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## Argumentation of introducing a discrete-continuous topology in the interests of algorithmization of complex functioning processes

Natalia L. Malinina 

Moscow Aviation Institute (National Research University), Moscow, Russia  
✉ malinina806@gmail.com

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**Abstract.** The main aim of the research is to show and prove the necessity of introducing a new, discrete-continuous topological structure to describe complicated systems and processes of their functioning. Currently, there are two topological structures: continuous and discrete. At the same time, there are functional approaches in order to describe complicated systems and processes of their functioning, based on continuous topology. Until now, it has not been possible to build full functionality for the design of complicated technical objects. Therefore, the functional approach does not fully correspond to the increasingly complicated tasks of our time. The introduction of discrete-continuous topology is especially important for the exploring and modeling of complicated systems and processes of their functioning. In order to prove this fact, the present study describes the properties of complicated processes using examples of the flight process and the design process. The examination of these processes, as the most complicated, proves that the complicated systems and processes are topological spaces with metric, so they can be represented in the form of an oriented progressively bounded graph. Also, it proves the topological invariants of complicated systems and the processes of functioning. Presentation of the complicated processes in the form of a directed graph allows getting shorter path to their algorithmicization and programming, which is necessary for existing practice. In addition, the presentation of a complicated process as a directed graph will allow using the apparatus of graph theory for such purpose and will significantly expand the capabilities of programmers.

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## Обоснование введения дискретно-непрерывной топологии в интересах алгоритмизации сложных процессов функционирования

Н.Л. Малинина 

Московский авиационный институт (национальный исследовательский университет), Москва, Россия

✉ malinina806@gmail.com

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сложный процесс, дискретно-непрерывная топология, модель, теория графов

**Аннотация.** Цель исследования – показать и доказать необходимость введения новой, дискретно-непрерывной топологической структуры для описания сложных систем и процессов их функционирования. В настоящее время существуют две топологические структуры: непрерывная и дискретная. Также имеются функциональные подходы к описанию сложных систем и процессов их функционирования, основанные на непрерывной топологии. До сих пор не удалось построить полный функционал для систем проектирования сложных технических объектов, по этой причине функциональный подход не в полной мере соответствует усложняющимся задачам современности. И поэтому введение дискретно-непрерывной топологии важно для исследования и моделирования сложных систем и процессов функционирования. В качестве доказательства описываются свойства сложных процессов на примерах процесса полета и процесса проектирования. Изучение этих процессов как самых сложных показывает, что они, при условии введения новой дискретно-непрерывной топологии, могут быть представлены в виде ориентированного графа. Обосновываются топологические инварианты сложных систем и процессов функционирования. Представление сложных процессов в виде ориентированного графа позволяет более основательно перейти к их алгоритмизации и программированию, что необходимо для существующей практики. Кроме того, представление сложного процесса как ориентированного графа позволит применить для этих целей аппарат теории графов, что позволит значительно расширить возможности программистов.

### Для цитирования

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### Introduction

To describe the real world and build its models, topologists use only two topologies: anti-discrete or discrete [1–3]. However, in the nowadays there are complicated systems and processes of functioning of complicated systems, which are also complicated. Whatever one may say, we cannot imagine an airplane, helicopter, car or submarine using continuous topology (structure). We cannot crush an airplane like clay and mold a helicopter or a submarine out of it in one continuous and one-to-one transformation, and, similarly, we cannot disassemble a complicated object into elements and create a new ob-

ject from these elements, also complying with the requirements of one-to-one conversion [1–3].

So, it is necessary to introduce one more topology (structure), which will help to cope with the problems that arise, when we are creating complicated systems or processes, namely, discrete-continuous topology. About 20 years ago V. Korukhov wrote about it [4; 5], but this topology was described in terms of philosophy.

Let's try to justify the introduction of such an abstraction from the standpoint of practical life on a specific example and see what this can give for the possibilities of modeling complicated processes and systems.

## 1. Complicated process as an object of modeling

The concept of a complicated process, like the concept of a complicated system, is not strictly defined. In technology, the word “system” is defined mainly as an aggregate number of objects, which activity is organized by some rules [6]. A feature of complicated systems nowadays is information processes, which are aimed to ensure optimal management [7]. Therefore, a complicated process should be understood as the process of functioning of a complicated system. Examples of such complicated processes are:

- construction, repair, operation of complex equipment;
- processing of parts, assembly of units or machines;
- command and control of troops and combat assets;
- flying of the aircrafts;
- management of transport systems;
- implementation of computer programs, mathematical modeling;
- functioning of finite state machines;
- public administration.

Any complicated process consists of a set of sequential or parallel operations (essentially technological, design, including creative), as well as of the moments of the beginning and end of these operations or the moments of transition from one operation to another.

Elements of the first subset are operations that take place in time and require energy expenditures for their implementation (elementary operations). These can be signals that determine the possibility (or necessity) of the beginning or end of elementary operations. Together, they constitute the physical content of the process under study.

The elements of the second subset are the moments of the beginning and the moments of the end of operations or the moments of transition from one operation to another. They can also include links between operations and logical conditions for moving from one operation to another. Elements of this subset require practically no time or energy consumption for their implementation. They represent the logical structure of a complicated process.

It is obvious that any of the above processes can be represented as a disjunctive sum of the above subsets of elements. Physically, this entire process as a whole can be represented as continuous, but with discrete transitions from one element of the process to another one. In other words, we have a physically continuous process with a discrete logical structure.

However, in various attempts to automate the complicated processes, many researchers define them as a functional, applying an anti-discrete structure to them. Although until now it has not been possible to develop, no matter how significant the functionality of a complicated process, which would describe the whole process, and not just a tiny part of it.

## 2. Analysis of the complicated process

Let’s analyze a complicated process using the example of an airplane flight (design) process. The constituent parts of a complicated process, usually called its elements, are considered to be relatively time-limited and fairly homogeneous in terms of physical content, operations that can be described by a relatively small number of mathematical expressions.

Flight elements for an airplane can include, for example, takeoff run, climb, level flight, individual aerobatics, air combat attacks, etc. The complexity of the flight process of modern aircraft and the need for its more detailed study lead to the fact that more and more small flight elements are being studied. Moreover, such elements can no longer only refer to the aircraft as a whole, but also to its individual units, systems and components, including the crew. The main fact is that any complicated process can be divided into elements, in each of which some particular tasks of the general problem of the complicated process are solved.

A feature of the design process is the development and creation of new, previously non-existent objects, processes or systems. Here human creativity is closely intertwined with engineering synthesis, which is currently carried out only by humans [8; 9]. The definition of the design process as a complex discrete-continuous process in no way contradicts the creative aspects of design [10; 11].

The constituent parts of a complex process, usually called its elements, are considered to be relatively time-limited and sufficiently homogeneous in terms of physical content, operations that can be described by a relatively small number of mathematical expressions.

The interrelationships of elements are more complex and are not always obvious from a simple listing of them. Many elements of a complicated process, in particular flight, especially very small ones, often have to be performed simultaneously. The practical application of the concept of “element” of flight in research and in flight practice allows us to draw some general conclusions in the transfer to the concept of a complex process:

1) each element of a flight (complicated process) can have a mathematical description or a mathematical model, which includes certain parameters of an aircraft (an object subjected to the process);

2) each element of a flight (complicated process) has such characteristics as time of execution, probability of successful completion, etc.;

3) each element of a real flight (complicated process) has quite definite moments of beginning and end;

4) as an element of flight (complicated process), such simple operations can serve, which, especially when a detailed study is required, can be considered separately from others. All of the same can be transferred both to the design process and to other complicated processes.

As a result, the analysis of any complicated process is necessary. It precedes the development of a complicated process model and includes determination of:

- the elements of the process;
- the list of elements of a complicated process included in each investigated stage of a complicated process;
- the main parameters and characteristics for each element of the process, including time and probabilistic characteristics;
- the system of connections and relations between the elements of the process, or otherwise: the system of binary relations on the set of these elements.

Similar considerations can be made for the design process. The goal of engineering design is always the development and creation of new, previously non-existent objects, processes or systems [8; 9]. Human creativity here is closely intertwined with engineering synthesis, which, falling into the category of *NP* – complete tasks<sup>1</sup>, is currently carried out practically only by humans, since there is no other way to check the correctness of certain creative solutions, as by “collecting” from parts the entire product being developed as a whole one, and having checked in practice its compliance with a given criterion of efficiency [10; 12].

So, any complicated process includes a number of sequential and parallel operations (essentially technological, design, including creative), as well as the moments of the beginning and end of these operations or the moments of transition from one operation to another. So, when software is created to automate the control of a complicated process, it is considered that a complex process consists of a large number of elementary operations. Each of these elementary operations has common properties, but some of them are unique. Let’s try to give a more precise definition of the concept of a process’ element.

Elements or elementary operations of a complex process are called time-limited and homogeneous in their physical content and functional purpose areas of a complex process.

All such operations each have a well-defined beginning – entry (start time) and a very definite end – exit (end time). The inputs and outputs of complicated process’ elements constitute the second of the subsets of complex process elements mentioned earlier. By connecting inputs and outputs, various elementary operations of a complicated process are combined into a single complicated process. Each of the elements can only be connected to the entire process through input and output.

Therefore, the dividing of any complicated process into stages and elements should be based on a functional and logical principles, according to which each stage of a complicated process will rep-

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<sup>1</sup> This refers to the complexity of  $n!$  It refers to the overkill problems of combinatorics.

resent a system of elements that will be aimed to solve one of the particular tasks of a complicated process and can be assessed by the corresponding partial criterion of efficiency for this complicated process. In this case, a particular task that is solved at this stage of a complicated process is at this stage the main functional task, on the solution of which the outcome of the complicated process as a whole one also depends. The implementation of the entire set of elements of a complicated process that make up this stage should be subordinated to the main functional task of a complicated process.

Obviously, any complicated process includes not only elementary operations and their aggregates (stages), but also the moments of the beginning and end of the elements (stages) of a complicated process and the moments of transition from some operations (elements, stages) to others.

Based on the concepts of an element and a stage of a complicated process that correspond to the content and goals of this study, we will accept the following, preliminary, definition of a complicated process: a complicated process is a discrete-continuous process flowing in time and space. It includes separate interrelated elements (stages) of this process, as well as the moments of the beginning, end and transitions between them.

Thus, the set of elements of a complicated process consists of two subsets interconnected:  $H_0 = Q_0 \times V_0$ .

The process is usually implemented with the help of some means, or a complicated system. Let us consider these provisions on the example of an analysis of an aircraft flight.

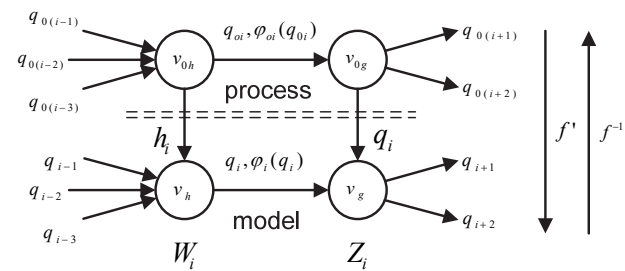
### 3. Physical and structural similarity of a complicated process and model

Modeling as a research method is based on the theory of similarity. With a sufficient degree of accuracy, it can be argued that modeling takes place only when there is a similarity.

However, rather often researchers, referring to physical or mechanical similarity, limit themselves only to the criteria of physical similarity, since it is far from always clear how to establish logical and structural similarity. This is because until now in

topology there are only two variants of the structure: anti-discrete and discrete. Complicated processes do not fit into both of these structures and remain outside the study using topological and combinatorial means, although they are partially used in the form of block diagrams or graph-diagrams. The issues of the physical similarity of the model and the process are well studied, however, attention should be paid to the issues of the relationship between structural and physical similarity.

From the concept of structural similarity, it follows that each element of a complicated process must be put in a one-to-one correspondence with an element of the model of this process using some mapping  $f$ . The physical similarity of the elements of the model and the complicated process is also directly related to the mapping  $f$  [2].



Rectangular diagram  $WZ$

*Note:*  $W_{0i}$  – set of parameters at the input of the element  $q_{0i}$ ;  $Z_{0i}$  – set of parameters at the output of the element  $q_{0i}$ ;  $W_i$  – set of parameters at the input of the element  $q_i$ ;  $Z_i$  – set of parameters at the output of the element  $q_i$ ;  $\varphi_{0i}(q_{0i}) = \varphi_{0i}$ ;  $W_{0i} \rightarrow Z_{0i}$  – transformation (display) of input parameters into output parameters, carried out when implementing an element of a real complex process  $q_{0i}$ ;  $\varphi_i(q_i) = \varphi_i$ ;  $W_i \rightarrow Z_i$  – transformation (mapping) of input parameters into output parameters, carried out when simulating an element of a complex process  $q_i$ ;  $h_i: W_{0i} \rightarrow W_i$  – “scale factor” between the inputs of the process and model elements;  $q_i: Z_{0i} \rightarrow Z_i$  – “scale factor” between the outputs of the process and model elements.

Consider a rectangular diagram  $WZ$  (Figure). In this diagram, the arrow  $q_{0i}$  denotes an element of the real process, to which, by means of the mapping  $f$ , the model element  $q_i$  is put in a one-to-one correspondence. The beginning of the implementation of the element  $q_{0i}$  is indicated by the vertex  $v_{0h}$ , which contains the ends of the arrows (the ends of the elements)  $q_{0(i-1)}, q_{0(i-2)}$ , etc.

The end of the realization of the element  $q_{0i}$  is indicated by the vertex  $v_{0g}$ , where the arrows

(the beginning of the realizations of the elements)  $q_{0(i+1)}, q_{0(i+2)}, \dots$ , begin. In the model, these vertices correspond to the vertices  $v_h$  and  $v_g$ , which indicate the beginnings and ends of the implementation of the  $q_i$  element of the model.

Due to structural similarity

$$q_i = f^1(q_{0i}) \text{ and } q_{0i} = f^{-1}(q_i). \quad (1)$$

Let's denote:  $k_{0i} = \{k_{0j}\}_i$  is a set of physical similarity criteria for the element  $q_{0i}$ ;  $j = 1, 2, \dots$  – indices of criteria;  $k_i = \{k_{0j}\}_i$  is a set of physical similarity criteria for the element  $q_i$ ;  $j = 1, 2, \dots$  – indices of criteria.

When modeling, the criteria for the physical similarity of a real complex process and its model should be equal:

$$k_{0i} = k_i \text{ for everyone } i = 1, 2, \dots, n_Q. \quad (2)$$

To ensure the physical similarity of the elements  $q_{0i}$  and  $q_i$ , the mappings  $\varphi_{0i}, \varphi_i, h_i, g_i$  must meet the following condition:

$$\left. \begin{aligned} h_i \circ \varphi_i \circ g_i^{-1} \circ \varphi_{0i}^{-1} &\equiv \text{idem.} \\ \varphi_{0i} \circ g_i \circ \varphi_i^{-1} \circ h_i^{-1} &\equiv \text{idem.} \end{aligned} \right\} \quad (3)$$

This is possible only if the mappings  $\varphi_{0i}, \varphi_i, h_i, g_i$  are one-to-one. From equalities (3) it follows that

$$\left. \begin{aligned} h_i \circ \varphi_i &\equiv \varphi_{0i} \circ g_i \\ h_i^{-1} \circ \varphi_{0i} &\equiv \varphi_i \circ g_i^{-1} \end{aligned} \right\} \quad (4)$$

This means that the diagram  $WZ$  is bicommutative [2]. It follows from this that, subject to the structural and physical similarity between the elements of the process and the model:

- the input parameters of the process element uniquely determine the output parameters of the model element and vice versa;
- the parameters of the input of the model element uniquely determine the parameters of the output of the process element and vice versa.

Insofar as

$$\begin{aligned} h_i^{-1} \circ \varphi_{0i} &\equiv \varphi_i \circ g_i^{-1}, \\ \text{then } \varphi_i &\equiv h_i^{-1} \circ \varphi_{0i} \circ g_i. \end{aligned} \quad (5)$$

Expression (5) means that the mapping  $\varphi_i$  is determined not only by the mapping  $\varphi_{0i}$ , but also by the mappings  $h_i^{-1}$  and  $g_i$ . The latter are the particular values of the mapping  $f$ , which reflects the structural similarity of a real complex process and its model (systems  $H_0$  and  $H$ ).

Thus, if the mapping  $\varphi_{0i}$  is a transformation of the real physical characteristics of the process'  $q_{0i}$  element, then the model of this transformation  $\varphi_i$  is determined not only by the transformation  $\varphi_{0i}$ , but also depends on the "scale factors"  $h_i$  and  $g_i$ , which in turn depend on the structural similarity of the process and his models. Consequently, expression (5) establishes in the most general form the connection between the structural and physical similarity of the process and the model. In the particular case when the coefficients  $h_i$  and  $g_i$  are identical transformations (scale 1: 1), we have

$$\varphi_i \equiv \varphi_{0i}. \quad (6)$$

Hence formula (1) follows.

Let us consider the flight process, and in its representation, any complicated process, as a topological  $F$  space. The study of elements and relationships between them in complicated processes using the examples of flight and design processes shows that complicated processes meet three conditions, which are called the axioms of the topological structure [3], and therefore a complicated process is a topological space. In addition, the complicated process considered as the system  $H_0 = Q_0 \times V_0$  is metric. The metric of this space is given by a temporary program or the order in which the process is executed.

Any model of a complicated process should be formed as some kind of mathematical structure. This mathematical structure turns into a model when some physical interpretation is given to the elements of the model. The question of the similarity or equivalence of a complicated process and its model rests on the problem of similarity or equivalence of their structures, and for this it is necessary to establish their topological similarity and find a system of topological invariants in order to be able to compare a process and a model accurately.

If we talk about the flight process of any aircraft, then it is not enough to describe the dynamics

of its flight to form its model. Any flight is also characterized by a time program or flight schedule, the processes of forming not only control signals, but also the moments of their implementation, etc. If we proceed from the requirements of similarity, then this, and a very large, set of characteristics and properties of flight should also be considered.

So, it becomes necessary to consider not only the physical similarity of the elements of a complicated process and the elements of its model, but also the structural similarity of the process and its model. Let's try to consider the topological invariants of a complicated process, using, as an example, two such complicated processes as the design or flight process of an aircraft and try to find common properties and characteristics between them.

#### 4. Topological invariants of the complicated process and its model

The introduction of the discrete-continuous topology on the sets  $Q_0$  and  $V_0$  makes it possible to describe the properties of these sets using topological invariants.

Each of the sets  $Q_0$  and  $V_0$  is finite. Therefore, one of the invariants of the system  $H_0 = Q_0 \times V_0$  is the finiteness of sets, which is numerically expressed by their cardinality:

- $n_{0Q}$  – is the cardinality of the set  $Q_0$ ;
- $n_{0V}$  – is the cardinality of the set  $V_0$ .

Of these two quantities, the cardinality  $n_{0Q}$  of the set  $Q_0$  is a topological invariant of the system  $H_0$ , while the cardinality  $n_{0V}$  of the set  $V_0$  is not, since it is completely determined through the cardinality  $n_{0Q}$  of the set  $Q_0$  and its ordinal type  $R_0$ .

One of the main topological properties is the compactness property. When applied to a complicated process, the compactness property means that any sequence of elements has at least one limit point beyond which there can be no process elements. The compactness property is equivalent to the property of a set to be closed and bounded. Indeed, the design flight process as a set  $H_0$  includes boundary points (closure); moreover, no sequence of either the flight process or the design process can go to infinity.

All elements of a complex process have a well-defined orientation in time and space. Each elemen-

tary process can be realized only in one direction: from the beginning to the end, since the passage of time is irreversible.

The properties of the system  $H_0 = Q_0 \times V_0$  listed above: finiteness, closedness, boundedness and directionality allow representing the design process model in the form of a finite, progressively bounded, directed graph  $H = Q \times V$ .

The graph  $H = Q \times V$  has a finite number of arcs (oriented edges) and vertices; it can have cycles, but it has no contours. A real complicated process (design or flight) cannot be imagined as consisting of completely independent, unrelated elements. This means that the corresponding graph cannot also consist of unrelated components, i.e. the graph  $H = Q \times V$  has one connected component.

So, the following topological properties of the set  $H_0$  have been established as a system of sets  $Q_0$  and  $V_0$ :

- limb (finiteness);
- limitedness (boundedness);
- isolation;
- connectivity.

Consequently, the model of a complicated process can be represented in the form of a graph  $H = Q \times V$ . Graphs, in addition to the specified properties and the number of elements included in them, can have different characteristic numbers. In this case the cyclomatic number of the graph  $H = Q \times V$  becomes very interesting. It is determined by the following formula:

$$v(H) = m_H - n_H + 1, \quad (7)$$

where  $m_H$  is the number of arcs (edges), and  $n_H$  is the number of vertices of the graph  $H$ .

The cyclomatic number is equal to the largest number of independent cycles of the graph [13; 14] and determines the complexity of the graph's structure, and, consequently, the complexity of both a complicated process (system) and its model. The cyclomatic number is the next and very important invariant of the system  $H_0 = Q_0 \times V_0$ .

The study of the properties of the order defined on the sets  $Q_0$  and  $V_0$  shows that these sets are linearly ordered, inductive and can always be completely

ordered, that is, they can have ordinal types, and therefore can be numbered.

It turns out that the ordinal type  $F_0$  of the set  $V_0$  is completely determined by the ordinal type  $R_0$  of the set  $Q_0$ . This means that the way of ordering the set  $V_0$  is completely determined by the way of ordering the set  $Q_0$ .

Thus, to ensure the similarity of a complicated process and its model, it is necessary that the ordinal type  $R_0$ , given on the set  $Q_0$  of elements of the complicated process, and the ordinal type  $R$ , defined on the set of arcs of the graph  $H(V, Q)$ , that is  $R_0 = R$ . Equality  $F_0 = F$  is ensured in this case automatically.

Theorems proving topological properties, namely, finiteness, limitedness (boundedness), isolation, and connectedness, are below.

**Properties of the sets  $Q_0, V_0$  and  $H_0 = Q_0 \oplus V_0$**

*Theorem 1.* The set  $Q_0 = \{q_{0i}\}$  is finite [11].

*Proof.* Since each element  $q_{0i} \in Q_0$  has a finite implementation time ( $0 < \Delta t_i < \infty$ ), the number of simultaneously executed process elements does not exceed a finite number of possible executors (in the process of designing a software product, these can be systems, programs, units, people), then in total the flow time of the entire process as a whole is also finite. The theorem is proved.

The cardinality of the set  $Q_0 = \{q_{0i}\}$  is equal to the number  $n_{0Q}$  of the elements  $q_{0i}$ .

*Theorem 2.* The set  $V_0 = \{v_{0h}\}$  is finite [11].

*Proof.* Since the set  $Q_0 = \{q_{0i}\}$  is finite, and each element  $q_{0i} \in Q_0$  can be assigned exactly two elements  $v_{0h} \in V_0$ . Let us denote these elements  $v_h^{(+)}(q_{0i})$  and  $v_h^{(-)}(q_{0i})$  respectively. We have  $V_0 = \{v_h^{(+)}(q_{0i})\} \cup \{v_h^{(-)}(q_{0i})\}$ . Obviously,  $\overline{\{v_h^{(+)}(q_{0i})\}} \cup \overline{\{v_h^{(-)}(q_{0i})\}} = \emptyset$  and, therefore,  $V_0$  is finite. The theorem is proved.

The cardinality of the set  $V_0$  is equal to the number  $n_{0V}$  of elements  $v_{0h}$ .

It follows from the theorems proved that the set  $H_0 = Q_0 \oplus V_0$  is finite.

*Theorem 3.* The set  $V_0 = \{v_{0h}\}$  is a metric space [11].

*Proof.* Since  $v_{0h} \in V_0$  are the points on the time axis (numerical axis), then each pair  $(v_{0x}, v_{0y})$  can

be associated with the number  $\rho(v_{0x}, v_{0y}) > 0$ , called the interval (distance) and is satisfying the axioms of the metric:

- 1)  $\rho(v_{0x}, v_{0y}) \geq 0$ ;
- 2)  $\rho(v_{0x}, v_{0y}) = 0$ ;
- 3)  $\rho(v_{0x}, v_{0y}) = 0 \leftrightarrow x = y$ ;
- 4)  $|\rho(v_{0x}, v_{0y})| = |\rho(v_{0y}, v_{0x})|$  – symmetry;
- 5)  $|\rho(v_{0x}, v_{0z})| \leq |\rho(v_{0x}, v_{0y})| + |\rho(v_{0y}, v_{0z})|$  – triangle rule.

The theorem is proved.

*Corollary 1.* The metric on the set  $Q_0$  is a consequence of the metric on the set  $V_0$ .

*Corollary 2.* The sets  $V_0 = \{v_{0h}\}$ ,  $Q_0 = \{q_{0i}\}$  and  $H_0 = Q_0 \oplus V_0$  are topological spaces [1–3].

The introduction of topology on the set  $H_0 = Q_0 \oplus V_0$  allows us to consider the question of topological invariants of the system  $H_0$ . One of the main topological properties of a space is the compactness property.

*Theorem 4.* If  $Q_0 = \{q_{0i}\}$  is a set of process elements, and  $V_0 = \{v_{0h}\}$  is a set of moments of their beginning and end, then the set  $H_0 = Q_0 \oplus V_0$  is compact. To prove Theorem 4, we show first that the sets  $Q_0$  and  $V_0$  are compact [11].

As the first part of the proof, we use the achievements of Theorem 6.1 by N. Steenrod and W. Chinn [15] and introduce a lemma.

*Lemma 4.1.* Any closed element  $\Delta v_{0h} \in V_0$  is compact. The proof of the lemma is given in [15]. The result of the proof: the element  $\Delta v_{0h}$  is compact. For further reasoning, we use Theorem 6.4 of Steenrod and W. Chinn [15] and reintroduce the lemma.

*Lemma 4.2.* Let  $X$  be a compact space, and let the function  $f: X \rightarrow Y$  be continuous, then the image  $f_x$  is compact [15]. The proof of the lemma is given in [15]. It obviously follows from Lemma 4.1 and Lemma 4.2 that any closed element of a complex process  $q_{0i} \in Q_0$  is compact. Therefore, the set  $Q_0 = \{q_{0i}\}$  is compact.

Let us proceed to the proof of the second part of the theorem, that the set  $V_0 = \{v_{0h}\}$  is compact. To do this, compose the set  $V_0^* = \{v_h^{(+)}(q_{0i}); v_h^{(-)}(q_{0i})\}$ . Obviously,  $V_0^*$  is finite and completely covers the set, that is,  $V_0^*$  is a finite cover of  $V_0$ . Therefore,  $V_0$  is compact [11]. Since  $Q_0$  is compact and  $V_0$  is compact, it is quite obvious that  $H_0 = Q_0 \oplus V_0$  is compact. The theorem is proved.



Since the topological space  $H_0 = Q_0 \oplus V_0$  is metric and compact, it is compact in itself. Applied to the design process or a flight process (as well as to any complex process flowing over time), this means that the endpoints of a process belong to the process itself. The compactness property is equivalent to the property of a set to be closed and bounded.

*Definition.* A set  $X = R^m$  is called bounded if it is contained in some sufficiently large ball, that is, if there are points  $x_0$  and a number  $r > 0$  such that:  $X \subset N(x_0, r)$  [15].

*Definition.* Let  $X$  be a set in  $R^m$ . A subset  $A \subset X$  is called closed in  $X$  if its complement in  $X$  is an open set in  $X$ . In short,  $A$  is closed in  $X$  if  $X - A$  is open in  $X$  [15].

*Theorem 5.* The compact set  $H_0 = Q_0 \oplus V_0$  is bounded and closed [11].

The first part of Theorem 6.1 from was used as Lemma 4.1 [15]. The full theorem says: every compact subset is bounded and closed in  $R^m$ . This theorem implies the validity of Theorem 5. Thus,  $H_0 = Q_0 \oplus V_0$  is a closed and bounded set. The theorem is proved.

As applied to the considered complex process, the property of boundedness means that no sequence of process elements can go to infinity. Closure, in turn, means that a complex process as a set includes all its boundary points (closure of the set). The boundedness and closedness of the set  $H_0 = Q_0 \oplus V_0$  are topological invariants of a complex process and allow one to establish some properties of a directed graph, which can be used to represent a model of such a complex process as a design process.

Let us show further that the set  $H_0 = Q_0 \oplus V_0$  as a mathematical object is a graph.

*Definition.* A graph is a pair consisting of a set  $X$  and a mapping  $\Gamma: X \rightarrow X$ , or, which is the same, a pair  $G(X, \Gamma)$  is a graph in which  $X$  is a set of vertices and  $\Gamma: X \rightarrow X$  is a set of edges.

*Theorem 6.* If the set  $Q_0 = \{q_{0i}\}$  is a set of elements of a complex process, and  $V_0 = \{v_{0h}\}$  is a set of moments of their beginning and ends, then the pair  $H_0 = Q_0 \oplus V_0$  is the essence of a graph [11].

*Proof.* Obviously, each  $q_{0i} \in Q_0$  corresponds to a pair  $(v_h^{(+)}(q_{0i}); v_h^{(-)}(q_{0i}))$  and, if  $V_0^* =$

$\{v_h^{(+)}(q_{0i}); v_h^{(-)}(q_{0i})\}$  is the set of all such pairs, then  $\overline{V_0^*} = \overline{\{v_h^{(+)}(q_{0i}); v_h^{(-)}(q_{0i})\}} = \emptyset$ .

This implies that  $V_0^* \equiv f(Q_0)$  and that there exists a single-valued mapping  $\Gamma_V: V_0 \rightarrow V_0$  such that all  $q_{0i} \in \Gamma_V(v_{0h})$ , and there exists an  $f$  – a single-valued mapping of the set  $Q_0$  onto the set  $V_0$  such that all  $f_{q_{0i}} \in V_0$ .

Then, by definition,  $H_0 = Q_0 \oplus V_0$  is a graph in which  $V_0$  is a set of vertices, and  $Q_0$  is a set of edges. The theorem is proved.

The properties of the graph  $H_0(V_0, Q_0)$  are determined, in particular, by the fact that the set  $H_0 = Q_0 \oplus V_0$  is closed and bounded.

*Definition.* A graph is called progressively bounded at the vertex  $v_{0h}$  if there is an integer  $m$  such that the length of each path starting at the vertex  $v_{0h}$  does not exceed  $m$ ; the graph is progressively bounded at each of its points, progressively bounded [13].

This allows us to say that, in addition to the fact that the graph  $H_0(V_0, Q_0)$  is progressively bounded, it is also progressively finite, although the converse is not true [13]. All elements of the design process (or the process of flight) have a well-defined orientation in time and space.

Let's move on to considering the question of the connectedness of the graph.

*Definition.* A topological space  $(X, Y)$  is called connected if and only if the set cannot be represented as a union of two disjoint closed sets [1; 3].

The real design process (flight or design) differs from an arbitrary set of its elements, first of all, by the mutual dependence of its individual elements and sections among themselves. The design process (flight or design) cannot be imagined in the form of completely independent sections, despite the fact that when studying the design process (flight process) and forming its model, it can never be argued that we know all the connections of its elements with each other. The design (flight) process always consists of one and only one connectivity component, which, in turn, means that the graph  $H_0(V_0, Q_0)$  must always be connected.

*Definition.* A graph  $H_0(V_0, Q_0)$  is called complete if  $(v_{0y}; v_{0x}) \notin Q_0 \Rightarrow (v_{0x}; v_{0y}) \in Q_0$ , that is,

if any two vertices are connected in at least one direction [13; 14].

*Definition.* A graph  $H_0(V_0, Q_0)$  is called strongly connected when for any two vertices  $v_{0x}$  and  $v_{0y}$  ( $v_{0x} \neq v_{0y}$ ) there is a path going from  $v_{0x}$  to  $v_{0y}$  (or vice versa) [13; 14].

*Theorem 7.* Graph  $H_0(V_0, Q_0)$ , in which  $V_0 = \{v_{0h}\}$  is the set of moments of the beginning and end of the design process elements, and  $Q_0$  is the set of all pairs  $(v_{0x}; v_{0y})$  such that  $v_{0x} < v_{0y}$  (including transitivity) – strongly connected [11].

*Proof.*

*By definition.* However, the design process, more precisely, its model, may not include as elements all the pairs  $(v_{0x}; v_{0y})$  such that  $v_{0x} < v_{0y}$ , but only the necessary part of them, which ensures the connectivity of the graph  $H_0$ .

*Definition.* The partial graph of the graph  $H'_0(V'_0, Q'_0) = H'_0(V'_0, \Gamma)$  is the graph  $H_0(V_0, Q_0) = H_0(V_0, \Delta)$ , where  $v_{0h} = \Gamma_{v_{0h}}$  for all  $v_{0h} \in V_0$  [16].

It follows from Theorem 7 and the above definition, that the graph  $H_0(V_0, Q_0)$  is a partial graph of the graph  $H'_0(V'_0, Q'_0)$ .

Obviously, in the considered partial graph  $H_0(V_0, Q_0)$  not necessarily all  $(v_{0h}; v_{0g}) \in Q_0$ , that is, generally speaking, if the graph  $H'_0(V'_0, Q'_0)$  is connected, then from this statement it does not yet follow, that the graph  $H_0(V_0, Q_0)$  is connected. However, bearing in mind the definition adopted for everywhere dense and everywhere not dense elements of the complicated process, we can supplement the set  $Q_0$  with such a number of everywhere not dense elements  $(v_{0x}; v_{0y}) \in Q_0$  so that the graph  $H_0(V_0, Q_0)$  will be always connected [11].

Consider the conditions necessary to ensure the connectivity of the graph  $H_0(V_0, Q_0)$ .

The maximum possible number of arcs in a connected graph without loops is determined by Theorem 2.2.4. Ore [14]:

$$N_Q(n_V, 1) = \frac{1}{2}(n_V - 1) * n.$$

where  $n_V$  is the number of vertices in the graph.

It follows from Theorem 2.2.5 [14] that if in a graph with  $n_V$  vertices there are more arcs than  $N_Q(n_V, 1) = \frac{1}{2}(n_V - 1) * (n_V - 2)$ , then the graph

is connected [11]. Thus, the following theorem holds.

*Theorem 8.* In order for the graph  $H_0(V_0, Q_0)$ , in which  $Q_0 = \{q_{0i}\}$  is the set of everywhere dense elements of the complicated process, and  $V_0 = \{v_{0h}\}$  is the set of moments of their beginnings and ends, it is sufficient to supplement the set  $Q_0$  – everywhere is not dense elements of the complicated process so that the condition is met:

$$N_q(n_V, 1) \geq n_{0Q} + \Delta N_Q(v_{0x}, v_{0y}) \geq N_Q(n_V, 2),$$

where  $n_{0Q}$  – the number of elements of the design process (everywhere dense) or the cardinality of the set  $Q_0$ ;  $\Delta N_Q(v_{0x}, v_{0y})$  – an additional number of everywhere not dense elements of the design process;  $n_V$  is the number of start and end points of the design process elements [11].

*Corollary 8.1.* The minimum required number of additional everywhere not dense elements of the complicated process that ensure the connectivity of the graph  $H_0(V_0, Q_0)$  does not exceed:

$$N_{Q_{\min}}(v_{0x}; v_{0y}) = \frac{1}{2}(n_V - 1) * (n_V - 2) - n_{0Q}.$$

*Corollary 8.2.* The maximum possible addition of the graph  $H_0(V_0, Q_0)$  everywhere not dense elements of the complicated process is determined by the formula

$$N_{Q_{\min}}(v_{0x}; v_{0y}) = \frac{1}{2}(n_V - 1) * n_V - n_{0Q}.$$

*Corollary 8.3.* The connectedness condition of the graph  $H_0(V_0, Q_0)$ , defined by Theorem 10, can always be satisfied [11].

Indeed,

$$N_{Q_{\max}}(v_{0x}; v_{0y}) - N_{Q_{\min}}(v_{0x}; v_{0y}) = n_V - 1 > 0.$$

This allows us to formulate the following theorem.

*Theorem 9.* The complicated process, as a discrete-continuous process, considered in the form of a system  $H_0 = Q_0 \oplus V_0$ , can always be represented by a connected graph  $H_0(V_0, Q_0)$  [11].

So, if the design process as a discrete-continuous process is considered as a set  $H_0 = Q_0 \oplus V_0$ , where  $Q_0 = \{q_{0i}\}$  is a set of elements of the complicated process, and  $V_0 = \{v_{0h}\}$  is a set of moments of their

beginning and ends, then the set  $H_0$  – of course, is bounded and closed, and can always be represented as a connected progressively bounded, directed graph.

**Properties of the system of relations defined on the sets  $Q_0, V_0$  and  $H_0 = Q_0 \oplus V_0$**

Binary relations of the form  $a_x \leq a_y$  are defined on the sets  $Q_0, V_0$  and  $H_0 = Q_0 \oplus V_0$ . Consider the properties of these relations (order relations).

*Definition.* A relation of type  $a_x < a_y$  is called a partial ordering or an inclusion relation when it has the following properties [1; 3]:

- 1)  $a_x \leq a_y$  – reflexivity;
- 2) if  $a_x \leq a_y$  and  $a_y \leq a_x$ , then  $a_x = a_y$  – antisymmetric;
- 3) if  $a_x \leq a_y$  and  $a_y \leq a_z$  – transitivity.

*Definition.* A relation  $a_x < a_y$  is called a strictly ordering or strictly inclusive relation if it satisfies two conditions [1; 3]:

- 1)  $a_x < a_y$  and  $a_y < a_x$  do not take place at the same time;
- 2) transitivity.

A strictly ordering relation is also called a linear ordering. A linearly ordered set is called well-ordered if any non-empty subset in  $X$  has the first element.

Further, a number of theorems will be proved that will allow, taking into account the fact that the design process can be represented by a directed graph  $H_0(V_0, Q_0)$ , to establish the following properties of the sets  $Q_0$  and  $V_0$  of the system  $H_0 = Q_0 \oplus V_0$  and their order:

- 1) the set  $V_0$  can be defined if and only if the set  $Q_0$  and the order relation on the set  $Q_0$  are given;
- 2) the way of ordering (numbering of elements) of the set  $V_0$  is completely determined by the way of ordering (numbering of elements) of the set  $Q_0$ ;
- 3) for the set  $V_0$  to be linearly ordered, it is necessary and sufficient that the set  $Q_0$  be linearly ordered. It follows that if the set  $V_0$  is linearly ordered, then the set  $Q_0$  is also linearly ordered.

The real design process is in reality always, one way or another, organized, that is, ordered. The theorem follows from this.

*Theorem 10.* The set  $V_0$  is linearly ordered [11].

*Proof.* Since all  $v_{0h} \in V_0$  for a real discrete-continuous process are points on the time axis or

$V_0 \subset R^1$ , then for any pair the relation is defined:  $v_{0x} \leq v_{0y}$  or  $v_{0y} \leq v_{0x}$ . If  $v_{0x} \leq v_{0y}$  and  $v_{0y} \leq v_{0x}$ , this clearly proves that  $v_{0x} = v_{0y}$ . Thus, for all  $v_{0h} \in V_0$ , the linear ordering conditions are met. So  $V_0$  is linearly ordered. The theorem is proved.

The fact that the set is linearly ordered follows from said above and Theorem 10. This, in particular, means that in the directed graph  $H_0(V_0, Q_0)$  cannot have contours. In the actual design process, the possible presence of contours means that sequential execution of the same designated elements can take place, but this does not mean repetition of the same elements. Indeed, if during the design some elements are repeated, then such repetition, in fact, is the execution of a new element, since this inevitably changes, at least, the implementation time interval and the area of the space in which the element is “repeated”. Contours in iterative computational processes and in feedback systems that implement discrete-continuous processes have a similar content. Representation of such a “repetition” in the form of a graph with contours can significantly reduce the dimension of the graphical or matrix representation of the graph.

Since the sets  $Q_0$  and  $V_0$  are linearly ordered, each of them can have ordinal types [2]. The ordinal type of the set  $Q_0$  can be defined as a plurality of ordering options on the set  $Q_0$  or a plurality of design process options.

Let us introduce the notations:  $R_0$  is the ordinal type of the set  $Q_0$ ,  $F_0$  is the ordinal type of the set  $V_0$ . Ordinal types can be specified in different ways. It is convenient to define ordinal types in matrix form using adjacency matrices or sequence relation matrices.

Consider the questions of complete ordering of the sets  $Q_0$  and  $V_0$ . Based on Zermelo’s theorem [14], the following statement is true: the set  $Q_0 = \{q_{0i}\}$  can be completely ordered. Therefore, the set  $Q_0$  has a minimum element, and any subset  $X_0$  of the set  $Q_0$  has the first element. Indeed, the complicated process, as a whole, and any of its sections or stages have a minimum or first element. The ordinal type of a well-ordered set is called an ordinal number [2].

Let us introduce the notation  $\Omega_0$  – the ordinal number of the set  $Q_0$ . The presence of the ordinal

number  $\Omega_0$  means that the elements  $q_{0i} \in Q_0$  can be renumbered in several ways. Indeed, all elements of any particular design process can be renumbered, moreover, in several versions, in accordance with the implementation of a particular design process.

*Theorem 11.* The set  $Q_0 = \{q_{0i}\}$  is inductive [11].

*Proof.* Indeed, among the elements of the design process, you can always specify two such elements  $q_{0x}$  and  $q_{0y}$ , and choose the third  $q_{0z}$  so that the following conditions will take place:  $q_{0x} \leq q_{0z}$  and  $q_{0y} \leq q_{0z}$ . The theorem is proved.

Then, based on the Zorn lemma [2; 13] or the Hausdorff – Kuratowski maximality principle [2], the set  $Q_0$  has a maximal element, and on the basis of Zorn's theorem, any non-empty subset of the set  $Q_0$  has at least one maximal element. Indeed, for a real design process, you can always specify an element that defines the beginning of the design and elements that determine the beginning and end of any part of the design process.

Since the set  $V_0$  is linearly ordered, the relations of antisymmetry and transitivity are defined on the set  $V_0$ . Just like the set  $Q_0$ , the set  $V_0$  can be quite ordered and, therefore, the set  $V_0$  has a minimum element. Any subset  $Y_0$  of the set  $V_0$  has the first element. The set  $V_0$  has an ordinal number. The elements of the set  $V_0$  can be renumbered in several ways, however, the way of numbering the elements of the set  $V_0$  is determined by the way of numbering the elements of the set  $Q_0$ . Let us denote the ordinal number of the set  $V_0$  by  $\Lambda_0$ .

*Theorem 12.* The set  $V_0 = \{v_{0h}\}$  is inductive [11].

The theorem follows from the fact that  $H_0(V_0, Q_0)$  is a graph, and the set  $Q_0 = \{q_{0i}\}$  is inductive. Since the set  $V_0 = \{v_{0h}\}$  is inductive, it has a maximal element, and any nonempty subset of it has at least one minimal and one maximal element. Indeed, for a real design process, you can always indicate the moment of its end, and for any of its sections – the moments of the beginning and end of this section.

It follows from the above theorems that the order relations on the sets  $Q_0$  and  $V_0$ , determined by the ordinal types  $R_0$  and  $F_0$ , cannot be independent, that is, the ordinal type  $F_0 = V_0 \times V_0$  cannot be specified independently of the set  $Q_0$  and its ordinal type  $R_0 = Q_0 \times Q_0$ .

## Conclusion

The main result of the study is to show that an intermediate, discrete-continuous structure is necessary. Only with the introduction of such a topological abstraction is it possible to prove that a complex process or system, as well as a model, is a progressively bounded directed graph.

Thus, we get the opportunity to transfer the study of complex processes to the field of combinatorics and take advantage of the achievements of graph theory and topology, etc., in order to establish topological invariants or criteria for structural similarity for the possibility of comparison with each other as various complex processes, as well as products created for their implementation.

So, the complicated processes like discrete-continuous processes and their models have the following topological invariants or criteria for structural similarity:

1) the cardinalities of the sets are  $Q_0$  and  $Q$ . Let us denote them by  $n_{Q_0}$  and  $n_Q$  respectively;

2) the limitedness (boundedness) of the sets  $Q_0$  and  $Q$ . This property has no numerical characteristics, but it shows that all the boundary points of a complicated process belong to this process. Each boundary point of a complicated process is in one-to-one correspondence with one of the boundary points of the graph  $H(V, Q)$ , which is a model of a complicated process;

3) the boundedness (limitedness) of the sets  $Q_0$  and  $V_0$  impose on the graph  $H(V, Q)$  the requirement that there are no contours;

4) ordinal type of the set  $Q_0$ . The design process and its model will be similar only when the ordinal types of the sets of elements  $Q_0$  and  $Q$  are equal, that is  $R_0 = R$ ;

5) the cyclomatic number of the system  $H_0 = Q_0 \times V_0$ , represented on a model as a graph  $H(V, Q)$ , is determined by the formula  $\nu(H) = m_H - n_H + 1$ . This invariant is special. The requirement of equality of cyclomatic numbers of a complicated process and model is not always necessary:

a) the time program of a complex process is nothing more than a variant of the ordinal  $F_0$  type of the  $V_0$  set. Since  $F_0$  depends on  $R_0$ , it is impossible

to form a time program for a complex process regardless of the ordinal type of  $R_0$ . So, the presence of a logical structure of a complex process that determines the ordinal type  $R_0 = Q_0 \times Q_0$  is a necessary condition for the formation of a time program for a complex process;

b) since the ordinal type  $F_0$  is completely determined by the ordinal type  $R_0$  for a given  $Q_0$ , there must be a one-to-one correspondence  $f$  such that  $f: R_0 \rightarrow F_0$ . Therefore, for the formation of a time program of a complex process with known  $Q_0$  and  $R_0$ , rules (algorithms) formalized in full form can be defined and set, allowing automated application;

c) if the ordinal  $R_0$  type is not set, then the formation of a temporary program of the design process, that is, the definition of  $F_0$  is possible only with the simultaneous definition of  $R_0$  and, therefore, cannot be performed according to the rules, which can be formalized in the final form, but will require the use of some iterative methods.

The cyclomatic number of the system  $H_0 = Q_0 \oplus V_0$ , represented on the model as a graph  $H(V, Q)$ , is determined by the formula  $\nu(H) = m_H - n_H + 1$ . This invariant is special. The requirement of equality of cyclomatic numbers of a complex process and model is not always necessary.

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## About the author

Natalia L. Malinina, Candidate of Physical and Mathematical Sciences, Associate Professor of the Department 604, Aerospace Faculty, Moscow Aviation Institute (National Research University), 4 Volokolamsk Shosse, Moscow, 125993, Russian Federation. ORCID: 0000-0003-0116-5889, eLIBRARY AuthorID: 502378. E-mail: malinina806@gmail.com