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Research article

Geometric nonlinear analysis of thin elastic paraboloid of revolution shaped shells with radial waves

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Abstract. From the old ancient types of roof and dome construction, various forms of shells have been discovered which attract special attention. A shell is a structure composed of sheet material so that the curvature plays an important role in the structural behaviour, realizing its spatial form. There are different types of shells, namely thick and thin shells. G. Brankov, S.N. Krivoshapko, V.N. Ivanov, and V.A. Romanova made interesting researches of shells in the form of umbrella and umbrella-type surfaces. The term "nonlinear" refers to a given structure undergoing a change in stiffness in its loaded state. There are basically three different types of nonlinearities: geometric, physical and contact (boundary condition nonlinearity). For further analysis of the stress-strain state, a paraboloid with an inner radius of 4 m and an outer radius of 20 m and the number of waves equal to 6 was considered. The test shell is made of reinforced concrete. The minimum load parameter at which the shell loses stability indicates a more than three times the margin.

Introduction

A shell is a structure composed of sheet material so that the curvature plays an important role in the structural behaviour, realizing its spatial form. There are different types of shells, namely thick and thin shells. A shell is called thin if

$$\frac{1}{1000} \leq \frac{h}{R} \leq \frac{1}{20},$$

where R is the radius of curvature of the middle surface and h is its thickness.

Hence, shells for which this inequality if violated are referred to as thick shells.

From the old ancient types of roof and dome construction, various forms of shells have been discovered which attract special attention. Until present day in the centre of city of Rome stands a masterpiece of ancient shell construction and has withstood for almost two-thousand years called Panthe-

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on. Its span of 43 meters still impresses the modern engineering profession. The Hagias Sophia is a second example of the structural capacity of the classical builders. It was built in the 6th century, however, damages from earthquakes and fires have drastically altered the structure giving the building the appearance that it has today.

The modern era of shell structures started in 1925 with the completion of the first thin reinforced concrete shell covering the Zeiss planetarium in Jena, Germany, this delayed the development of methods of strength analysis. It was, however, a few years earlier, in the beginning of the 20th century that throughout Europe several reinforced concrete shell structures arose [1], inspired by the new material reinforced concrete, patented and promoted by Joseph Monier, who was then a French gardener. These early 'thick' shells are mostly documented in national literature only, and, therefore, less accessible for historical research. An example is the dome of the 1914 Cenakel church by J.G. Wiebenga, constructed in Nijmegen in the east of the Netherlands.

1. Literature overview

The most complete work on the history of the development of umbrella shells in modern architecture is a monography of G. Brankov [2]

In a paper [3], S.N. Krivoshapko explains in detail the different types of paraboloid umbrella-type structures and their formation. Six different types of umbrella-type surfaces with parabolic meridians are known. Different types of identical fragments form each shell type with different number of waves. He gives a comparative analysis on the differences of the umbrella-type surfaces with parabolic meridians with radial waves.

In a review article [4], principal achievements of science and engineering in the sphere of design, construction, and static, vibrational, and buckling analysis of thin-walled structures and buildings in the shape of paraboloid surfaces of revolution are summarized. These shells are useful as roof structures, TV towers, reinforced concrete water tanks, and dams. They are also used as supports for electric power transmission lines and as high chimneys. Several public and industrial buildings having the paraboloid form are described in the review. The basic results of theoretical and experimental investigations of stress-strain state, buckling, and vibration are presented. The influence of temperature and moisture on the stress-strain state of the shells in question is also analysed.

In a book [5] J.N. Reddy presents the theory and computer implementations of the finite element method as applied to nonlinear problems of heat transfer and similar field problems, fluid mechanics, and solid mechanics (elasticity, beams and plates). Both geometric as well as material nonlinearities are considered, and static and time-dependent responses are studied.

The information on the application of shells in the form of the paraboloid of revolution with radial waves is given in a paper [6]. V.N. Ivanov [7] was the first who used a finite difference energy method for the determination of stress-strain state of an umbrella-type shell. But A.S. Chepurnenko with colleagues [8] applied a finite element method for this aim. V.A. Ro-anova [9] offered a method of visualization of generating umbrella-type and umbrella surfaces with radial damping waves in the central point. O. Ariarskyi with colleagues [10] investigated the advantages of umbrella-type shells in comparison with the shells in the form of their base surfaces of revolution.

Shells of this type attract the attention of students of the Department of Civil Engineering of the Academy of Engineering of the RUDN University. They take them for the investigation in their degree works [11]. This paper will help them to continue their researches.

2. Types of nonlinearities

The term "nonlinear" refers to a given structure undergoing a change in stiffness in its loaded state.

There are three different types of nonlinearities: geometric, material, contact (boundary condition nonlinearity).

In FEA, nonlinearity means that the stiffness matrix (K) changes with the solution, K is a function of the nodal displacements; as K changes, a new solution must be attained until the thus iterated solution stops changing within tolerances.

Materials nonlinearity. Material non-linearity is the transfer of material behaviour from elastic to plastic (where elastic corresponds to a linear behaviour and plastic corresponds to a nonlinear behaviour). Even for a plastic material, initial portions of the stress strain graph tend to be linear and then become nonlinear post the Proportional limit (as shown in the Figure 1). From the graph in that figure, it's observed that the stiffness changes because of material property. Simply speaking, if the relation stress-strain is nonlinear we have material nonlinearity.

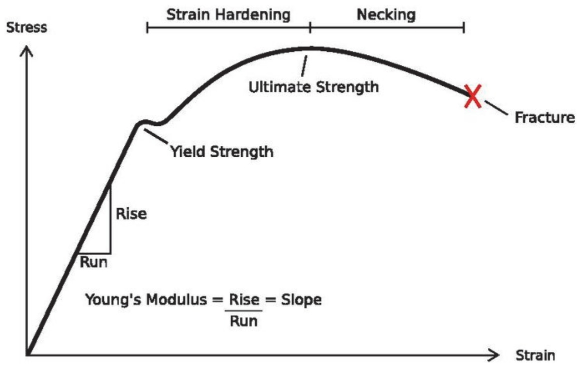


Figure 1. Material stress-strain diagram

Boundary conditions nonlinearity. This is when the boundary conditions defined in the simulation change during an analysis. For example, consider a deflecting cantilever beam (Figure 2, *a*).

Now, consider an object in the path of the deflection (Figure 2, *b*). Between the two above figures, the boundary conditions have changed. The cantilever beam can no longer deflect as the object prevents its deflection. This is what is meant by boundary condition nonlinearity. Here a boundary condition may be added or removed during the finite element run like gap analysis. Other examples include analyses having friction contacts.

Contact nonlinearity occurs when, due to the deformation of one or more parts in contact (pushing or pulling on other) produces a deformation leading to a change in the geometry of the part that translates into a change on K or on the forces (action and reaction) between the parts in contact forcing another iteration on approaching the solution.

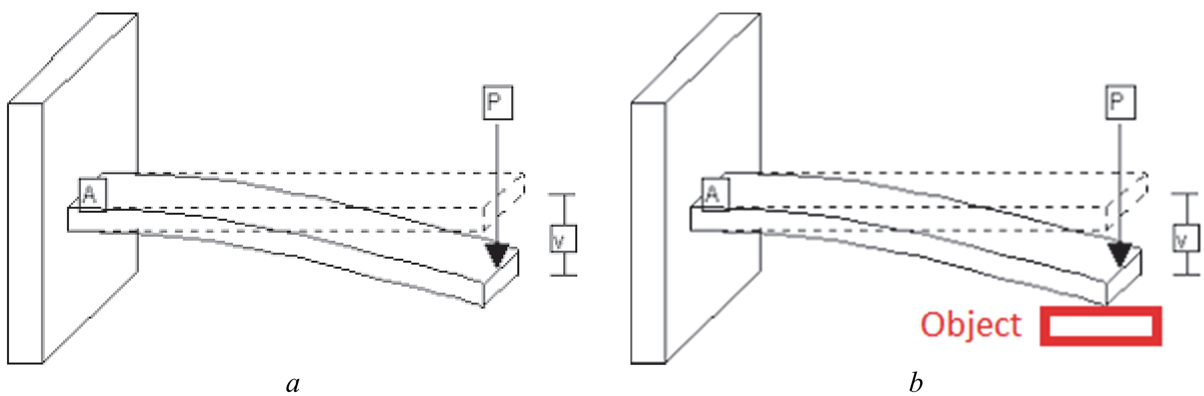


Figure 2. Deflecting cantilever beam

Geometric nonlinearity. Simply speaking, if the relation displacement-strain is nonlinear we have geometrical nonlinearity. As a simple example of geometrical nonlinearity, one can mention the case of elastic buckling of a column and the corresponding Euler equation. In this case, the deformation of column (when it buckles) is so high that the effect of secondary moments due to axial force have to be taken into consideration.

3. The parametric definition of the median surface

A paraboloid of revolution with radial waves is generated by plane parabolas the picks of which coincide with a central fixed point. Tangents drawn through the central points of parabolas must be in the same plane. Any cross section of the surface by the plane passing through the z-axis will be a parabola. The parametrical equations of this surface of umbrella type can be represented as [12]

$$x(u, v) = u \cos v; \; y(u, v) = u \sin v;$$

$$z(u, v) = [a \sin(nv) + b]\, u^2,$$

where v is an angle taken from the x-axis in the direction of the y-axis; a = const is an amplitude of the wave; n is a number of peaks of the waves; b is a constant parameter of the base paraboloid of evolution.

The curvilinear coordinate lines u, v are not lines of principle curvatures. If we take $a = b$ then lower wave picks will be on the xOy plane as show in the plotted surface below (Figure 3).

For further analysis of the stress strain state, we consider a paraboloid with an inner radius of 4 m and an outer radius of 20 m and the number of waves equal to 6 (Figure 4). Suppose the test shell is made of reinforced concrete [13–15].

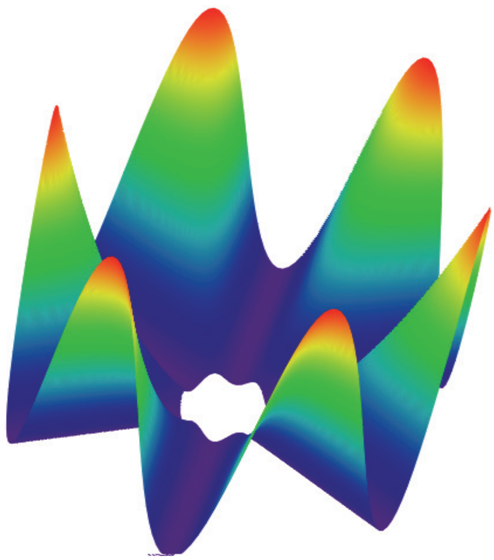


Figure 3. Paraboloid of revolution with radial waves

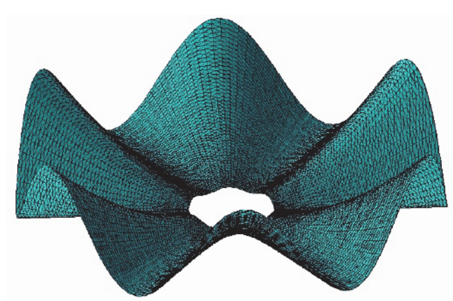


Figure 4. The finite element model of the paraboloid shell

The finite element analysis of the shell was performed in the certified LIRA-SAPR software package. The calculation was performed in a linear setting.

The finite element model of the shell (Figure 4) consists of 15 000 elements and 7650 nodes, the total number of nodal unknowns is 43 953. Flat surface triangular shell elements with six degrees of freedom in the node were used to model the surface.

The boundary conditions corresponded to free support on the inner ring and lower radial generatrices.

The stability calculation was carried out under the assumption of elastic work of the material in accordance with the graph shown in Figure 5. In this case, the redistribution of forces and the possibility of cracking were taken into account approximately by lowering the stiffness of the shell. The elastic modulus E, taken in the calculation, was determined by multiplying the initial elastic modulus of concrete E_0 by a factor of 0.3.

Thus, the calculation was carried out with the following material parameters: shell thickness $h = 16$ cm, elastic modulus $E = 9000$ MPa; Poisson's ratio $\nu = 0.2$; specific gravity $\gamma = 25$ kN/m^3.

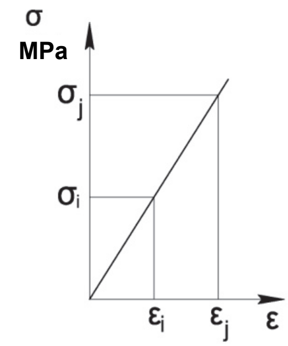


Figure 5. To linear calculation: diagram σ–ε of the material

The shell stability calculations were carried out on the main combination of loads:

$$q = 1.1g + 1.4S \text{ [кN/m}^2\text{]}, \quad (1)$$

where g is the normative value of the shell self-weight ($g = 3.5$ kN/m^2); S is the normative value of the snow load ($S = 1.8$ kN/m^2).

Deflections (Figure 3) were determined from the calculation of the shell for the normative combination of loads as a result of solving the resolving system of equations of the form

$$[K]\{z\} = \{P\}, \quad (2)$$

where $[K]$ is the structural rigidity matrix; $\{z\}$ is the vector of nodal displacements; $\{P\}$ – vector of nodal loads.

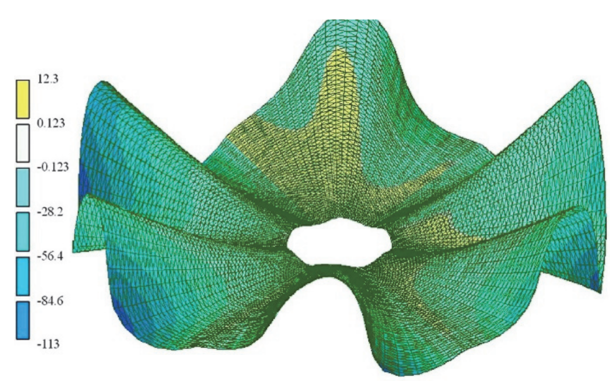


Figure 6. Fields of vertical displacements of the shell

The maximum deflections of the radial generatrix do not exceed 1/150 of the span of the shell (Figure 6).

Due to the symmetry of the shell, it is sufficient to investigate the stress state of its single wave.

Figures 7–12 show the fields of normal and tangential stresses, as well as linear moments obtained as a result of calculating the shell for the calculated combination of loads.

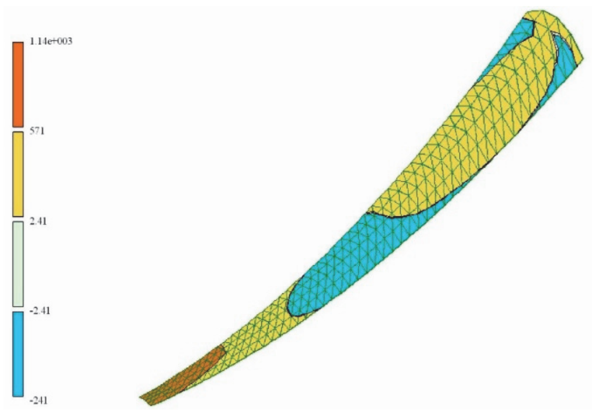


Figure 7. Fields of normal stresses in the radial direction, kN/m^2

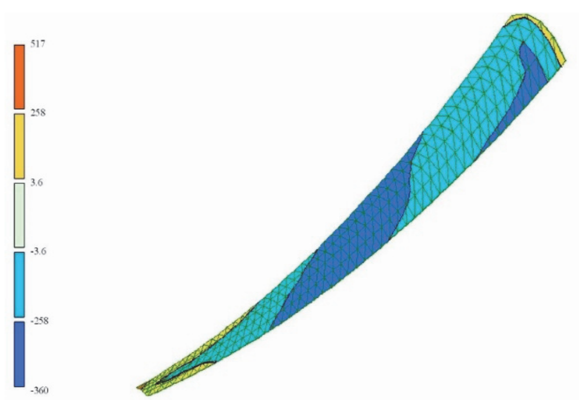


Figure 8. Fields of normal stresses in the circumferential direction, kN/m^2

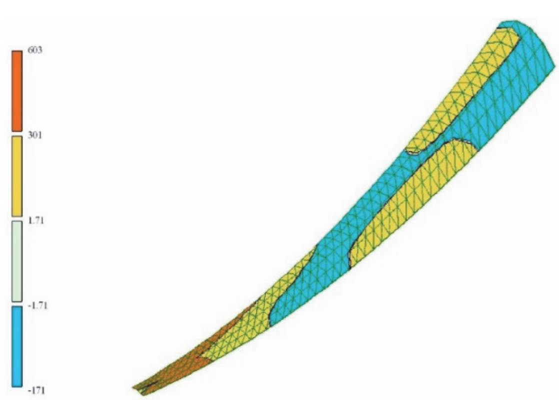


Figure 9. Fields of tangential stresses, kN/m^2

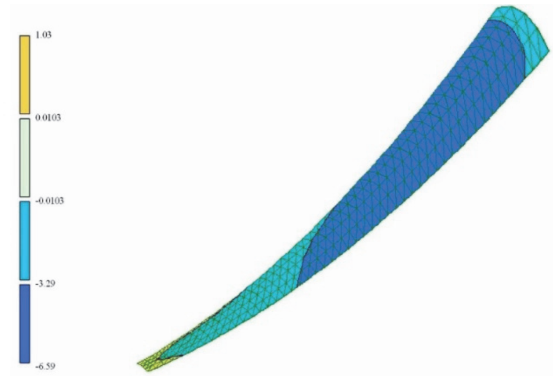


Figure 10. Fields of bending moments in the radial direction, kNm/m

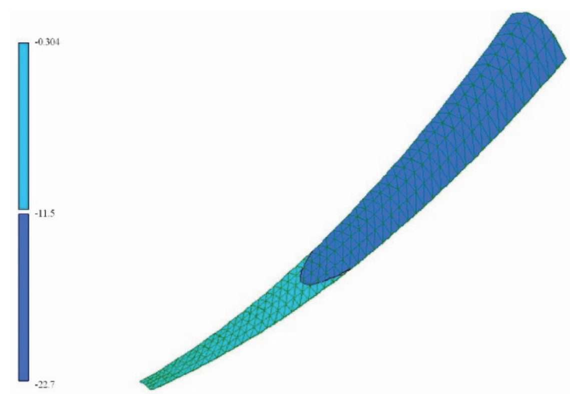


Figure 11. Fields of bending moments in the circumferential direction, kNm/m

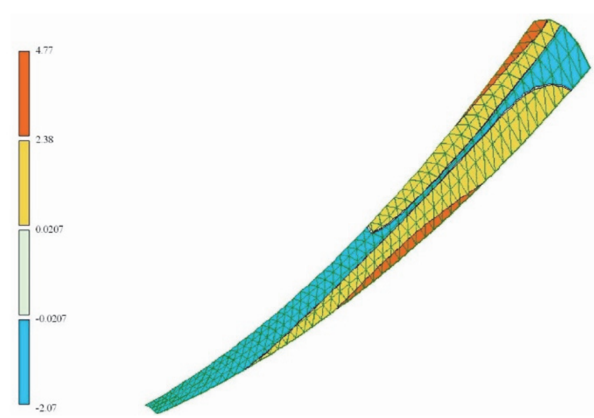


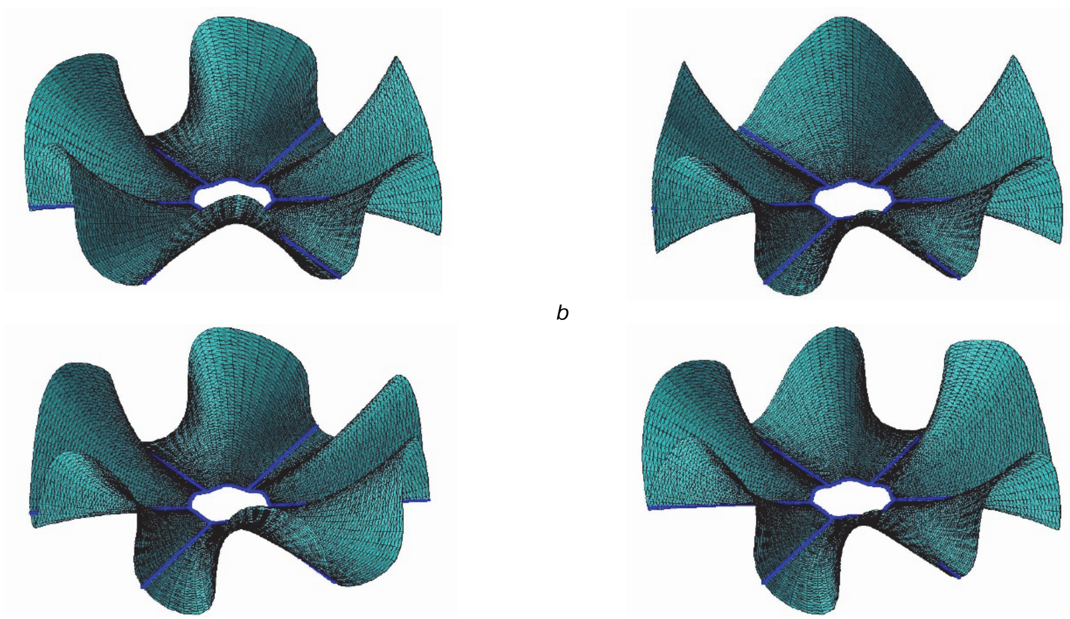
Figure 12. Fields of torques, kNm/m

It is known, the solution of the stability problem by the finite element method reduces to a generalized eigenvalue problem

$$[K - \lambda K_\sigma]\{\varphi\} = 0, \quad (3)$$

where λ is the load parameter at which the structure goes into an adjacent equilibrium state; $\{\varphi\}$ is the eigenvector corresponding to λ; [*K*] – structural rigidity matrix; [K_σ] is the matrix of initial stresses determined from a linear calculation.

Figure 13 show the forms of shell stability loss and the load parameter λ corresponding to each form.



a b c d

Figure 13. The buckling of the shell:
a – the form 1(λ = 3,768); *b* – the form 2 (λ = 4,000); *c* – the form 3 (λ = 4,285); *d* – the form 4 (λ = 5,000)

The minimum load parameter at which the shell loses stability indicates more than three times the margin.

Conclusion

Thus, it was established that a significant contribution to the overall stress state of the shell of the considered form is made by normal and tangential stresses. In addition, the influence of bending forces acting in the circumferential direction is also great. The values of moments in the radial direction were lower than those in the circumferential direction by 50–75%.

The minimum load parameter at which the shell loses stability indicates a more than three times the margin.

May be, the results of these researches will attract Russian and foreign engineers and architects for additional working out.

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Научная статья

Геометрически нелинейный расчет тонких упругих оболочек в форме параболоида вращения с радиальными волнами

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Ключевые слова:
параболоид вращения с радиальными волнами, материальная нелинейность, контактная нелинейность, нелинейность граничных условий, тонкая оболочка

Аннотация. Древние крыши и купола привлекают особое внимание, благодаря разнообразию форм оболочек. Оболочка – это конструкция, состоящая из листового материала, ее кривизна играет важную роль в конструктивном поведении, реализуя пространственную форму. Существуют два типа оболочек: толстые и тонкие. Г. Бранков, С.Н. Кривошапко, В.Н. Иванов и В.А. Романова провели интересные исследования параболоидных оболочек в форме зонтичных и зонтичного типа поверхностей. К данной конструкции, подвергающейся изменению жесткости в ее нагруженном состоянии, применим термин «нелинейный». Существует три основных типа нелинейностей: геометрическая, материальная и контактная (нелинейность граничных условий). В качестве объекта исследования для определения напряженно-деформированного состояния был выбран параболоид с внутренним радиусом 4 м, внешним радиусом 20 м и числом волн, равным 6. Тестовая оболочка изготавливалась из железобетона. Параметр минимальной нагрузки, при котором оболочка теряет стабильность, показал запас более чем в три раза.

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