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Hydrodynamic surfaces with midsection in the form of Lame curve

Valery V. Karnevich

Peoples' Friendship University of Russia (RUDN University), Moscow, Russia
✉ valera.karnevich@gmail.com

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Abstract. General representation of ship geometry is given by the method of slicing the ship hull by three mutually perpendicular planes: vertical symmetry plane which runs along the middle of hull width, horizontal plane which divides the hull into underwater and abovewater parts, and vertical plane perpendicular to the other two which coincides with midsection. By taking the same three predefined sections of the theoretical hull shape, it is possible to obtain three algebraic surfaces of different order, which are called hydrodynamic in this article. By introducing alphabetic parameters to signify orders of ship skeleton main curves and then by giving them various numerical values, it is possible to consider a large number of hull shapes, having only three explicit surface equations. Method of deriving the equations, obtained by other authors, using only three explicit algebraic equations is demonstrated. The proposed technique is illustrated on six new ship hull shapes.

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Гидродинамические поверхности с мидель-шпангоутом в форме кривых Ламе

В.В. Карневич

Российский университет дружбы народов, Москва, Россия
✉ valera.karnevich@gmail.com

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Аннотация. Общее представление о геометрических характеристиках формы корпуса судна дает метод сечения корпуса тремя взаимно перпендикулярными плоскостями, а именно вертикальной плоскостью симметрии, проходящей вдоль корпуса посередине ее ширины, горизонтальной плоскостью, делящей судно на надводную и подводную части, и вертикальной плоскостью, перпендикулярной первым двум и совпадающей с мидель-шпангоутом. Имея одну и ту же тройку наперед заданных плоских сечений теоретической формы судна этими плоскостями, можно получить три алгебраические поверхности разного по-

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рядка, названные в статье гидродинамическими. Вводя буквенные показатели степеней главных плоских кривых каркаса судна, а затем давая им разные цифровые значения, можно ввести в рассмотрение большое количество форм корпусов при наличии лишь трех явных уравнений поверхностей. Продемонстрировано, как получить уравнения, представленные другими авторами, используя только три явных алгебраических уравнения, описанные в статье. Предложенная методика проиллюстрирована на шести новых формах корпусов судов.

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Introduction

Choosing the outline of ship hull surface is one of the main problems of naval architects. A number of factors has to be considered. Often the hull shape is chosen based on designer intuition or with the help of empirical formulas or substantially expensive full-scale experiments [1]. Authors [2] recommend to conduct experiments on the models after selecting theoretical hull shape for optimization of ship geometric dimensions with regards to its mobility, decreasing water resistance and other navigational features. It was pointed out that the best sample was selected from 496 options. There are articles, for example, [3] and [4], which consider applied problems concerned with the capabilities of COMPAS-3D CAD program in geometric modelling of ship hull shapes. Such papers investigate methods of modelling hulls based on plane curves.

Some insight about modern cargo ship hull shapes for Russian river fleet can be gathered from the article by G.V. Egorov [5]. According to some sources that there 11 types of ships, and for each one there have been published a large number of research papers devoted to requirements on optimal shapes of the corresponding ships [6].

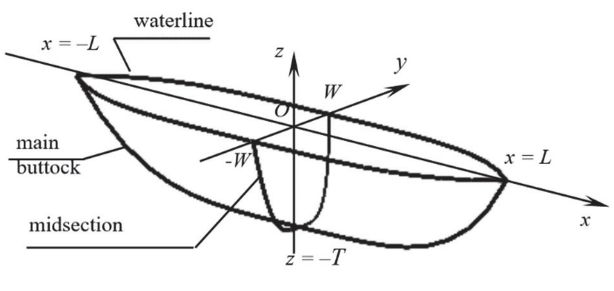


Figure 1. Hydrodynamic surface skeleton consisting of three plane curves

Some designers show that for the purpose of generating optimal form of ship hull, algebraic surfaces with predefined skeleton consisting of three plane curves, lying in mutually perpendicular cross-sections of the ship, are quite suitable (Figure 1). By having three plane curves coinciding with the midsection (*yOz* plane), main buttock (*xOz* plane), and waterline (*xOy* plane), one can construct three different algebraic surfaces [7]. One of them may be taken as the hull shape in the first approximation, then a cylindrical section may be inserted. Alternatively, the hull shape may consist of two different algebraic surfaces, where the bow and stern parts of the ship are joined smoothly at the midsection.

The obtained surfaces are called algebraic in certain papers [8], in others – hydro/aerodynamic [9], and sometimes simply hydrodynamic [7]. In this article, similarly to [7], the surfaces are named hydrodynamic with predefined waterline, midsection and buttock line.

Hydrodynamic surfaces with parabola of order *t*, Lame curve and parabola of order *k*

Thus, we have the waterline in the form of parabola of order *t*:

$$y = \pm W\left(1 - \frac{x^t}{L^t}\right), \tag{1}$$

midsection in the form of Lame curve:

$$|z|^n = T^n\left(1 - \frac{|y|^m}{W^m}\right), \tag{2}$$

main buttock in the form of parabola of order *k*:

$$z = -T\left(1 - \frac{x^k}{L^k}\right). \tag{3}$$

Taking the hull skeleton shape constructed from the three plane curves (Figure 1) by equations (1)–(3) allows to obtain the required formulas for a number of hull shapes owing to arbitrary powers *t, n, m, k.* For example, by plugging different values into parameters *n* and *m* in formula (2), we obtain different outlines of midsections (Figure 2).

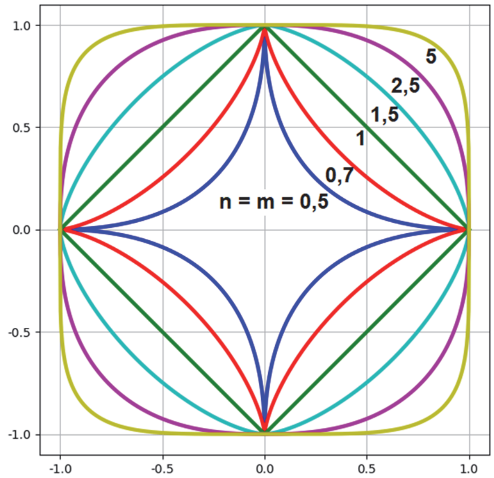


Figure 2. Lame curves at different values of parameters *n* = *m* = 0.5, 0.7, 1, 1.5, 2.5, 5

It is assumed that the main geometric parameters of the ship are defined as: T – hull draft, $2W$ – hull width, $2L$ – hull length (Figure 1).

Let us consider a section of hydrodynamic surface on z = const plane. Its equation is derived similarly to (1):

$$y = \pm W(z)\left(1 - \frac{x^t}{L(z)^t}\right), \qquad (4)$$

where L(z) – is x from formula (3):

$$L(z) = L\sqrt[k]{1 + \frac{z}{T}},$$

and $W(z)$ – is y from formula (2), therefore

$$W(z) = W\sqrt[m]{1 - \frac{|z|^n}{T^n}}.$$

Plugging the previous two expressions into formula (4), we obtain

$$y = \pm W\sqrt[m]{1 - \frac{|z|^n}{T^n}}\left[1 - \frac{x^t}{L^t(1 + z/T)^{t/k}}\right]. \qquad (5)$$

Equation (5) is the equation of the first hydrodynamic surface of interest.

Let us rewrite this equation in parametric form by taking

$$z = z(u) = -uT, \;\; x = x(u, v) = vL\sqrt[k]{1 - u},$$

$$y = y(u, v) = \pm W\sqrt[m]{1 - u^n}(1 - |v|^t), \qquad (6)$$

where $0 \le u \le 1$; $-1 \le v \le 1$; *t, k, n, m* – are positive numbers.

To obtain the second hydrodynamic surface with main skeleton (1)–(3), but constructed by the envelope of sections in x = const plane, we take

$$|z|^n = T(x)^n\left(1 - \frac{|y|^m}{W(x)^m}\right), \qquad (7)$$

where from formula (1)

$$W(x) = W\left(1 - \frac{x^t}{L^t}\right),$$

from formula (3)

$$T(x) = -T\left(1 - \frac{x^k}{L^k}\right).$$

Plugging in the last two expressions into (7), we obtain

$$|z|^n = T^n\left(1 - \frac{x^k}{L^k}\right)^n\left[1 - \frac{|y|^m}{W^m\left(1 - \frac{x^t}{L^t}\right)^m}\right]. \qquad (8)$$

Equation (8) is the algebraic equation of the second hydrodynamic surface of interest. Parametric equations for the second surface are

$$x = x(u) = uL, \;\; y = y(u, v) = vW(1 - u^t),$$

$$|z|^n = |z(u, v)|^n = -T^n\left(1 - u^k\right)^n(1 - v^m), \qquad (9)$$

where $-1 \le u \le 1$; $-1 \le v \le 1$; *t, k, n, m* – are positive numbers.

The third hydrodynamic surface is formed by sections in y = const plane, thus, according to formula (3),

$$z = -T(y)\left(1 - \frac{x^k}{L(y)^k}\right), \qquad (9)$$

where from formula (2)

$$T(y) = T\sqrt[n]{1 - \frac{|y|^m}{W^m}}$$

and from formula (1)

$$L(y) = L\sqrt[t]{1 \mp \frac{y}{W}}.$$

Plugging in the last two expressions into (9), we get

$$z = -T\sqrt[n]{1 - \frac{|y|^m}{W^m}}\left[1 - \frac{x^k}{L^k\left(1 \mp \frac{y}{W}\right)^{k/t}}\right]. \quad (10)$$

Parametric equations for the third hydrodynamic surface are:

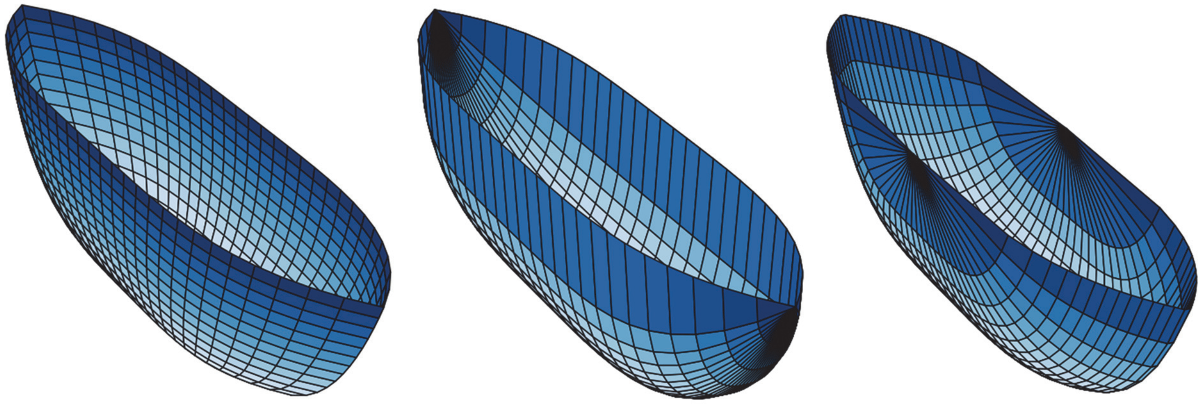
$$y = y(u) = uW,$$

$$x = x(u,v) = vL\sqrt[t]{1 \mp u},$$

$$z = z(u,v) = -T\sqrt[n]{1 - |u|^m}\left(1 - v^k\right), \quad (11)$$

where $-1 \le u \le 1$; $-1 \le v \le 1$; t, k, n, m – positive numbers.

Example 1. The three hydrodynamic surfaces defined by equations (6), (9), (11) are rendered taking $t = 4$, $m = n = 5/2$, $k = 4$. In this case the midsection is in the form of Piet Hein's superellipce. All three hydrodynamic surfaces are shown on Figure 3.

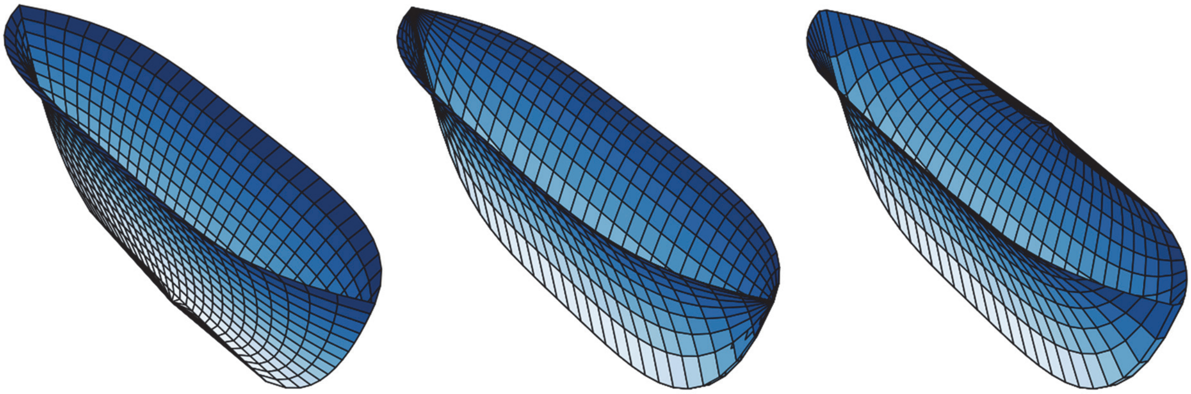


1st surface defined by equation (6)

2nd surface defined by equation (9)

3rd surface defined by equation (11)

Figure 3. Three hydrodynamic surfaces with $T = 1$ m, $W = 0.5$ m, $L = 5$ m



1st surface defined by equation (6)

2nd surface defined by equation (9)

3rd surface defined by equation (11)

Figure 4. Three hydrodynamic surfaces with $T = 1$ m, $W = 0.5$ m, $L = 5$ m

Example 2. Let us render the three hydrodynamic surfaces defined by equations (6), (9), (11) taking $t = 4$; $m = n = 0{,}7$; $k = 4$. All three hydrodynamic surfaces are shown on Figure 4.

Results

Plane curves (1)–(3), which coincide with the main skeleton of the ship, describe a large number of plane algebraic curves and allow to construct numerous groups of three hydrodynamic surfaces. Figures 3 and 4 show hydrodynamic surfaces with convex (Figure 3) and concave (Figure 4) midsections. The groups of three surfaces are rendered with the help of Matplotlib plotting library in Python programming language.

Previously, each hydrodynamic surface used to be constructed for predefined curves (1)–(3) separately. For example, encyclopedia [10] presents hydrodynamic surfaces for $t = k = m = 4$, $n = 1$ case and $t = k = 4$, $n = 2$, $m = 1/3$ case. Explicit and parametric equations for two groups of three hydrodynamic surfaces for $t = m = n = k = 2$ and for $t = 2$, $m = k = 4$, $n = 1$ are derived in article [7]. Several surfaces are considered in papers [9; 11].

Discussion

Hydrodynamic surfaces with the skeleton from predefined plane algebraic curves cannot have Gaussian curvature equal to zero, i.e. cannot be developable surfaces. They will need to be approximated when applying sheet metal to the hull, that is the hull designed at the first stage will need to be replaced by the hull from developable surface sheets (Figure 5).

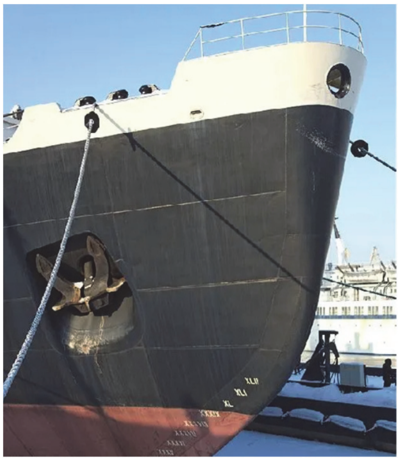


Figure 5. Replacing theoretical hull shape with developable surface sheets

Considering that ship mobility is affected mostly by the outline of the waterlines, the approximated developable surface needs to be preferably constructed as a surface on two sections in z = const plane [12]. These sections will lie in the planes parallel to the waterline plane (1). Approximation may not be needed, if the hull is made of composite materials. Some useful suggestions on geometric modelling of ships hulls can be found in [13]. H. Tober [14] studied the influence of ship hull geometry on water resistance in motion. However, the author considered merely geometrical problems of hull design.

Conclusion

Since the hull of a real ship has very complex form and depends on many factors, the results presented in this article can only be applied at the early stage of ship hull design. Analytical method of hull shape definition, as opposed to the graphical one, will allow to extensively apply computer modelling, for which it is quite easy to implement a program accounting even for cylindrical insertions.

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About the author

Valery V. Karnvich, PhD student, Department of Civil Engineering, Engineering Academy, Peoples' Friendship University of Russia (RUDN University), 6 Miklukho-Maklaya St, Moscow, 117198, Russian Federation. ORCID: 0000-0002-6232-2676, eLIBRARY SPIN-code: 4233-3099. E-mail: valera.karnevich@gmail.com