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АНАЛИТИЧЕСКИЕ И ЧИСЛЕННЫЕ МЕТОДЫ РАСЧЕТА КОНСТРУКЦИЙ
ANALYTICAL AND NUMERICAL METHODS OF ANALYSIS OF STRUCTURES

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Stress state analysis of an equal slope shell under uniformly distributed tangential load by different methods

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Abstract. Nowadays there are various calculation methods for solving a wide range of problems in construction, hydrodynamics, thermal conductivity, aerospace research and many other areas of industry. Analytical methods that make up one class for solving problems, and numerical calculation methods that make up another class, including those implemented in computing complexes, are used for the design and construction of various thin-walled structures such as shells. Due to the fact that thin-walled spatial structures in the form of various shells are widely used in many areas of human activity it is useful to understand and know the capabilities of different calculation methods. Research works on the study of the stress-strain state of the torse shell of equal slope with an ellipse at the base are not widely available at the moment. For the first time the derivation of the differential equations of equilibrium of momentless theory of shells to determine the normal force N_u from the action of uniformly distributed load tangentially directed along rectilinear generatrixes to the middle surface of the torse of equal slope with a directrix ellipse is presented in this article. The parameters of the stress state of the studied torse are also obtained by the finite element method and the variational-difference method. The SCAD software based on the finite element method and the program SHELLVRM written on the basis of the variational-difference method are used. The numerical results of the parameters of the stress state of the studied torse are analyzed, and the advantages and disadvantages of the analytical method and two numerical calculation methods are determined.

Keywords: thin shell theory, analytical method, momentless state, torse shell, surface of equal slope, finite element method, variational-difference method, SCAD Office computing system, Mathcad system

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Анализ напряженного состояния оболочки одинакового ската при действии равномерно распределенной касательной нагрузки различными методами

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Аннотация. На сегодняшний день существуют различные методы расчета для решения широкого спектра задач в строительстве, гидродинамике, теплопроводности, космических исследованиях и других отраслях. Для проектирования и возведения разнообразных тонкостенных конструкций типа оболочек применяются аналитические методы, составляющие один класс для решения задач, и численные методы расчета, составляющие другой класс, в том числе реализованные в вычислительных комплексах. В связи с тем, что тонкостенные пространственные конструкции в форме разнообразных оболочек широко используются во многих сферах деятельности человека, полезно понимать и знать возможности различных методов расчета. Работы по исследованию напряженно-деформированного состояния торсовой оболочки одинакового ската с эллипсом в основании представлены на данный момент в малом объеме. В статье впервые приводится вывод дифференциальных уравнений равновесия безмоментной теории оболочек для определения нормального усилия N_u от действия равномерно-распределенной нагрузки, направленной по касательной вдоль прямолинейных образующих к срединной поверхности торса одинакового ската с направляющим эллипсом. Также получены параметры напряженного состояния исследуемого торса методом конечных элементов и вариационно-разностным методом. Используются вычислительный комплекс SCAD Office на основе метода конечных элементов и программа SHELLVRM, написанная на базе вариационно-разностного метода. Выполнен анализ числовых результатов параметров напряженного состояния исследуемого торса, установлены плюсы и минусы применения аналитического метода и двух численных методов расчета.

Ключевые слова: теория тонких оболочек, аналитическое решение, безмоментное состояние, торсовая оболочка, поверхность одинакового ската, метод конечных элементов, вариационно-разностный метод, вычислительный комплекс SCAD Office, система Mathcad

Introduction

For the design of diverse engineering structures, various calculation methods are used, such as analytical, numerical and numerical-analytical. In the practice, to get the general parameters of the stress-strain state of spatial-structures, engineers use automated numerical calculation methods because analytical calculation methods are quite complex and time consuming.

The most common numerical calculation method is the finite element method (FEM). Originally, FEM was used for solving mathematical problems in a simpler form. The subsequent development of FEM and automated software systems based on this method such as SIMULIA (www.3ds.com), ANSYS (www.ansys.com), SAP2000 (www.csiamerica.com), SCAD (www.scadsoft.com), PROKON (www.prokon.com) and others, made it

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possible to apply it to solve a wide range of problems in aerospace research, to model and take into account dynamic loads, to solve various problems in thermal conductivity, hydrodynamics, construction and many other areas.

The idea of discretization on which the FEA is based is very old. Before 1922, Courant used the finite element ideas in Dirichlet's principle. The period of 1962–1972 is known as the golden age of FEM [1]. There are five groups of papers (Courant, Argyris, Turner et al., Clough and Zienkiewicz) which may be considered in the development of the FEM and in one of these the name originated [2]. Clough coined the term "finite elements", Turner perfected the direct stiffness method and the works of Huges, Bathe and Zienkiewicz [3] laid the foundation for further progress of the FEM [1]. In [4], a method for calculating bending plates by the finite element method in stresses is proposed, and a comparison with the results of the finite element method in displacements is made. The solution of plane problems of the theory of elasticity based on the approximation of stresses is considered, the calculations of a cantilever beam and a plate with a hole are performed for various finite element meshes, and comparison is made with solutions by the method of finite elements in displacements and with exact solutions in the work [5]. A special issue including 35 papers is devoted to research in the field of development and application of FEM [6].

The finite-difference energy method (FDEM) [7–10] or so called variational-difference method (VDM) [9–14] is also referred to numerical calculation methods [15; 16]. This method takes into account the geometric characteristics of the middle surface of the shell, which allows a more accurate representation of the stress-strain state of thin-walled structures of complex geometry. The VDM (FDEM) is based on the idea put forward by Courant in 1943 [9; 17; 18], which was continued by Houbolt in 1958 [8], who performed static analysis of beams and plates combining finite difference analog of derivatives with a variational formulation [19]. Further developed by Griffin and Varga in 1963 [20] who introduced finite difference into the variational formulation of strain compatibility and boundary conditions for the analysis of plane elasticity problems [19]. Further Bushnell in 1973, and Brush and Almroth in 1975 [21] who extended the approach to the analysis of other type of structures [22].

The successful application of VDM largely depends on how well the system of basic functions allows the qualitative characteristics of the solution. Consequently, it can be expected that the efficient solution of these variational problems will require numerical schemes that differ from traditional techniques based on continuous approximations [23].

In the Department of Construction of the Academy of Engineering of the RUDN University of Russia, the Doctor of Technical Sciences, Professor V.N. Ivanov together with his postgraduate students (currently PhD) Nasr Younis Ahmed Abboushi (Palestine), Muhammad Rizwan (Pakistan), Bock Hyeng Christian Alain (Cameroon), Govind Prasad Lamichhane (Nepal) led the development of SHELLVRM, a new Variational-Difference Method based program. This program allows to determine the stress-strain state of plates and various types of shells with an orthogonal coordinate system, which middle surfaces are described by analytical equations. The program includes such classes of shells as: flat shells on rectangular and curved planes, shells of revolution, shells in the form of Joachimsthal's channel surfaces, shells in the form of Monge surfaces and normal cyclic surfaces. The program includes a system of plane curves, on the basis of which sections of surface classes are formed and coefficients of quadratic forms are calculated. The basics of the VDM and the text of the program for plate calculations are given in [15].

Analytical calculation methods are used for spatial structures in the form of various surfaces [24]. Analytical methods are quite complex and time-consuming. More than 600 analytical surfaces are described in the Encyclopedia of Analytical Surfaces [25]. The geometry of surfaces and automated possibilities of their construction are considered in the monograph [26].

Among an extensive variety of analytical surfaces, the torse shells of equal slope possesses the ability to unfold onto a plane without folds and breaks [27], and this type of shells are widely used in many areas of industry and manufacturing [28–31].

This article is part of a series of research papers devoted to the study of the geometry and stress state of torse shell of equal slope with an ellipse at the base under the action of different loads. In previous works, the authors have performed calculations this shell under the action of a linear load on the upper edge and under the action of self-weight [32; 33] and with a different restraint of the base ellipse [34]. Also, a design of an awning in the shape of a torse of equal slope was proposed and new results were obtained in the field of geometric studies [35; 36]. In this article, we consider the uniformly distributed load directed along rectilinear generatrixes of the torse. The choice of the load type is determined by the possibilities of the momentless shell theory. The main task of this article is to find an analytical solution and determine the parameters of the stress state of the torse by the momentless theory (MLT), followed by comparison with the results of two numerical methods (the finite element method and the variational-difference method).

The surface of equal slope is a ruled surface generated by a straight line moving in the normal plane of a flat directrix-curve with a constant angle of inclination to the plane of the directrix. If we take an ellipse as a flat directrix-curve, then straight lines of equal inclination to the plane of the ellipse will generate the torse surface of equal slope (Figure 1). The surfaces of equal slope are surfaces of zero Gaussian curvature (K = 0). The papers [37; 38] describe the basic properties of these surfaces. The equal slope surface also belongs to the class of Monge surfaces [24; 27].

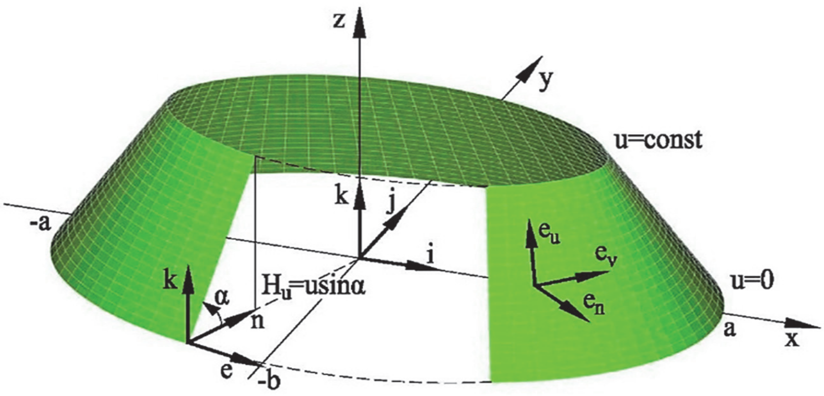

Figure 1. Torse shell of equal slope with an ellipse at the base

As its shown in [27], the directrix ellipse can be defined by parametric equations (1):

$$x = x(v) = a\cos v\,,\;\; y = y(v) = b\sin v, \tag{1}$$

where a and b are the dimensions of the semi-axes of the directrix ellipse at the base of the torse, and the parameter v must be in the limits $0 \le v \le 2\pi$.

According to [27], the parametric form of setting the torse surface with a directrix ellipse is:

$$x = x(u,v) = a\cos v - \frac{ub\cos\alpha\cos v}{\sqrt{a^2\sin^2 v + b^2\cos^2 v}};$$

$$y = y(u,v) = b\sin v - \frac{ua\cos\alpha\sin v}{\sqrt{a^2\sin^2 v + b^2\cos^2 v}};$$

$$z = z(u) = u\sin\alpha. \tag{2}$$

The family of u lines is the rectilinear generatrixes of the torse surface of equal slope, while the coordinate line u = 0 coincides with the ellipse at the base, α is the angle between the principal normal $\boldsymbol{n} = -\boldsymbol{e} \times \boldsymbol{k}$ directed inwards of the directrix ellipse and the straight generatrix u (Figure 1).

The coefficients of the basic quadratic forms of a given surface and its main curvatures are [27]:

$$A = 1; \qquad B = \mu^{1/2} - u\frac{\beta}{\mu}; \qquad F = 0; \qquad L = M = 0; \quad N = B\frac{ab\sin\alpha}{\mu};$$

$$k_1 = k_u = 0; \qquad k_2 = k_v = \frac{ab\sin\alpha}{B\mu}, \tag{3}$$

where $\mu = \mu(v) = a^2\sin^2 v + b^2\cos^2 v$, $\beta = ab\cos\alpha$.

Let us consider the application of the momentless theory of shell calculation, the finite element method and the variational-difference method on the example of a thin torse shell of equal slope with an ellipse at the base under the action of a uniformly distributed load q = 1 kN/m^2 tangentially directed along rectilinear generatrixes to the middle surface of the torse (Figure 2). Thus, the external surface load is $X = -q$, $Y = Z = 0$. The geometric parameters of the torse are: a = 3 m, b = 2 m, α = 60°, the length of the straight generatrixes is u = 2 m. The boundary conditions at the level of the directrix ellipse (u = 0 m) are simple (movable) supports, and at the top (u = 2 m) the edge is free.

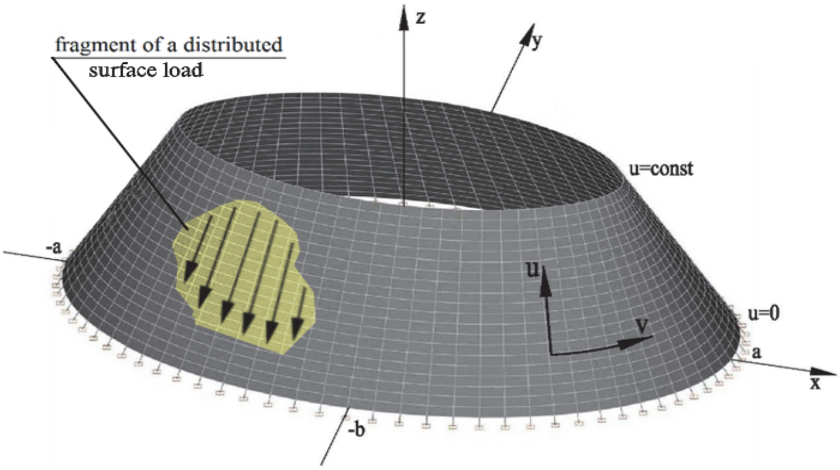

Figure 2. Torse under the action of external distributed surface load

To determine the parameters of the stress state of the considered torse, the momentless theory of shell calculation [24; 27], the SCAD integrated system for finite element structural analysis (FEA), and the SHELLVRM program based on the variational-difference method [15; 16] are used.

The differential equilibrium equations of the momentless theory are obtained from the general equilibrium equations of the moment shell theory [24; 27].

Differential equations of equilibrium of the momentless torse shell

The momentless theory is a simplified version of the general theory of thin elastic shells, which neglects the influence of transverse forces and moments. At the same time, the possibility of existence of the momentless stress state of the shell depends on a number of conditions [24; 27]. The shell should have the form of a smoothly changing continuous surface, also the load on the shell should be continuous and smooth, and the supports of the edges should allow the shell to move freely in the direction normal to the middle surface, normal movements and rotation angles at the edges of the shell should not be restrained.

We obtain differential equations of equilibrium for determining the normal force under the action of a uniformly distributed load acting in the direction of a tangent along rectilinear generatrixes to the middle surface of the considered torse.

General differential equations of equilibrium of the momentless theory [24; 27] have the form:

$$\frac{\partial}{\partial u}(BN_u) - \frac{\partial B}{\partial u}N_v + \frac{1}{A}\frac{\partial}{\partial v}(A^2S) + ABX = 0;$$

$$\frac{\partial}{\partial v}(AN_v) - \frac{\partial A}{\partial v}N_u + \frac{1}{B}\frac{\partial}{\partial u}(B^2S) + ABY = 0;$$

$$\frac{N_v}{R_v} + \frac{N_u}{R_u} - Z = 0. \qquad (4)$$

For the considered case of load application (Figure 2), we obtain $X = -q$ and $Y = Z = 0$. The differential equations of equilibrium (4), taking into account expressions (3), are transformed as following:

$$\frac{\partial}{\partial u}(BN_u) - \frac{\partial B}{\partial u}N_v + \frac{\partial S}{\partial v} + XB = 0;$$

$$\frac{\partial N_v}{\partial v} + \frac{1}{B}\frac{\partial}{\partial u}(B^2 S) = 0;$$

$$\frac{N_v}{R_v} = 0. \qquad (5)$$

The resulting system of differential equations (5) is of second order. To solve it, it is sufficient to have one boundary condition at each point of the torse shell contour. Thus, at the top of the shell at $u = 2$ m the force $N_u = 0$. Moreover, from the second and third equations of system (5) the forces $N_v = 0$ and $S = 0$.

By integrating the first equation of system (5), we obtain the expression for the values of normal force N_u along the rectilinear generatrixes u:

$$N_u = \frac{1}{B(u,v)}\left[\int qB(u,v)\,du + X_1(v)\right]. \qquad (6)$$

Here $X_1(v)$ is an arbitrary function of integration.

Then, by integrating of (6):

$$\int B(u,v)\,du = u\mu^{1/2} - \frac{u^2\beta}{2\mu} = \frac{u}{2}\left(B(u,v) + \mu^{1/2}\right) = \frac{\mu}{2\beta}\left(\mu - B^2(u,v)\right). \qquad (7)$$

To satisfy the boundary condition $N_u = 0$ on the upper free edge under $u = \eta = 2$ m, the arbitrary function of integration $X_1(v)$ in (7) must be equal to:

$$X_1(v) = -q\left(\eta\mu^{1/2} - \frac{\eta^2\beta}{2\mu}\right). \qquad (8)$$

The equation (6) for the calculation of numerical values of the normal forces N_u along the rectilinear generatrixes taking into account the value $X_1(v)$ of the arbitrary integration function (8) takes the following form:

$$N_u = \frac{q}{B(u,v)}\left[\mu^{1/2}(u - \eta) - \frac{\beta}{2\mu}(u^2 - \eta^2)\right]. \qquad (9)$$

To find numerical results of normal force N_u (9) we use the engineering math software Mathcad.

Numerical methods for investigation of the stress state of the shell

The investigation of the stress state of the torse of equal slope was performed by the finite element method and the variational-difference method. The first calculation is performed by using SCAD software. The view of the 3D computational model when approximating the middle surface by a set of quadrangular planar shell elements is shown in Figure 2. The maximum distance between the nodes of the finite elements of the computational model is 0.228 m. The number of finite elements is 1680 and of nodes is 1760.

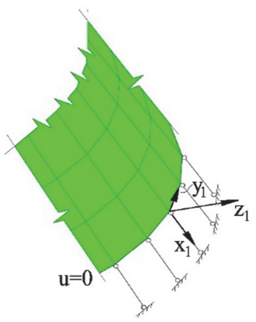

Figure 3. Implementing of momentless state in SCAD software

For the implementation of simple (movable) supports, which is a necessary condition for the momentless work of the torse, the SCAD program has added short bar elements with hinges (Figure 3). The introduction of hinges in these support rod elements releases linear movements along the normal to the torse middle surface (Figure 3, direction z_1), angular movements tangent to the surface (Figure 3, direction y_1) and normal to the surface of the shells (Figure 3, direction z_1), as well as angular movements in the direction of rectilinear generatrixes u (Figure 3, direction x_1).

The second calculation is performed in the program SHELLVRM, based on the variational-difference method. The calculated grid is similar to the grid in FEM. This calculation also takes into account and implements all the necessary conditions for the momentless state of the shell. The calculation is performed for a 1/4 segment of the torse shell, taking into account two planes of symmetry.

Results and discussion

The obtained results of the analytical calculation are compared with the results of numerical methods (by the finite element method and the variational-difference method) for 11 cross-sections (Figure 4).

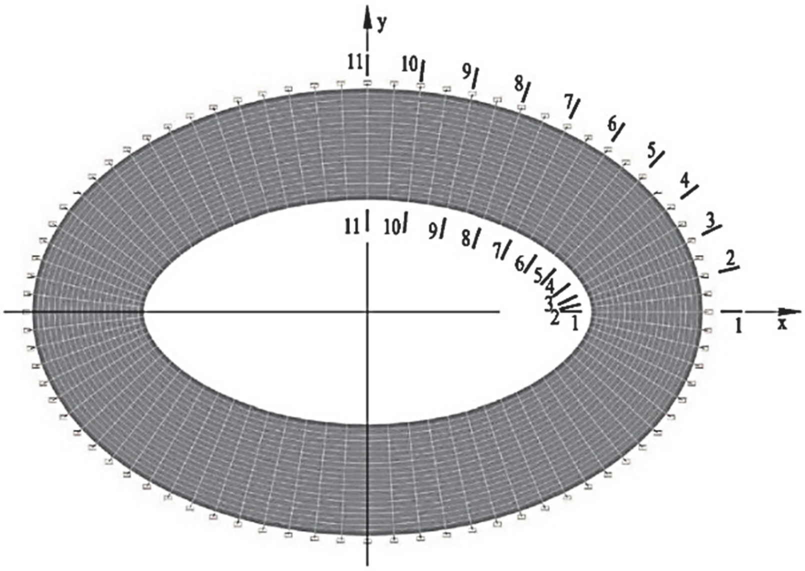

Figure 4. Cross-sections of the torse to compare the results

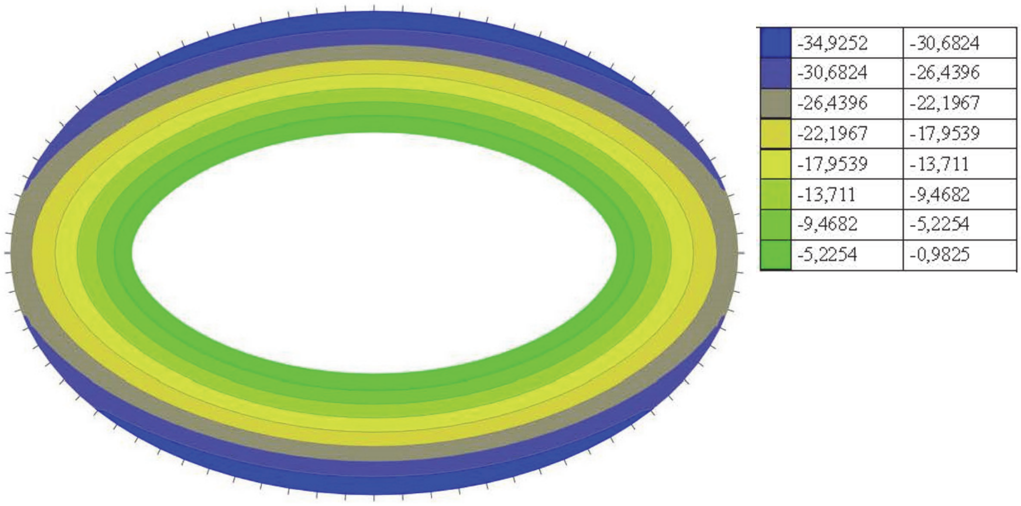

Figure 5. Normal stress σ(N_u) by FEM, kN/m^2

The maximum deviations of the analytical results of normal force N_u along the rectilinear generatrixes from the results of two numerical methods are: 7.4% in section 1–1 (Table 1), 5.0% in section 2–2, 1.9% in section 3–3, 3.7% in section 4–4, 4.1% in section 5–5 (Table 2), 3.6% in section 6–6, 2.8% in section 7–7, 2.2% in section 8–8, 2.0% in section 9–9, 1.9% in section 10–10, and 1.9% in section 11–11 (Table 3). At nodes of coordinates u = 2.00 m in FEM and VRM the values are different from zero when compared with MLT.

For an overall picture of the stress state of torse shell under the action of uniformly distributed load q tangentially applied along rectilinear generatrixes to the torse middle surface, the contour graph distribution of normal stress σ(N_u) obtained in the SCAD software is shown in Figure 5.

Table 1

Results of normal force N_u: cross section 1–1

U-axis coordinate, m	N_u, MLT, section 1–1, kN/m	N_u, FEM, section 1–1, kN/m	Deviation, MLT and FEM, section 1–1, %	N_u, VDM, section 1–1, kN/m	Deviation, MLT and VDM, section 1–1, %
0.000	–1.2500	–1.2438	0.50	–1.2980	3.70
0.200	–1.1432	–1.1906	3.98	–1.1920	4.09
0.400	–1.0353	–1.0841	4.50	–1.0860	4.67
0.600	–0.9258	–0.9757	5.11	–0.9767	5.21
0.800	–0.8143	–0.8640	5.75	–0.8645	5.81
1.000	–0.7000	–0.7477	6.39	–0.7480	6.42
1.200	–0.5818	–0.6253	6.96	–0.6255	6.99
1.400	–0.4579	–0.4942	7.35	–0.4944	7.38
1.600	–0.3250	–0.3503	7.22	–0.3504	7.25
1.800	–0.1769	–0.1871	5.45	–0.1870	5.40
2.000	0.0000	–0.0563	–	–0.0002	–

Table 2

Results of normal force N_u: cross section 5–5

U-axis coordinate, m	N_u, MLT, section 5–5, kN/m	N_u, FEM, section 5–5, kN/m	Deviation, MLT and FEM, section 5–5, %	N_u, VDM, section 5–5, kN/m	Deviation, MLT and VDM, section 5–5, %
0.000	–1.5623	–1.5004	4.12	–1.5340	1.84
0.200	–1.4292	–1.4004	2.06	–1.4010	2.01
0.400	–1.2930	–1.2647	2.23	–1.2650	2.21
0.600	–1.1531	–1.1262	2.39	–1.1270	2.32
0.800	–1.0090	–0.9840	2.54	–0.9842	2.52
1.000	–0.8599	–0.8375	2.68	–0.8376	2.66
1.200	–0.7050	–0.6859	2.78	–0.6860	2.77
1.400	–0.5432	–0.5284	2.80	–0.5285	2.78
1.600	–0.3731	–0.3637	2.58	–0.3638	2.56
1.800	–0.1928	–0.1892	1.88	–0.1899	1.53
2.000	0.0000	–0.0572	–	–0.0002	–

Table 3

Results of normal force N_u: cross section 11–11

U-axis coordinate, m	N_u, MLT, section 11–11, kN/m	N_u, FEM, section 11–11, kN/m	Deviation, MLT and FEM, section 11–11, %	N_u, VDM, section 11–11, kN/m	Deviation, MLT and VDM, section 11–11, %
0.000	–1.7778	–1.7439	1.95	–1.7870	0.51
0.200	–1.6159	–1.6240	0.50	–1.6240	0.50
0.400	–1.4512	–1.4585	0.50	–1.4590	0.53
0.600	–1.2833	–1.2899	0.51	–1.2900	0.52
0.800	–1.1122	–1.1178	0.50	–1.1180	0.52
1.000	–0.9375	–0.9420	0.47	–0.9420	0.48
1.200	–0.7590	–0.7622	0.42	–0.7622	0.42
1.400	–0.5763	–0.5783	0.35	–0.5784	0.36
1.600	–0.3892	–0.3901	0.23	–0.3902	0.26
1.800	–0.1972	–0.1973	0.07	–0.1974	0.10
2.000	0.0000	–0.0553	–	0.0000	–

Comparison of the obtained results of normal force N_u by three calculation methods shows good convergence. The concentration of the largest deviations of the numerical values of the normal force N_u by the momentless theory from the VDM and FEM is in the region of the shell with the largest change in the radius of curvature along the curvilinear directrices, i.e., in the upper nodes of sections 1–1 and 2–2 (Figure 4).

According to the Theory of Strength of Materials, the numerical values of the normal force N_u under the action of uniformly distributed load tangentially along rectilinear generatrixes to the torse middle surface at the nodes of all sections at coordinate u = 2.00 m must be $N_u = 0$. However, the values of the normal force N_u in the FEM and VDM are different from zero, and the results of the VDM are more accurate compared to the FEM. It is well known that the accuracy of the results of FEM and VDM calculations depends on the correct choice of the size of the finite elements (mesh). Moreover, it is noted in [15] that a comparison of the results of VDM and FEM calculations at the same mesh shows close accuracy, and in some cases, VDM gives even higher accuracy results.

FEM and VDM allow obtaining numerical values also for normal forces N_v along curved directrices, bending moments M_u, M_v, tangential forces S and shear forces Q_u, Q_v. The normal forces N_v by VDM range from –0.0246 to 0.0148 kN/m, and by FEM ranges from 0.0490 to 0.0216 kN/m. The shear forces Q_u, Q_v range from –0.01 to 0.01 kN/m. The tangential forces S ranges from –0.0354 to 0.0354 kN/m by FEM, and from –0.0162 to 0.0067 kN/m by VDM. The values of bending moment M_u range from –0.0261 to 0.4244 N·m/m by VDM, and by FEM from –0.1143 to 0.4733 N·m/m. The values of bending moment M_v by VDM range from –0.4140 to 1.4030 N·m/m, and by FEM range from –0.4398 to 1.5562 N·m/m.

The bending moments M_u and M_v are of particular interest, since the values of bending stresses when compared with normal stresses can be used to infer the bending state of the torse shell under the action of the considered load.

The normal stress σ_M and σ_N from normal forces $N_{u,v}$ and moments $M_{u,v}$ are determined as follows:

$$\sigma_N = \frac{N_{u,v}}{h}\ ;\ \sigma_M = \frac{6M_{u,v}}{h^2}. \qquad (10)$$

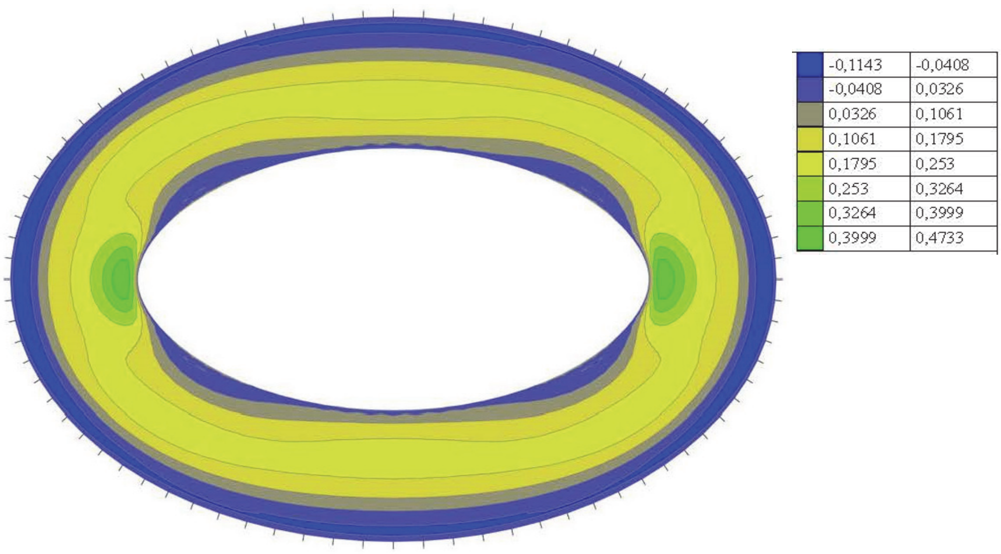

Figure 6. Bending moment M_u by FEM, N·m/m

The results of the VDM for the maximum ratio of stresses σ_{Mu} to σ_{Nu} are: in the cross section 1–1 is 156.5% in the node of coordinate u = 2.00 m, 27.2% in the node with the coordinate u = 1.80 m, 13.1% in the node with the coordinate u = 1.60 m, 7.2% in the node with the coordinate u = 1.40 m, in other nodes does not exceed 4.4%. In cross section 2–2 is 61.9% in the node of coordinate u = 2.00 m, 25.0% in the node with coordinate u = 1.80 m, 12.3% in the node with the coordinate u = 1.60 m, 6.8% in the node with the coordinate u = 1.40 m, other nodes do not exceed 4.2%. In section 3–3 is 25.9% in the node with the coordinate u = 2.00 m, 18.5% in the node with the coordinate u = 1.80 m, 9.9% in a node with coordinate u = 1.60 m, and in the other nodes does not exceed 5.8%. In section 4–4 is 7.0% in node with coordinate u = 2.00 m, 9.3% in the node with coordinate u = 1.80 m, 6.7% in the node with coordinate u = 1.60 m, and in all other nodes does not exceed 4.5%. In section 5–5 is 45.5% in the node with the coordinate u = 2.00 m; in section 6–6 is 13.3% in the node with the coordinate u = 2.00 m; in section 7–7 is 17.4% in the node with the coordinate u = 2.00 m; in section 8–8 is 10.9% in the node with the coordinate u = 2.00 m; in section 10–10 is 8.9% in the node with the coordinate u = 2.00 m; in section 11–11 is 34.8% in the node with the coordinate u = 2.00 m, 5.8% in the node with the coordinate u = 1.80 m. In other nodes of sections 5–5 to 11–11, the stress ratio does not exceed 5.2%.

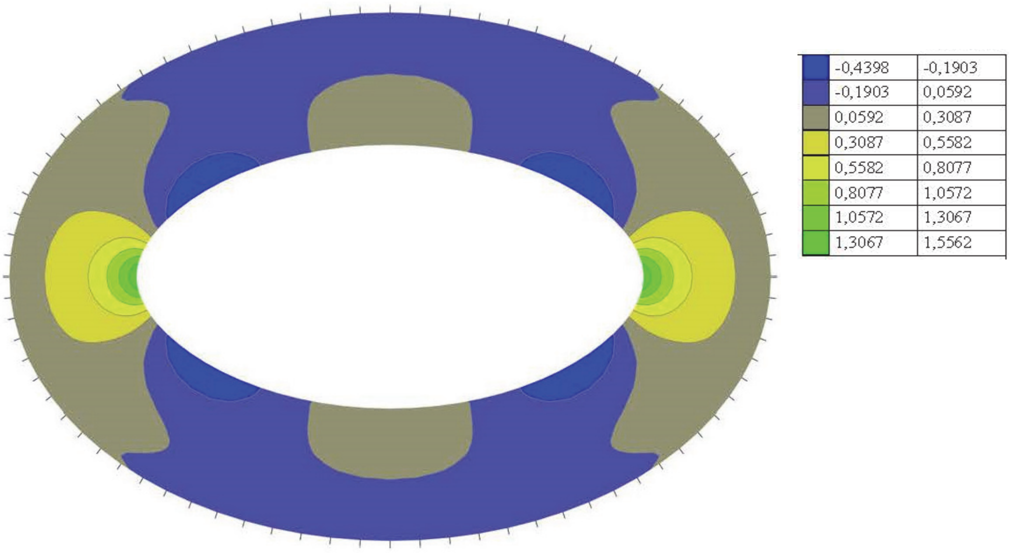

Figure 7. Bending moment M_v by FEM, N·m/m

The bending stresses σ_{Mv} arising from a uniformly distributed load directed tangentially along rectilinear generatrixes to the middle surface, in the VDM have an even greater influence on the bending state of the considered torse shell with a directrix ellipse at the base.

The results of studying the influence of bending stresses σ_{Mu} and σ_{Mv} in FEM show a similar character. Figures 6 and 7 show the contour graph distribution of bending moments M_u and M_v obtained in the SCAD software.

Conclusion

The research is carried out at the Academy of Engineering of the Peoples' Friendship University of Russia (RUDN University). In the field of geometry and stress-strain state of various shells, in particular torse shells class, works at RUDN University have been carried out since 1960's. An undeniable contribution to modern theory of shells was made by Prof. V.G. Rekach, Prof. S.N. Krivoshapko and Prof. V.N. Ivanov and their postgraduate students (today PhD in Technical Sciences). Currently, S.N. Krivoshapko and V.N. Ivanov continue their research in the field of shell theory [39–41].

This paper for the first time presents the differential equations of equilibrium for a torse shell of equal slope with a directrix ellipse and the expression for the normal force N_u determination under the action of uniformly distributed load tangentially directed along rectilinear generatrixes to the torse middle surface.

Determination of the internal force N_u of the investigated torse shell by the analytical method is a complex and time-consuming task that requires a lot of time and increased concentration of attention on its implementation, since a slight inaccuracy can lead to incorrect results. The comparison of the results of the momentless theory with the results of the finite element method and the variational-difference method shows good convergency, which indicates the correctness of the obtained differential equilibrium equations and the expression for determining the values of the normal force N_u. The use of SHELLVRM and SCAD programs simplifies the solution of this task. However, the calculation in the SHELLVRM program is possible if there is the program text for its implementation, and in the SCAD program it becomes difficult to implement a momentless state (introduction of simple-movable supports). When choosing a method of solving the problem, SCAD program, based on the finite element method, is the simplest and most versatile way for solving the research problem.

The values of the normal force N_u along the rectilinear generatrixes of the shell indicate that the considered shell is working in compression. Thus, it is a big plus when selecting materials for the design and manufacture of torse shells. Considering the property of this class of shells to be flattened on the plane without folds and breaks, this is also an advantage when selecting torse shells among similar shaped.

Due to the results of the FEM and VDM, it was found that the bending stresses σ_{Mu} and σ_{Mv} have a significant influence on the torse shell stress state. Therefore, it is necessary to consider the bending moments M_v and M_u when designing different structures in the form of this class of shells. The momentless theory does not allow us to obtain these parameters of the stress state of the torse. Thus, it may be concluded that the momentless theory is not applicable for the considered torse shell of equal slope with ellipse directrix.

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