

ТЕОРИЯ ТОНКИХ УПРУГИХ ОБОЛОЧЕК THEORY OF THIN ELASTIC SHELLS

DOI 10.22363/1815-5235-2021-17-2-165-174
UDC 514.75/77:514.8:72.01

RESEARCH ARTICLE / НАУЧНАЯ СТАТЬЯ

The pendulum type surfaces with congruential cross sections

Sergey N. Krivoshapko, Svetlana L. Shambina*

Peoples' Friendship University of Russia (RUDN University), 6 Miklukho-Maklaya St, Moscow, 117198, Russian Federation
*shambina_sl@mail.ru

Article history

Received: December 12, 2020
Revised: February 18, 2021
Accepted: March 12, 2021

For citation

Krivoshapko S.N., Shambina S.L. The pendulum type surfaces with congruential cross sections. *Structural Mechanics of Engineering Constructions and Buildings*. 2021;17(2): 165–174. <http://dx.doi.org/10.22363/1815-5235-2021-17-2-165-174>

Abstract. The article discusses new kinematic surfaces that can be attributed to the class of surfaces of congruent cross sections. The surfaces of congruent cross sections were first identified in a separate class by Professor I.I. Kotov. Circular, elliptical and parabolic cylinders are taken as the guiding surfaces, and circles and parabolas are taken as generating plane curves, which can be located in the plane of the generating curve of the guiding cylinder or in a plane parallel to its longitudinal axis. The introduction of a new independent parameter helped to solve the set geometric problems. The analytical formulas are presented in generalized form, so the shape of the flat generatrix curve can be arbitrary. Two types of surfaces are considered: 1) when the local axes of the generating curves remain parallel during their movement; 2) when these axes rotate. The resulting surfaces can be of interest to architects, or can find application in machine-building thin-walled structures or in the study of the trajectories of bodies during their oscillatory-translational motion.

Keywords: computer aided design, pendulum type surface, surface of congruent cross sections, helical movement, kinematic surface, free form architecture

Поверхности конгруэнтных сечений маятникового типа

С.Н. Кривошапко, С.Л. Шамбина*

Российский университет дружбы народов, Российская Федерация, 117198, Москва, ул. Миклухо-Маклая, д. 6
*shambina_sl@mail.ru

История статьи

Поступила в редакцию: 12 декабря 2020 г.
Доработана: 18 февраля 2021 г.
Принята к публикации: 12 марта 2021 г.

Аннотация. Рассматриваются новые кинематические поверхности, которые можно отнести к классу поверхностей конгруэнтных сечений. Поверхности конгруэнтных сечений впервые были выделены в отдельный класс профессором И.И. Котовым. В качестве направляющей поверхности принимаются

Sergey N. Krivoshapko, Professor of the Department of Civil Engineering, Academy of Engineering, DSc, Professor; eLIBRARY SPIN-code: 2021-6966, Scopus Author ID: 6507572305, ORCID iD: <https://orcid.org/0000-0002-9385-3699>.

Svetlana L. Shambina, Associate Professor of the Department of Civil Engineering, Academy of Engineering, PhD; eLIBRARY SPIN-code: 5568-0834, Scopus Author ID: 57060572700, ORCID iD: 0000-0002-9923-176X, WoS ResearcherID: V-9606-2017.

Кривошапко Сергей Николаевич, профессор департамента строительства, Инженерная академия, доктор технических наук, профессор; eLIBRARY SPIN-код: 2021-6966, Scopus Author ID: 6507572305, ORCID iD: <https://orcid.org/0000-0002-9385-3699>.

Шамбина Светлана Львовна, доцент департамента строительства, Инженерная академия РУДН, кандидат технических наук, eLIBRARY SPIN-код: 5568-0834, Scopus Author ID: 57060572700, ORCID iD: 0000-0002-9923-176X, WoS ResearcherID: V-9606-2017.

© Krivoshapko S.N., Shambina S.L., 2021

This work is licensed under a Creative Commons Attribution 4.0 International License
<https://creativecommons.org/licenses/by/4.0/>

Для цитирования

Krivoshapko S.N., Shambina S.L. The pendulum type surfaces with congruent cross sections // *Строительная механика инженерных конструкций и сооружений*. 2021. Т. 17. № 2. С. 165–174. <http://dx.doi.org/10.22363/1815-5235-2021-17-2-165-174>

круговой, эллиптический и параболический цилиндры, а за образующие плоские кривые – окружности и параболы, которые могут быть расположены в плоскости образующей кривой направляющего цилиндра или в плоскости параллельной его продольной оси. Решению поставленных геометрических задач помогло введение нового независимого параметра. Формулы приведены в обобщенном виде, поэтому форма плоской образующей кривой может быть произвольной. Рассматриваются два типа поверхностей: когда местные оси образующих кривых остаются параллельными при движении и когда они поворачиваются. Полученные поверхности могут быть интересны архитекторам, найти применение в машиностроительных тонкостенных конструкциях или при изучении траекторий движения тел при их колебательно-поступательном движении.

Ключевые слова: компьютерное проектирование, поверхность маятникового типа, поверхность конгруэнтных сечений, винтовое движение, кинематическая поверхность, архитектура свободных форм

Introduction

The surface of congruent cross sections is a surface that carries a continuous one-parameter family of plane lines [1]. Such a surface is obtained by movement of a flat line (generator).

The selection of the surfaces under consideration in a separate class simplified the presentation of methods for constructing these surfaces by means of computer graphics and descriptive geometry of surfaces. The simplest types of surfaces of congruent sections are *surfaces of plane-parallel transfer* relative to the projection plane. *Surfaces of revolution* can also be assigned to the class of surfaces of congruent sections [2]. *Monge carved surfaces* fit the definition of congruent section surfaces [3]. All *cyclic surfaces* with a generating circle of constant radius can be included in the class of surfaces of congruent sections [4]. *Rotative surfaces* are included in one of the groups of surfaces of congruent sections [5]. Ordinary *helical surfaces* are formed by the helical movement of any rigid line. Hence, they can be included in the class of surfaces of congruent cross sections on a circular cylinder [6]. Full information about all the surfaces listed above can be found in the encyclopedia [7].

Purpose of the study

In this era of innovative ideas, the existing well-studied analytical surfaces are no longer sufficient for the implementation of the creative ideas of architects and engineers [8–10]. Architects and mechanical engineers require the creation and study of new shapes and surfaces described by analytical equations in order to introduce them into various branches of science and technology. Therefore, the authors had the idea to expand the class of surfaces of congruent cross sections adding *surfaces of pendulum type on a circular cylinder with congruent circles* [11]. The present paper will submit for consideration additional surfaces of congruent pendulum-type cross sections on circular, elliptical and parabolic cylinders. These surfaces, perhaps, will satisfy some of the architects' needs in new forms and will allow mechanical engineers to study the process of oscillatory motion of bodies of the considered forms in space.

Methodology of the study

Methods of analytical and differential geometry are used to study the declared analytical surfaces. To visualize these surfaces, the MathCad and AutoCad systems are used. For one family of plane coordinate lines of the surfaces under consideration, the rigid generating curves themselves are taken, and the other family of curvilinear coordinates is the trajectories of points of congruent generating curves [12].

Surfaces of congruent cross sections of pendulum type on a circular cylinder

We assume that the center of a circle of constant radius r moves in the coordinate plane xOy along a fixed circle of radius R , and at the same time, it moves along the axis z (Figure 1). The cyclic surface with the parallelism plane xOy formed in such a way can be called a *right circular helical surface on the cylinder* (Figure 2). The parametric equations of this right circular surface on a cylinder were obtained in a paper [11].

Let's slightly change the conditions of the problem. Suppose that the generating circle lies in a plane which is parallel to the yOz coordinate plane (Figure 3). In this case, the parametric equations of the surface of congruent cross sections of the pendulum type take the form

$$\begin{aligned}
 x &= x(t) = R \sin \alpha = R \sin(c + b \sin t), \\
 y &= y(t, \beta) = R \cos \alpha + r \cos \beta = R \cos(c + b \sin t) + r \cos \beta, \\
 z &= z(t, \beta) = at + r \sin \beta.
 \end{aligned}
 \tag{1}$$

Here

$$\alpha = c + b \sin t,
 \tag{2}$$

where α is the angle varying according to a given law (Figure 3); β is the central angle of the generatrix circle, which is measured from the y -axis towards the z -axis, $0 \leq \beta \leq 2\pi$; t is a variable parameter; a is a constant that determines the length of the cyclic surface in the direction of the z axis; b is the amplitude of the sinusoid, according to the law of which the angle α changes (Figure 3); c is a constant that determines the position of the sinusoid $\alpha = \alpha(t) = c + b \sin t$ in the direction of the α axis.

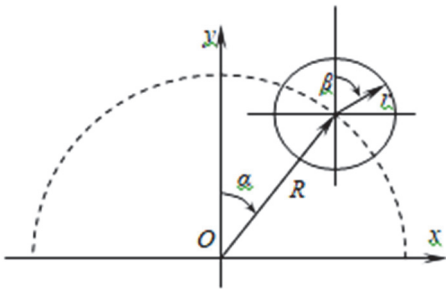


Figure 1. A method of forming of surface of congruent circular cross sections of pendulum type on circular cylinder

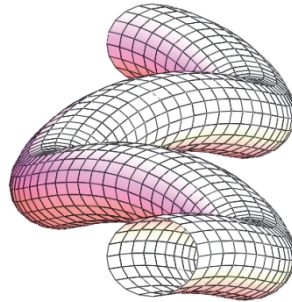


Figure 2. A surface of congruent circular cross sections of pendulum type on a circular cylinder

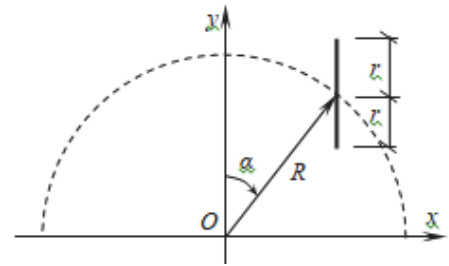


Figure 3. A method of forming of surface of congruent cross sections of pendulum type on circular cylinder with constant circular generatrices which are disposed in the planes parallel to the yOz coordinate plane

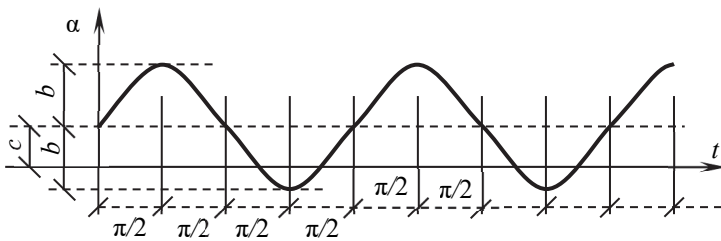


Figure 4. A diagram of changing of the central angle α

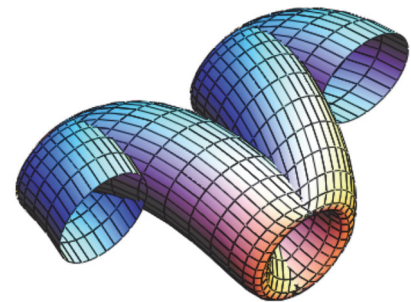


Figure 5. A surface of congruent cross sections of pendulum type on circular cylinder with the constant circular generatrices which are disposed in the planes parallel to the yOz coordinate plane

According to Figure 4 we have

$$\alpha_{\max} = c + b, \alpha_{\min} = c - b.
 \tag{3}$$

In Figure 5 a cyclic surface with the plane of parallelism is shown. For this surface we have

$$R = 3 \text{ m}; r = 1 \text{ m}; 0 \leq \beta \leq 2\pi; 0 \leq t \leq 3\pi.$$

Let the angle α varies within the limits $-\pi/4 \leq \alpha \leq \pi/4$, therefore, according to formulas (3), we have $c = 0$; $b = \pi/4$. If the surface's length in the z direction is equal to $9 \text{ m} + 2r$, then

$$z = at = a3\pi = 9 \text{ or } a = 3/\pi \text{ [m]}.$$

It can be seen from Figure 5 that the presented surface intersects itself in the area of the cross sections $t = \pi/2 + n\pi$ ($n = 1, 2, \dots$).

Surfaces of congruent cross sections of pendulum type on an elliptical cylinder

Let’s suppose that the center of a circle with constant radius r moves in the xOy plane along a fixed ellipse with R and simultaneously moves along the z -axis (Figure 6), where

$$R = R(\alpha) = pd/[p^2\sin^2\alpha + d^2\cos^2\alpha]^{1/2}. \tag{4}$$

The cyclic surface formed in such a way with the xOy plane of parallelism can be called a *right circular helical surface on an elliptical cylinder* (Figure 7).

The parametric equations of this right circular surface on an elliptical cylinder can be written as follows:

$$\begin{aligned} x &= x(t, \beta) = R\sin\alpha + r\sin\beta = R\sin(c + b\sin t) + r\sin\beta, \\ y &= y(t, \beta) = R\cos\alpha + r\cos\beta = R\cos(c + b\sin t) + r\cos\beta, \\ z &= z(t) = at, \end{aligned} \tag{5}$$

where $R = R(\alpha)$ is determined by a formula (4), angles α and β are shown in Figure 6, angle α is presented in the form (2), parameters b and c are shown in Figure 4, a is a constant that determines the length of the cyclic surface in the direction of the z axis.

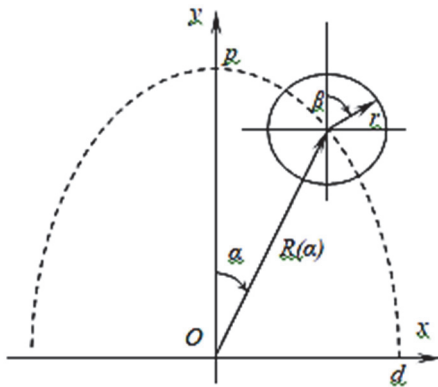


Figure 6. A method of forming surface of congruent circular cross sections of pendulum type on elliptical cylinder

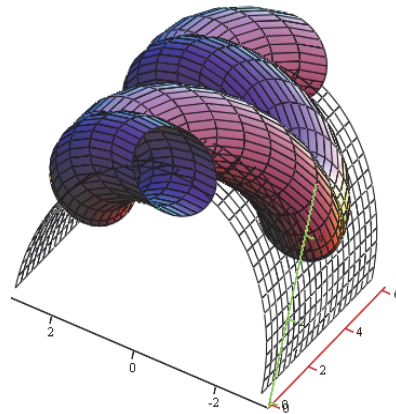


Figure 7. A surface of congruent circular cross sections of pendulum type on an elliptical cylinder

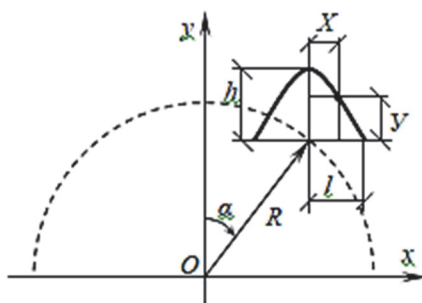


Figure 8. A method of forming surface of congruent parabolic cross sections of pendulum type on circular cylinder

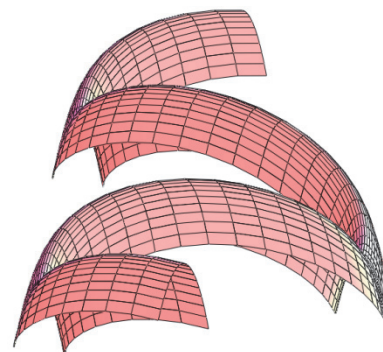


Figure 9. A surface of congruent parabolic cross sections of pendulum type on a circular cylinder

Let the angle α varies within the limits $-\pi/4 \leq \alpha \leq \pi/4$, therefore, according to formulas (3) we have $c = 0$; $b = \pi/4$. Let us take the length of the surface in the direction of the z -axis equal to 6 m, therefore $z = at = a3\pi = 6$

or $a = 2/\pi$ [m]. We shall take an elliptical cylinder with $p = 4$ m, $d = 3$ m, and $r = 1$ m as an example. The surface with the accepted geometric parameters is shown in Figure 7.

In an article [11], it is studied the surface formed by a parabola

$$Y = h - (h / l^2) X^2, \tag{6}$$

which moves along a fixed circle of radius R and simultaneously moves along the z -axis (Figure 8).

As a result, the surface of congruent parabolic cross sections of the pendulum type on a circular cylinder was obtained (Figure 9).

If we take an elliptical cylinder with R (Figure 10), determined by a formula (4), instead of a circular cylinder with $R = \text{const}$, then we obtain a surface of congruent parabolic cross sections of pendulum type on an elliptical cylinder.

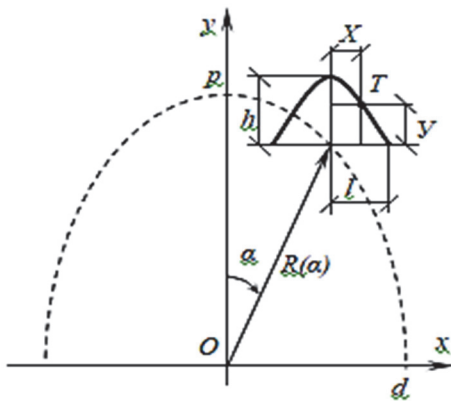


Figure 10. A method of forming of surface of congruent parabolic cross sections of pendulum type on elliptical cylinder

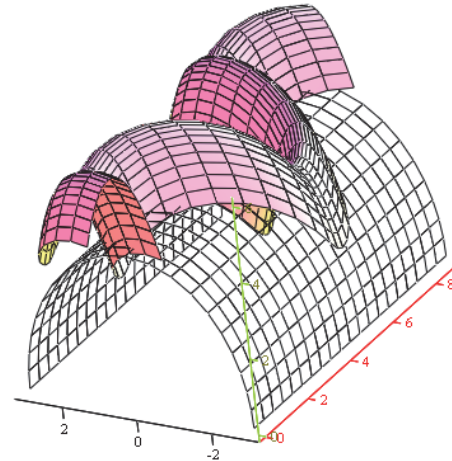


Figure 11. A surface of congruent parabolic cross sections of pendulum type on an elliptical cylinder

The parametric equations of this surface with the plane of parallelism xOy on an elliptical cylinder can be written as

$$\begin{aligned} x &= x(t, X) = R \sin \alpha + X = R \sin(c + b \sin t) + X, \\ y &= y(t, X) = R \cos \alpha + Y = R \cos(c + b \sin t) + Y, \\ z &= z(t) = at, \end{aligned} \tag{7}$$

where $R = R(\alpha)$ is determined by a formula (4), the angle α is shown in Figure 10, the angle α is presented in the form (2), parameters b and c are shown in Figure 4, a is a constant that determines the length of the cyclic surface in the direction of the z axis.

Let the angle α varies within the limits $-\pi/4 \leq \alpha \leq \pi/4$, therefore, according to formulas (3), one can obtain $c = 0$; $b = \pi/4$. Let us assume the length of the surface in the direction of the z -axis to be 9 m, therefore, $z = at = a3\pi = 9$ or $a = 3/\pi$ [m]. For this example, we take an elliptical cylinder with $p = 4$ m, $d = 3$ m, and $l = 1$ m, $h = 2$ m. The surface with the accepted geometric parameters is shown in Figure 11.

Surfaces of congruent cross sections of pendulum type on a parabolic cylinder

Let there be a square parabola in the cross section of a parabolic cylinder (Figure 12)

$$y = H - (H / L^2)x^2, \tag{8}$$

and another parabola performs oscillatory motion along this parabola with simultaneous movement along the cylinder's axis Oz

$$Y = h - (h / l^2)X^2. \tag{9}$$

The geometric parameters of both parabolas are shown in Figure 12. In addition, we obtain

$$x = x(\alpha) = -\frac{L^2}{2H \tan \alpha} \left(1 - \sqrt{1 + 4 \frac{H^2}{L^2} \tan^2 \alpha} \right), \quad (10)$$

$$R = R(x) = \sqrt{x^2 + \frac{H^2}{L^4} (L^2 - x^2)^2}, \quad (11)$$

$$\tan \alpha = \frac{x}{y} = \frac{xL^2}{H(L^2 - x^2)},$$

$$\sin \alpha = \frac{x}{R} = \frac{x}{\sqrt{x^2 + \frac{H^2}{L^4} (L^2 - x^2)^2}},$$

$$\cos \alpha = \frac{y}{R} = \frac{H(L^2 - x^2)}{L^2 \sqrt{x^2 + \frac{H^2}{L^4} (L^2 - x^2)^2}}.$$

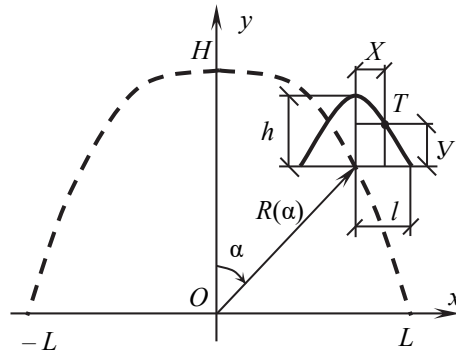


Figure 12. A method of forming surface of congruent parabolic cross sections of pendulum type on parabolic cylinder

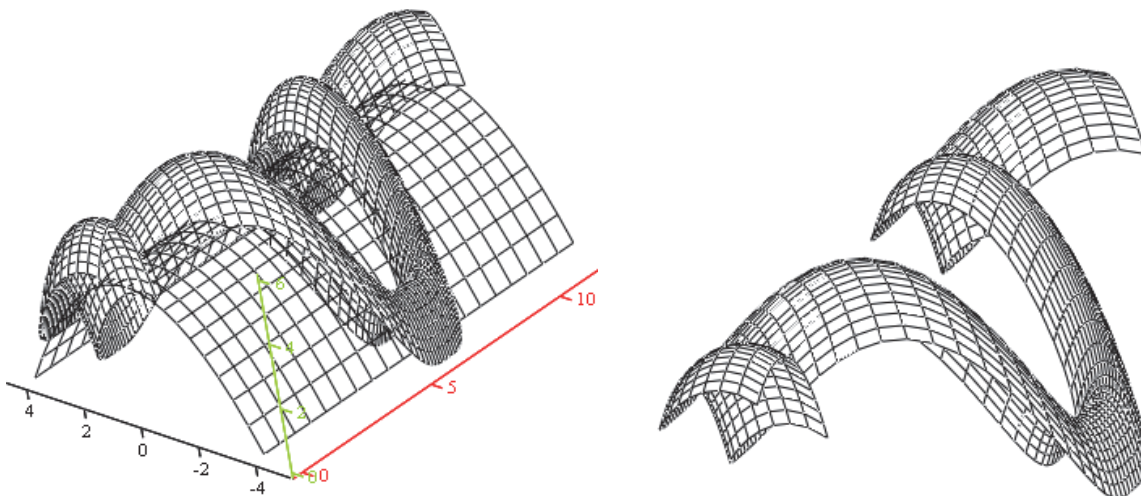


Figure 13. A surface of congruent parabolic cross sections of pendulum type on a parabolic cylinder

Now we can write the parametric equations of the sought-for surface:

$$\begin{aligned}x &= x(t, X) = R(\alpha)\sin\alpha + X = R(t)\sin(c + bsint) + X, \\y &= y(t, X) = R(\alpha)\cos\alpha + Y = R(t)\cos(c + bsint) + h - (h/l^2)X^2, \\z &= z(t) = at.\end{aligned}\tag{12}$$

We have $\alpha = c + bsint$, but $\alpha_{\max} = c + b = \pi/2$, $\alpha_{\min} = c - b = -\pi/2$, that is, $c = 0$, $b = \pi/2$.

Substituting $x = x(\alpha)$ from the formula (10) into the expression for $R = R(x)$, we obtain $R = R(\alpha)$. And taking into account the dependence (2), we obtain $R = R(t)$. However, the given parametric equations (12) can be applied in the interval $-\pi/2 < \alpha < \pi/2$, excluding the point $\alpha = 0$. Some uncertainty appears in the formula (10) at the points $\alpha = 0$ and $\alpha = \pm\pi/2$ if we use computer counting. This uncertainty can be avoided when we use manual counting.

When using a computer calculation, it is better to use the parametric equations of the desired surface in the form

$$\begin{aligned}x &= x(t, X) = L\sin\beta + X = L\sin(c + bsint) + X, \\y &= y(t, X) = H - H\sin^2\beta + Y = H - H\sin^2(c + bsint) + h - (h/l^2)X^2, \\z &= z(t) = at,\end{aligned}\tag{13}$$

where the angle β is related to the angle α (which is shown in Figure 12) in the following way:

$$\sin\alpha = (L/R)\sin\beta.$$

It is obvious that if $\beta = 0$, then $\alpha = 0$, but if $\beta = \pi/2$, then $x = L$. According to the formula (11), $R = L$ and then $\alpha = \pi/2$.

We have $\beta = c + bsint$, but $\beta_{\max} = c + b = \pi/2$, $\beta_{\min} = c - b = -\pi/2$, that is, $c = 0$, $b = \pi/2$.

Figure 13 shows a parabolic surface of a pendulum type on a parabolic cylinder. This surface is given by parametric equations (13) and has the following geometric parameters: $-\pi/2 \leq \alpha \leq \pi/2$, the same for β , $-l \leq X \leq l$, $l = 1$ m, $L = 4$ m, $H = 4$ m, $h = 2$ m.

The surface length is 12 m = $z_{\max} = a3\pi$, that is $a = 12/(3\pi) = 4/\pi$ [m].

Research results

Before the work of I.I. Kotov [1] was published, there was no definition of surfaces of congruent cross sections. These and some other surfaces, formed by the motion of a rigid curve, were included in the class of kinematic surfaces [13]. The authors followed the Kotov's method and have obtained for the first time the parametric equations of 5 surfaces with the congruent cross sections of pendulum type on circular, elliptic and parabolic cylinders, which can find application in technology and in architecture of free forms. A new subclass of surfaces of congruent sections of pendulum type on non-circular cylinders is introduced for the first time.

Most surfaces' formulas are presented in a generalized form, which makes it possible to expand the types of possible cylindrical guide surfaces and types of plane congruent curves. By changing the constants contained in the parametric equations of the surfaces under consideration, it is possible to visualize a whole set of surfaces defined by the same equation. The central angle of the guiding cylindrical surface is taken as one independent parameter in the parametric equations. The solution of the set geometric problems was helped by the introduction of a new independent parameter t , introduced by formula (2).

All the analytic surfaces considered in the presented article have so far been unknown to geometers, as well as their parametric equations.

All the equations obtained for the new surfaces of congruent sections on cylinders are verified using specific numerical examples. All surfaces in the article were constructed using the computer complex MathCad and AutoCad based on the *Bank of Surfaces and Curves* created at the Engineering Academy of the Peoples' Friendship University of Russia [14].

Discussion

Suggestions and recommendations for the use of congruent section surfaces

As mentioned earlier in the introduction, the selection of surfaces of congruent sections into a separate class helped to simplify the presentation of methods for constructing surfaces with a plane rigid generatrix. The surfaces presented may be of interest to architects, or can find application in machine-building thin-walled structures or in the study of trajectories of motion of bodies during their oscillatory-translational motion.

Some architects suggest using these surfaces in freeform architecture. The method of forming surfaces of congruent sections (profiles) makes it possible to actively use the methods of computer modeling in the creation and variant selection of the corresponding forms of structures and structures [15].

The article [16] also supports the idea of using congruent section surfaces in free-form architecture. Sometimes their application is caused by the need to solve structural and geometric problems. Sometimes the final choice is influenced by the lower cost of the project. But, in general, structures in the form of surfaces of congruent sections remain in the form of concept projects [16].

Free-form architecture contains many problems of a geometric nature that need to be solved, but their solution will create new opportunities for optimizing architectural designs in practice [10]. In solving some geometric problems, surfaces of congruent sections can help.

Several surfaces of congruent sections can be easily docked with each other and get a new innovative shape of the structure [13]. If you dock the two surfaces shown in Figure 5, you can create a new object for the Pivot Forms architecture (Figure 14).

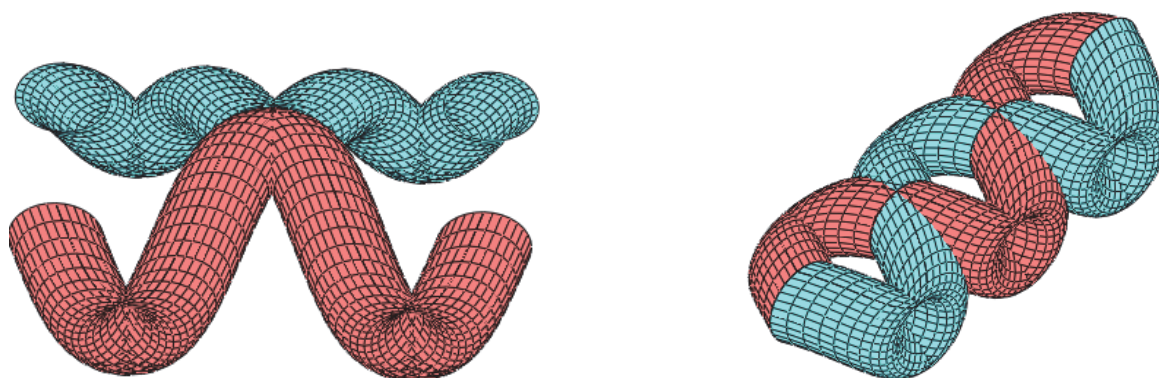


Figure 14. The computer models of the two joined cyclic surfaces of congruent circular sections of pendulum type

Curves of the second order are mainly used as movable generators of rigid plane curves [6–8; 13], but in some cases the necessity requires the use of more complex curves [13; 15; 17].

Conclusion

In the encyclopedia [7] it is shown that at the present time more than 600 analytical surfaces have been studied and proposed for use, which are grouped into 38 classes. Over the past decade, new analytical surfaces have emerged that are not included in the encyclopedia, but researchers of these surfaces are confident that they will be needed by engineers and architects. The authors are also sure of this, proposing for consideration new shapes of surfaces of congruent plane sections moving along a given circular, elliptical and parabolic cylinder along guiding sinusoidal curves lying on these cylinders. Circles, ellipses and parabolas are taken as rigid generating curves. Taking into account the results presented in their previous published work [11], the authors introduced 9 new surfaces of congruent sections on cylinders. In the future, it may be necessary to use other congruent curves, which is easy to implement using the materials of this article.

The geometry of surfaces with congruent curves was studied in works of M. Carmelo and M. Biagio [18], V.N. Ivanov [19], and S.N. Krivoshapko [20]. But they used a different, other than the authors, approach to the formulation and solution of selected geometric problems.

References

1. Kotov I.I. *Descriptive geometry: a course of lectures for students of the faculty of advanced training for teachers*. Moscow: MAI; 1973. (In Russ.)
2. Jasion P., Magnucki K. Buckling and post-buckling analysis of an untypical shells of revolution. *Insights and Innovations in Structural Engineering, Mechanics and Computation. Proceedings of the 6th International Conference on Structural Engineering, Mechanics and Computation*. CRC Press; 2016. p. 766–771. <https://doi.org/10.1201/9781315641645-125>.
3. Filipova J., Rynkovskaya M. Carved Monge surfaces as new forms in the architecture. *MATEC Web of Conferences*. 2017;95(12):5. <https://doi.org/10.1051/mateconf/20179517006>
4. Bock Hyeng Ch.A., Yamb E.B. Application of cyclic shells in architecture, machine design, and bionics. *Int. J. of Modern Engineering Research*. 2012;2(3):799–806.
5. Andrews J., Séquin C.H. Generalized, basis-independent kinematic surface fitting. *Computer-Aided Design*. 2013; 45(3):615–620. <https://doi.org/10.1016/j.cad.2012.10.047>
6. Savičević S., Ivandić Ž., Jovanović J., Grubiša L., Stoić A., Vukčević M., Janjić M. The model for helical shells testing. *Tehnički Vjesnik*. 2017;24(1):167–175. <https://doi.org/10.17559/TV-20150816201404>
7. Krivoshapko S.N., Ivanov V.N. *Encyclopedia of analytical surfaces*. Switzerland: Springer International Publishing; 2015. <https://doi.org/10.1007/978-3-319-11773-7>
8. Krivoshapko S.N., Bock Hyeng Ch.A. Geometrical research of rare types of cyclic surfaces. *International Journal of Research and Reviews in Applied Sciences*. 2012;12(3):346–359.
9. Čučaković A., Jović B., Komnenov M. Biomimetic geometry approach to generative design. *Periodica Polytechnica Architecture*. 2018;47(2):70–74. <https://doi.org/10.3311/PPar.10082>
10. Pottmann H., Eigensatz M., Vaxman A., Wallner J. Architectural geometry. *Computers & Graphics*. 2015;47:145–164. <https://doi.org/10.1016/j.cag.2014.11.002>
11. Krivoshapko S.N., Shambina S.L. On the question of surfaces of congruent sections of pendulum type on a circular cylinder. *Applied Geometry and Engineering Graphics*. 2011;88:196–200. (In Russ.)
12. Knott G., Viquerat A. Helical bistable composite slit tubes. *Composite Structures*. 2019;207(1):711–726. <https://doi.org/10.1016/j.compstruct.2018.09.045>
13. Mesnil R., Santerre Y., Douthe C., Baverel O., Leger B. Generating high node congruence in freeform structures with Monge’s surfaces. *IASS 2015: Future Visions*. Amsterdam; 2015.
14. Ivanov V.N., Romanova V.A. *Constructive forms of spatial structures (visualization of surfaces in MathCad, AutoCad)*. Moscow: ASV Publ.; 2016. (In Russ.)
15. Bartoň M., Pottmann H., Wallner J. Detection and reconstruction of freeform sweeps. *Computer Graphics Forum*. 2014;33(2). <https://doi.org/10.1111/cgf.12287>
16. Mesnil R., Douthe C., Baverel O., Léger B., Caron J.-F. Isogonal moulding surfaces: a family of shapes for high node congruence in free-form structures. *Automation in Construction*. 2015;59:38–47. <https://doi.org/10.1016/j.autcon.2015.07.009>
17. Abd-Ellah H.N., Abd-Rabo M.A. Kinematic surface generated by an equiform motion of astroid curve. *International Journal of Engineering Research & Science*. 2017;3(7):100–114. <https://doi.org/10.25125/engineering-journal-IJOER-JUL-2017-13>
18. Carmelo M., Biagio M. Quando due figure congruenti sono direttamente congruenti. *Boll. Unione Mat. Ital. A*. 1992;6(3):425–430.
19. Ivanov V.N. Geometry of the cyclic translation surfaces with generating circle and directrix of the base sphere. *Structural Mechanics of Engineering Constructions and Buildings*. 2011;(2):3–8. (In Russ.)
20. Krivoshapko S.N., Ivanov V.N. Surfaces of congruent sections on cylinder. *Vestnik MGSU*. 2020;15(12):1620–1631. (In Russ.) <https://doi.org/10.22227/1997-0935.2020.12.1620-1631>

Список литературы

1. *Котов И.И.* Начертательная геометрия: курс лекций для слушателей факультета повышения квалификации преподавателей. М.: МАИ, 1973. 200 с.
2. *Jasion P., Magnucki K.* Buckling and post-buckling analysis of an untypical shells of revolution // *Insights and Innovations in Structural Engineering, Mechanics and Computation: Proceedings of the 6th International Conference on Structural Engineering, Mechanics and Computation*. 2016. Pp. 766–771. <https://doi.org/10.1201/9781315641645-125>
3. *Filipova J., Rynkovskaya M.* Carved Monge surfaces as new forms in the architecture // *MATEC Web of Conferences*. 2017. Vol. 95. No. 12. P. 5. <https://doi.org/10.1051/mateconf/20179517006>
4. *Bock Hyeng Ch.A., Yamb E.B.* Application of cyclic shells in architecture, machine design, and bionics // *Int. J. of Modern Engineering Research*. 2012. Vol. 2. Issue 3. Pp. 799–806.
5. *Andrews J., Séquin C.H.* Generalized, basis-independent kinematic surface fitting // *Computer-Aided Design*. 2013. Vol. 45. Issue 3. Pp. 615–620. <https://doi.org/10.1016/j.cad.2012.10.047>
6. *Savičević S., Ivandić Ž., Jovanović J., Grubiša L., Stoić A., Vukčević M., Janjić M.* The model for helical shells testing // *Tehnički vjesnik*. 2017. Vol. 24. Issue 1. Pp. 167–175. <https://doi.org/10.17559/TV-20150816201404>

7. Krivoshapko S.N., Ivanov V.N. Encyclopedia of analytical surfaces. Switzerland: Springer International Publishing, 2015. 752 p. <https://doi.org/10.1007/978-3-319-11773-7>
8. Krivoshapko S.N., Bock Hyeng Ch.A. Geometrical research of rare types of cyclic surfaces // *International Journal of Research and Reviews in Applied Sciences*. 2012. Vol. 12. Issue 3. Pp. 346–359.
9. Čučaković A., Jović B., Komnenov M. Biomimetic geometry approach to generative design // *Periodica Polytechnica Architecture*. 2018. Vol. 47. Issue 2. Pp. 70–74. <https://doi.org/10.3311/PPar.10082>
10. Pottmann H., Eigensatz M., Vaxman A., Wallner J. Architectural geometry // *Computers & Graphics*. 2015. Vol. 47. Pp. 145–164. <https://doi.org/10.1016/j.cag.2014.11.002>
11. Кривошапко С.Н., Шамбина С.Л. К вопросу о поверхностях конгруэнтных сечений маятникового типа на круговом цилиндре // *Прикладна геометрія та інженерна графіка*. К.: КНУБА, 2011. Вип. 88. С. 196–200.
12. Knott G., Viquerat A. Helical bistable composite slit tubes // *Composite Structures*. 2019. Vol. 207. Issue 1. Pp. 711–726. <https://doi.org/10.1016/j.compstruct.2018.09.045>
13. Mesnil R., Santerre Y., Douthe C., Baverel O., Leger B. Generating high node congruence in freeform structures with Monge’s surfaces // *IASS 2015: Future Visions*. Amsterdam, 2015.
14. Иванов В.Н., Романова В.А. Конструкционные формы пространственных конструкций. Визуализация поверхностей в системах MathCad, AutoCad: монография. М.: Изд-во АСВ, 2016. 412 с.
15. Bartoň M., Pottmann H., Wallner J. Detection and reconstruction of freeform sweeps // *Computer Graphics Forum*. 2014. Vol. 33. Issue 2. <https://doi.org/10.1111/cgf.12287>
16. Mesnil R., Douthe C., Baverel O., Léger B., Caron J.-F. Isogonal moulding surfaces: A family of shapes for high node congruence in free-form structures // *Automation in Construction*. 2015. Vol. 59. Pp. 38–47 <https://doi.org/10.1016/j.autcon.2015.07.009>.
17. Abd-Ellah H. N., Abd-Rabo M. A. Kinematic surface generated by an equiform motion of astroid curve // *International Journal of Engineering Research & Science*. 2017. Vol. 3. Issue 7. Pp. 100–114. <https://doi.org/10.25125/engineering-journal-IJOER-JUL-2017-13>
18. Carmelo M., Biagio M. Quando due figure congruenti sono direttamente congruenti // *Boll. Unione Mat. Ital. A*. 1992. Vol. 6. No. 3. Pp. 425–430.
19. Иванов В.Н. Геометрия циклических оболочек переноса с образующей окружностью и направляющими меридианами базовой сферы // *Строительная механика инженерных конструкций и сооружений*. 2011. № 2. С. 3–8.
20. Кривошапко С.Н., Иванов В.Н. Поверхности конгруэнтных сечений на цилиндрах // *Вестник МГСУ*. 2020. Т. 15. Вып. 12. С. 1620–1631. <https://doi.org/10.22227/1997-0935.2020.12.1620-1631>