корпуса подводной лодки замкнутой поверхностью с отличающимися носовой и хвостовой частями, а также с параллельной центральной вставкой и проиллюстрированы результаты моделирования. Представленная методика может эффективно применяться на ранних этапах проектирования судна при выборе оптимальной формы поверхности корпуса, для чего необходимо рассмотреть ряд различных форм.

Для цитирования
Карневич В.В. Построение гидродинамических поверхностей каркасами из кривых Ламе на примере корпуса подводной лодки // Вестник Российского университета дружбы народов. Серия: Инженерные исследования. 2022. Т. 23. № 1. С. 30–37. http://doi.org/10.22363/2312-8143-2022-23-1-30-37

Introduction

The hull form of a vessel determines the key hydrodynamic properties: maneuverability and water resistance [1]. Therefore, hull shape optimization is one of the primary tasks of naval architects. The pioneer investigations in this field may be attributed to William Froude, who by order of the Royal Institution of Naval Architects worked on identifying the most efficient ship hull form. He was able to confirm the applicability of his theoretical findings by testing physical models of ships in model basins, which were constructed for the first time ever specifically for this purpose. He derived a formula for extrapolation of small-scale test results onto the real conditions and validated its efficiency by full-scale experiments [2]. However, building and testing ship models even at the present day is substantially expensive and time-consuming, not to mention full-scale tests [3]. For this reason, today, in studying and designing ships precedence is given to computer analysis, CFD (Computational Fluid Dynamics) in particular. Authors of [4] evaluate the accuracy of the CFD method in simulating model basin tests using various cases of geometric shapes of ship hulls.

The initial step in computer analysis is creating a geometric model. The most widespread practice in geometric modelling of ship hulls is the construction of hull wireframe by a set of contours in mutually perpendicular planes: by cross-section, buttock and waterline curves. Article [5] indicates that by defining a discrete wireframe, the hull surface may be constructed only approximately, since the points that do not belong to the wireframe may not be uniquely determined. A finer wireframe increases the accuracy of the generated surface, but at the same time increases the amount of computational labor required for the analysis of the model. The authors of [6] studied hull geometry optimization with the use of AI (artificial intelligence) and concluded that the non-parametric surface model (defined by an array of point coordinates) is prone to a local optimization trap, and that the optimization direction in the parametric model (algebraic surface with a finite number of parameters) is largely affected by the first calculation iteration, so a combined model was processed by AI. Article [7] also draws attention to the disadvantages of discrete surfaces, in dynamic problems in particular, and proposes a number of structurally complex, but effective techniques of transforming hull model from a set of contours to a whole surface. Nonetheless, a parametric model is exceptionally convenient at the preliminary design stage, when it is necessary to consider a vast variety of hull forms and to use a minimal number of variables for this purpose.

This paper considers a method of modelling a ship hull with the help of algebraic surfaces, which are defined by a skeleton of three plane curves. These three curves coincide with midsection (in yOz plane), main buttock (in xOz plane) and waterline (in xOy plane), as demonstrated in Figure 1. Articles [8; 9] refer to such surfaces as hydrodynamic and point out that given the same three curves it is possible to generate three different algebraic surfaces. Article [10] presents a technique for producing hydrodynamic surfaces, which are constructed by midsection line in the form of Lame curve and parabolas in the other two perpendicular planes and which model a hull of a symmetric surface vessel. This paper proposes a more general model, where all three curves are Lame curves, and the algorithm of constructing a multi-piece surface of a vessel with different fore and aft bodies and a parallel middle body. Such surfaces may be applied in submarine hull design; an example of geometric modelling of which is given further below in this paper.

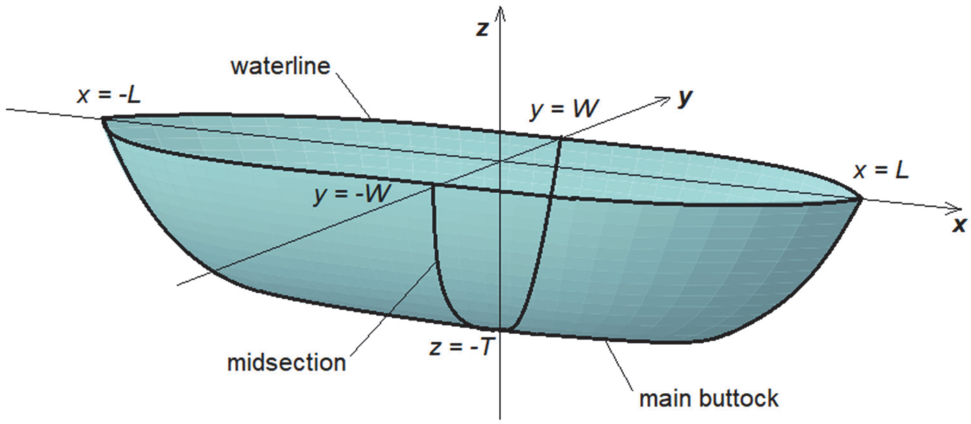


Figure 1. Hydrodynamic surface skeleton of three plane curves

Hydrodynamic surfaces generated by Lame curves

Let us assume the three lines, which form the hull skeleton, to be in the form of Lame curves in the corresponding planes:

– midsection:

$$|z|^n = T^n\left(1 - \frac{|y|^m}{W^m}\right); \tag{1}$$

– main buttock:

$$|z|^b = T^b\left(1 - \frac{|x|^a}{L^a}\right); \tag{2}$$

– waterline:

$$|y|^k = W^k\left(1 - \frac{|x|^j}{L^j}\right). \tag{3}$$

T, *W*, *L* are the geometric parameters of the hull (Figure 1), which are specified and which represent the height, width and length of the ship respectively.

Positive variable exponents *n, m, b, a, k, j* of three principal curves (1–3) allow to obtain a huge number of surface shapes. Lame curves, also known as superellipses, with different values of these exponents are shown in Figure 2. For example, the midsection line at $n = m$ between 0 and 1 is concave in the form of a four-armed star; at $n = m = 1$ it represents a rhombus; at $n = m > 1$ it is convex and represents a circle in particular case of $n = m = 2$. With increasing values of $n = m$, Lame curve approaches a rectangular shape.

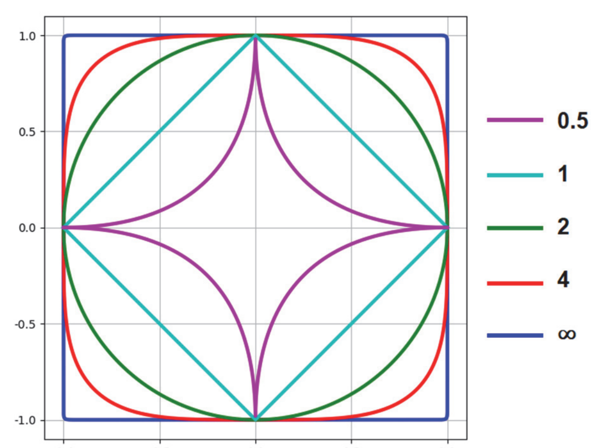


Figure 2. Shape of midsection line in the form of Lame curve at different values of parameters $n = m = 0.5; 1; 2; 4; \infty$

Since the hull surface may be generated by families of sections in three different planes, and all the three resulting surfaces will be different algebraically, let us consider the construction of each individual surface below.

Hydrodynamic surface generated by a family of sections in *yOz* plane

Let us take midsection equation (1), which defines the contour of the vessel section in *yOz* plane, as the surface generator:

$$|z|^n = T(x)^n\left(1-\frac{|y|^m}{W(x)^m}\right). \quad (4)$$

Midsection width *W* and height *T* vary depending on the particular section in *yOz* plane, i.e. depending on the *x*-coordinate, and they trace the waterline and main buttock respectively. Thus, variable width *W*(*x*) can be obtained using equation (3):

$$W(x) = W\left(1-\frac{|x|^j}{L^j}\right)^{1/k}, \quad (5)$$

and variable height *T*(*x*) can be obtained using equation (2):

$$T(x) = T\left(1-\frac{|x|^a}{L^a}\right)^{\frac{1}{b}}. \quad (6)$$

By substituting expressions (5) and (6) into equation (4), we obtain:

$$|z|^n = T^n\left(1-\frac{|x|^a}{L^a}\right)^{n/b}\left[1-\frac{|y|^m}{W^m\left(1-\frac{|x|^j}{L^j}\right)^{m/k}}\right]. \quad (7)$$

Equation (7) is the algebraic equation of the first hydrodynamic surface. This equation may be represented in parametric form:

$$x = x(u) = uL,$$

$$y = y(u,v) = vW\left(1-|u|^j\right)^{1/k}, \quad (8)$$

$$z = z(u,v) = \pm T(1-|u|^a)^{1/b}(1-|v|^m)^{1/n},$$

where $-1 \le u \le 1, -1 \le v \le 1$.

Hydrodynamic surface generated by a family of sections in *xOz* plane

The contour of the vessel section in *xOz* plane is the buttock line, so we take equation (2) as the generator:

$$|z|^b = T(y)^b\left(1-\frac{|x|^a}{L(y)^a}\right), \quad (9)$$

where height *T* and length *L* vary with *y* and trace the midsection and waterline respectively.

Then, *T*(*y*) is derived from equation (1):

$$T(y) = T\left(1-\frac{|y|^m}{W^m}\right)^{1/n}, \quad (10)$$

and *L*(*y*) is derived from equation (3):

$$L(y) = L\left(1-\frac{|y|^k}{W^k}\right)^{1/j}. \quad (11)$$

The complete equation of the surface is obtained by substituting expressions (10) and (11) into equation (9):

$$|z|^b = T^b\left(1-\frac{|y|^m}{W^m}\right)^{b/n}\left[1-\frac{x^a}{L^a\left(1-\frac{|y|^k}{W^k}\right)^{a/j}}\right]. \quad (12)$$

Parametric equations of the second hydrodynamic surface are:

$$y = y(u) = uW,$$

$$x = x(u,v) = vL\left(1-|u|^k\right)^{1/j}, \quad (13)$$

$$z = z(u,v) = \pm T(1-|u|^m)^{1/n}(1-|v|^a)^{1/b},$$

where $-1 \le u \le 1, -1 \le v \le 1$.

Hydrodynamic surface generated by a family of sections in *xOy* plane

The contour of the vessel section in *xOy* plane is the waterline, so we take equation (3) as the generator:

$$|y|^k = W(z)^k \left(1 - \frac{|x|^j}{L(z)^j}\right), \qquad (14)$$

where length L and width W vary with z and trace the main buttock and midsection respectively.

Therefore, $L(z)$ is derived from equation (2):

$$L(z) = L\left(1 - \frac{|z|^b}{T^b}\right)^{1/a}, \qquad (15)$$

and $W(z)$ is y from equation (1), so

$$W(z) = W\left(1 - \frac{|z|^n}{T^n}\right)^{1/m}. \qquad (16)$$

By substituting expressions (15) and (16) into equation (14)

$$|y|^k = W^k\left(1 - \frac{|z|^n}{T^n}\right)^{k/m}\left[1 - \frac{|x|^j}{L^j\left(1 - \frac{|z|^b}{T^b}\right)^{j/a}}\right], \qquad (17)$$

we obtain the complete equation of the surface. This equation in the parametric form is

$$z = z(u) = uT,$$

$$x = x(u,v) = vL\left(1 - |u|^b\right)^{1/a}, \qquad (18)$$

$$y = y(u,v) = \pm W(1 - |u|^n)^{1/m}\left(1 - |v|^j\right)^{1/k},$$

where $-1 \le u \le 1, -1 \le v \le 1$.

Algorithm of constructing submarine hull surface with different fore and aft bodies

Let us define variables L_f and L_a, which represent the lengths of fore and aft bodies of the vessel respectively. Height T and width W of both parts of the hull are the same for the purpose of joining the parts smoothly at $x = 0$ (Figure 3). The values of exponents m and n, which determine the surface shape in yOz plane, are also identical for both parts. The values of exponents a, b, j, k are specified for fore and aft parts separately.

The overall variation interval of the normalized independent variables is defined as $0 \le u, v \le 1$. Then, the complete surface consists of 8 fragments and is constructed by equations (8), (13) or (18) using Table 1.

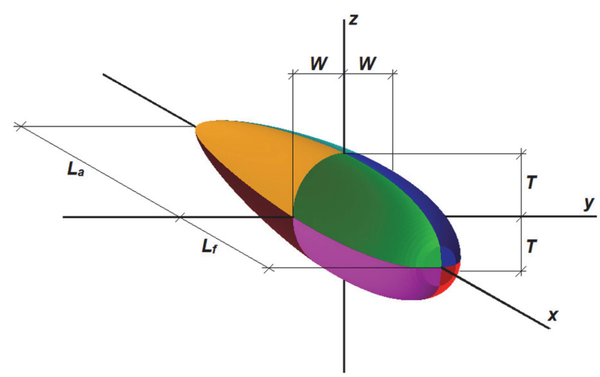


Figure 3. Surface fragments of submarine hull with different fore and aft bodies

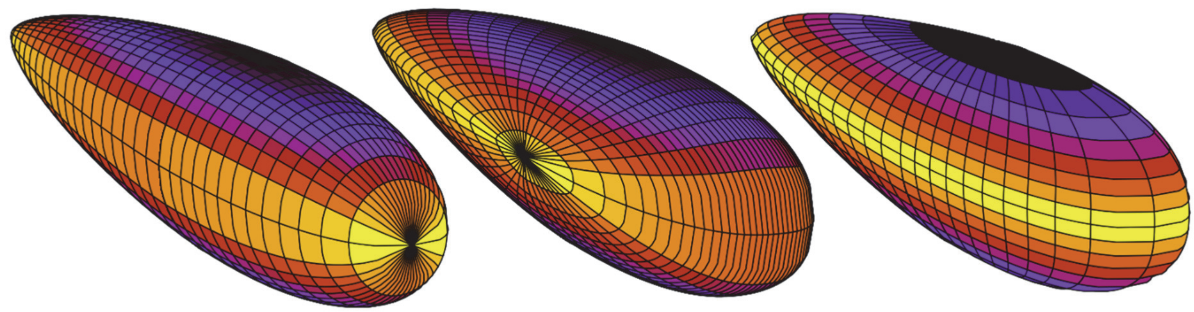
Table 1

Signs of expressions for surface fragments of submarine with different fore and aft body shapes

Vessel part		Fore body ($L = L_f$)				Aft body ($L = L_a$)			
Fragment		1	2	3	4	5	6	7	8
Expression sign	***x***	+	+	+	+	–	–	–	–
	y	+	+	–	–	+	+	–	–
	z	+	–	+	–	+	–	+	–

The visualization of the three hydrodynamic surfaces generated by families of sections in different planes is performed taking $m = n = 2$, so that the section perpendicular to the x-axis (midsection) is in the shape of a circle. The values of the remaining parameters are chosen such that they reflect the general shape of a submarine hull without a parallel middle body [11]. The three hydrodynamic surfaces are shown in Figure 4.

In order to increase usable space, submarine hulls are often designed with a cylindrical insert (parallel middle body) between the fore and aft bodies. Let us define the length of the parallel body as L_m and position it such that its middle point is located at $x = 0$. Then, the fore and aft bodies are displaced to locations $x = +L_m/2$ and $x = -L_m/2$ respectively (Figure 5).



Generated using equation (8) *Generated using equation (13)* *Generated using equation (18)*

Figure 4. Three hydrodynamic surfaces with $L_f = 40$ m, $L_a = 20$ m, $W = T = 5$ m; $m = n = 2$; $a = b = j = k = 2.5$ (for fore body); $a = b = j = k = 1.5$ (for aft body)

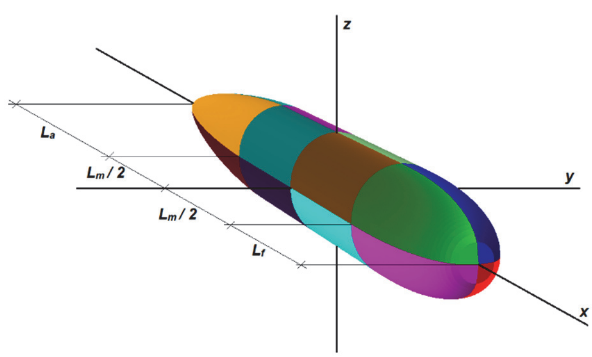


Figure 5. Surface fragments of submarine hull with a parallel middle body between fore and aft bodies

In the parametric equations of the fore and aft body fragments a term of $+L_m/2$ needs to be added to the expressions for x. Variation interval of parameters u and v remains to be from 0 to 1. Then, a fragment of the parallel middle body can be defined by the following parametric equations:

$$x = x(u) = uL_m,$$

$$y = y(v) = vW, \qquad (19)$$

$$z = z(u, v) = T(1 - |v|^m)^{1/n},$$

where the expression for z is obtained from equation (1), which specifies the shape of the midsection.

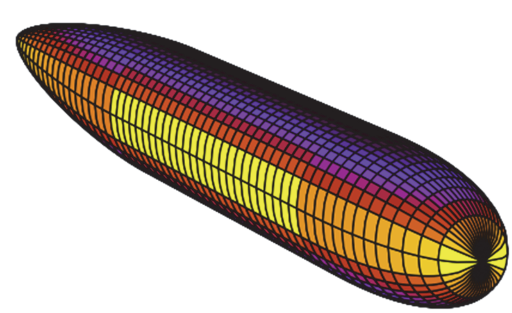
In other words, equations (19) may characterize not only cylindrical ($m = n = 2$) parallel middle bodies, but also any other arbitrary cross-section shape, which is allowed by the Lame curve exponents. With that, all the surface fragments of a submarine need to connect continuously and smoothly, so parameters m and n must be equal for all parts of the hull. The parallel middle body also consists of 8 fragments, the signs of expressions for which are taken according to Table 2.

Table 2

Signs of expressions for fragments of submarine parallel middle body

Vessel part		$0 \le x \le L_m/2$				$-L_m/2 \le x \le 0$			
Fragment		1	2	3	4	5	6	7	8
Expression sign	***x***	+	+	+	+	–	–	–	–
	y	+	+	–	–	+	+	–	–
	z	+	–	+	–	+	–	+	–

As an example, the visualization of a submarine hull with a parallel middle body is performed with the algorithm described above using equations (8) and (19) – family of sections in yOz plane. The geometric parameters are identical to the previously constructed surface (Figure 4), and parallel middle body length $L_m = 40$ m. The generated surface is shown in Figure 6.



Fore and aft bodies are generated by equations (8)

Figure 6. Surface of submarine hull with a parallel middle body and with $L_m = 40$ m, $L_f = 40$ m, $L_a = 20$ m, $W = T = 5$ m; $m = n = 2$; $a = b = j = k = 2.5$ (for fore body); $a = b = j = k = 1.5$ (for aft body)

Results

The hydrodynamic surfaces were visualized using Python programming language. Arrays containing the coordinates of points of the surfaces were computed according to the algorithms described above with the help of NumPy library. The surfaces were constructed using the arrays of points and visualized with the help of Matplotlib plotting library. This programming environment allows to achieve a high level of automation by leaving the geometric parameters, Lame curve exponents and 3D plotting format arguments as the only variables.

Figure 7. Submarine hull manufacturing using segments of developable skin (Crown Copyright 2013, by Andrew Linnett, www.defenceimages.mod.uk, photo 45155780.jpg)

Some studies consider individual cases of hydrodynamic surfaces: encyclopedia [12] presents hydrodynamic surfaces with buttock and waterline curves as the 4th order parabolas and midsection in the form of Lame curve with $m = 4$, $n = 1$, and also with $n = 2$, $m = 1/3$. Article [8] introduces explicit and parametric equations for two sets of three hydrodynamic surfaces with buttock and waterline in the form of the 2nd order parabola and midsection in the form of Lame curve with $m = n = 2$, and also with $m = 4$, $n = 1$. Several particular surfaces are considered in [13; 14]. Article [10] presents the expressions for hydrodynamic surfaces with variable values of the exponents of the parabola, which defines the shape of the main buttock and waterline, and the Lame curve exponents, which defines the shape of the midsection.

Hydrodynamic surfaces generated by families of plane algebraic curves cannot be developable surfaces, apart from the parallel middle body [10]. When manufacturing the hull, the theoretical surface must be approximated by segments of developable skin (Figure 7). If the hull is manufactured using composite material, approximation with developable segments may not be necessary.

Conclusion

The hull shape of real vessels is very complex and it must satisfy a number of design criteria. However, the presented equations for hydrodynamic surfaces and the algorithms of their construction may be considerably useful at the early stages of selecting the optimal hull shape.

It is worth to mention in regards to the versatility of the proposed model that in geometric modelling of submarines, the upper and lower sections may have different geometric form by specifying different heights T and different Lame curve exponents of the corresponding surface fragments.

The analytical method of describing the hull shape allows extensive use of computer modelling, as was demonstrated by the case of constructing and visualizing submarine hull surfaces.

References

1. Basin AM, Anfimov VN. *Hydrodynamics of a vessel: water resistance, propellers, controllability and pitching*. Leningrad: Rechnoi Transport Publ.; 1961. (In Russ.)

2. Brown DK. The way of a ship in the midst of the sea: the life and work of William Froude. Periscope Publishing Ltd.; 2006.

3. Doctors LJ. Optimization of marine vessels on the basis of tests on model series. *Journal of Marine Science and Technology*. 2020;25:887–900. https://doi.org/10.1007/s00773-019-00687-4

4. Tober H. *Evaluation of drag estimation methods for ship hulls.* Stockholm: KTH Royal Institute of Technology, School of Engineering Sciences; 2020.

5. Rychenkova AYu, Klimenko ES, Borodina LN. Geometric modeling and quality assessment of the hull frame surface in COMPASS-3D CAD. *Russian Journal of Water Transport.* 2020;62:71–90. (In Russ.) https://doi.org/10.37890/jwt.vi62.49

6. Li Z, Weimin C. Key Technology of Artificial Intelligence in Hull Form Intelligent Optimization. *ICMAI 2020: Proceedings of the 2020 5th International Conference on Mathematics and Artificial Intelligence*. New York; 2020. p. 167–171. http://doi.org/10.1145/3395260.3395296

7. Kwang Hee Ko. A survey: Application of geometric modeling techniques to ship modeling and design. *International Journal of Naval Architecture and Ocean Engineering.* 2010;2(4):177–184. http://doi.org/10.2478/IJNAOE-2013-0034

8. Krivoshapko SN. Hydrodynamic surfaces. *Shipbuilding.* 2021;(3):64–67. (In Russ.)

9. Krivoshapko SN. Tangential developable and hydrodynamic surfaces for early stage of ship shape design. *Ships and Offshore Structures.* Taylor & Francis; 2022. p. 1–9. https://doi.org/10.1080/17445302.2022.2062165

10. Karnevich VV. Hydrodynamic surfaces with midsection in the form of Lame curve. *RUDN Journal of Engineering Research.* 2021;22(4):323–328. https://doi.org/10.22363/2312-8143-2021-22-4-323-328

11. Jackson HA, Fast C, Abels F, Burcher R, Couch R. Fundamentals of submarine concept design. Discussion. *Transactions-Society of Naval Architects and Marine Engineers.* 1992;100:419–448.

12. Krivoshapko SN, Ivanov VN. *Encyclopedia of Analytical Surfaces*. Springer International Publishing Switzerland; 2015. https://doi.org/10.1007/978-3-319-11773-7

13. Krivoshapko SN. On aero-hydro-dynamical surfaces given by algebraic plane curves. *Structural Mechanics of Engineering Constructions and Buildings.* 2010;(2):3–4. (In Russ.)

14. Avdonev EYa. Analytical description of the ship hull surfaces. *Prikladnaya Geometriya i Inzhenernaya Grafika*. 1972;15:156–160. (In Russ.)

About the author

Valery V. Karnvich, Master of Technical Science, PhD student, Department of Civil Engineering, Academy of Engineering, Peoples' Friendship University of Russia (RUDN University), 6 Miklukho-Maklaya St, Moscow, 117198, Russian Federation. ORCID: 0000-0002-6232-2676, eLIBRARY SPIN-code: 4233-3099. E-mail: valera.karnevich@gmail.com