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RESEARCH PAPER

Simplified selection of optimal shell of revolution

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Abstract

Relevance. Architects and engineers, designing shells of revolution, use in their projects, as a rule, spherical shells, paraboloids, hyperboloids, and ellipsoids of revolution well proved themselves. But near hundreds of other surfaces of revolution, which can be applied with success in building and in machine-building, are known. ***Methods.*** Optimization problem of design of axisymmetric shell subjected to given external load is under consideration. As usual, the solution of this problem consists in the finding of shape of the meridian and in the distribution of the shell thickness along the meridian. In the paper, the narrower problem is considered. That is a selection of the shell shape from several known types, the middle surfaces of which can be given by parametrical equations. The results of static strength analyses of the domes of different Gaussian curvature with the same overall dimensions subjected to the uniformly distributed surface load are presented. Variational-difference energy method of analysis is used. ***Results.*** Comparison of results of strength analyses of six selected domes showed that a paraboloid of revolution and a dome with a middle surface in the form of the surface of rotation of the $z = -a\cosh(x/b)$ curve around the Oz axis have the better indices of stress-strain state. These domes work almost in the momentless state and it is very well for thin-walled shell structures. New criterion of optimality can be called "minimum normal stresses in shells of revolution with the same overall dimensions, boundary conditions, and external load".

Keywords: dome; shell of revolution; paraboloid of revolution; the forth order paraboloid of revolution; catenary line; variational-difference energy method of analysis; linear shell theory; geometrical modeling; optimal design

Introduction

V.V. Novozhilov [1] was one of the first scientists who began to seek for a shell of revolution with the most advantageous indices of stress-strain state. In particular, examining four different domes (spherical, parabolic, half-elliptical, and the lesser part of elliptical domes), he determined that a dome in the form of the lesser part of ellipsoid of revolution was the most advantageous one because it can work as momentless shell with comparatively slight rigidity of the support contour. Membrane strength theory of shells subjected to dead load was used.

Now, the protection of erections from the terrorist attacks and, thus, search of effective structural shapes to mitigate the blast energy is very important problem. The influence of inside blast pressure was studied for six shapes of domes of the same weight and thickness in a paper [2]. The study shows that the parabolic

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and bowl shape of domes could withstand the blast load with the least top displacement.

It is necessary to pay attention to the investigation [3], carried out with five types of shells that are spherical, elliptical, parabolic, and hyperbolic shells of revolution as well as combined shells consisting of two shell fragments with middle hyperbolic and parabolic surfaces of revolution. The combined dome, put together from the lower hyperbolic dome and the upper paraboloid of revolution, recommended itself best of other four types of shells.

The determination of optimal geometrical parameters of a reinforced concrete elliptical shell, used as a covering of the round building with the diameter of the base equal to 27 m, is a purpose of a paper [4]. In general, well known domes of revolution are studied in great number of published works. Considerably less number of researches is devoted to shells of revolution of untypical forms, for instance, to egg-shaped shells [5] or to toroidal domes with elliptic cross section [6]. In papers [7; 8], stress-strain state of the well-known shells as well as the shells of negative Gaussian curvature with the opening at the top, that did not find application else, is examined with the help of finite difference energy method. Several shells of revolution of untypical forms are offered for the application in a manuscript [9] and a monography [10]. Thus, choosing the geometry of dome covering, it is necessary to take into account architectural, technological, technical, and economic demands.

Diversity of opinion of famous architects and civil engineers on the rank of shell structures in modern architecture is very broad, beginning from enthusiastic reviews and optimistic expectations in the 50–60th years of the last century until negation of progressive role of these structures at the end of the 20th century [11]. The well-known Portuguese architect Eduardo Elísio Machado Souto de Moura said: "I don't think that any global new forms will appear but new technologies and materials will be arisen". R. Buckminster Fuller answered about his architectural creations in such style: "Let architects tell about aesthetics… I shall prefer dome where stresses and strains are going away".

In a paper, the illustration of the simplified selection of optimal shell of revolution is carried out for domes of untypical forms. It should be noted that 24 criteria of optimality are known now [12]. The investigation of several untypical domes of revolution of positive and alternating Gaussian curvatures which did not find else the wide-spread application in architecture, construction, and machine-building is an aim of the presented paper too. Simplified selection of optimal shell of revolution begins with geometrical modelling of a necessary shell.

1. Geometrical modelling of shells with similar overall dimensions

In figure 1, six types of shells of revolution are shown:

– a paraboloid of revolution (figure 1, *a*);

– a shell in the form of the fourth order paraboloid of revolution with a $x^4 = cz$ meridian (figure 1, *b*);

– a shell with the middle surface called "Soucoupoid" which can be traced by a curve $z = (b/a^3)(a^2 - x^2)^{3/2}$ in the process of its rotation about an axis Oz (figure 1, *c*);

– a shell with a middle surface of rotation of a curve $z = be^{-(ax)2}$ about an axis Oz (figure 1, *d*);

– a shell in the form of a surface of rotation of a curve $z = -a\cosh(x/b)$ (figure 1, *e*);

– a shell in the form of "fairing of cycloidal type" (figure 1, *f*).

Let these shells with middle surfaces of revolution have three identical overall dimensions R, f, and h, where R is the radius of a base, f is a shell rise (figure 2), h is the constant shell thickness.

All surfaces taken for consideration can be given by parametric equations

$$x = x(r,\beta) = r\cos\beta,\ y = y(r,\beta) = r\sin\beta,\ z = z(r), \quad (1)$$

where the coordinate lines r and β (parallels and meridians) are the lines of principal curvatures, $0 \le \beta \le 2\pi$, $0 \le r \le R$. Hence, for paraboloid of revolution (figure 2, *a*),

$$z = z(r) = f\cdot(1 - r^2/R^2),$$

for the fourth order paraboloid of revolution (figure 2, *b*)

$$z = z(r) = f\cdot(1 - r^4/R^4),$$

for a surface of revolution "Soucoupoid" (figure 2, *c*)

$$z = z(r) = f\cdot(1 - r^2/R^2)^{3/2},$$

for a surface of rotation of a curve $z = be^{-(ax)2}$ about an axis Oz (figure 2, *d*)

$$z = z(r) = b/\exp\{(r^2/R^2)\ln[b/(b-f)]\},\ b > f,$$

for a surface of rotation of a curve

$$z = -a\cosh(r/b) \quad (2)$$

about an axis Oz (figure 2, *e*)

$$z = z(r) = -a\cosh(r/b),$$

where $b = R/\mathrm{Arcosh}\,(1 + f/a)$, a is an arbitrary number.

A surface of revolution shown in figure 1, *f* can be expressed by parametric equations as [13]

$$x = x(t,\gamma) = a(t + \sin t)\cos\gamma,$$

$$y = y(t,\gamma) = a(t + \sin t)\sin\gamma,$$

$$z = z(t) = c(1 + \cos t), \quad (3)$$

where γ is the angle taken from the coordinate axis *Ox* into the direction of the *Oy* axis, $0 \le \gamma \le 2\pi$; $0 \le t \le \pi$. The surface "fairing of cycloidal type" (figure 1, *f*; figure 2, *f*) must have a given *R* radius of a base and a given shell rise *f*, that is why it is necessary to take in formulas (3)

$$c = f/2, \quad a = R/\pi.$$

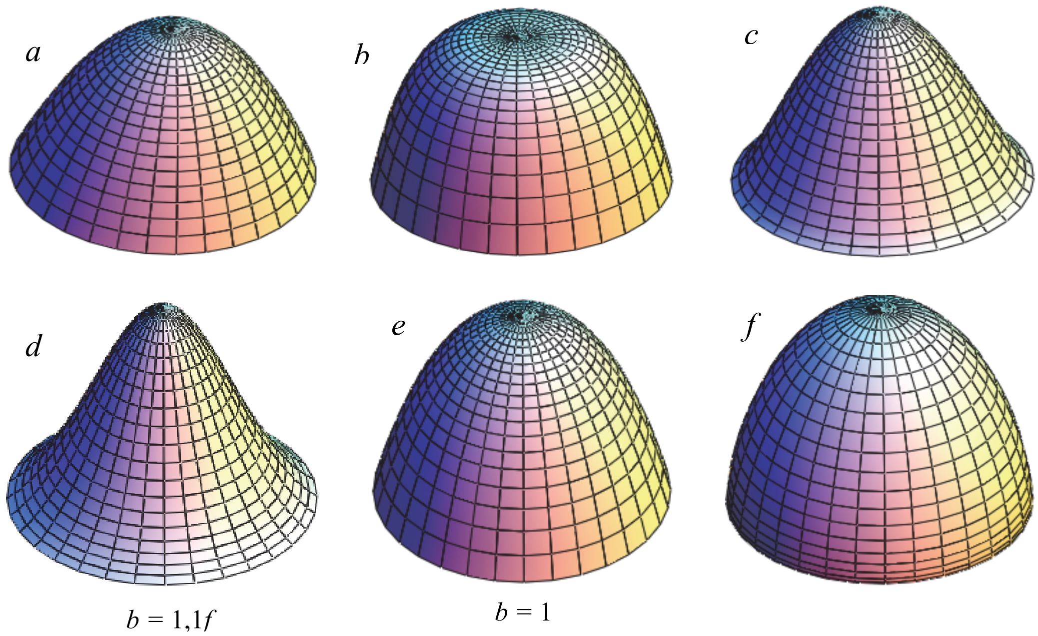


Figure 1. Six types of surfaces of revolution:
a – paraboloid of revolution; *b* – the fourth order paraboloid of revolution; *c* – a surface of revolution "Soucoupoid";
d – a surface of rotation of a curve $z = b\exp(-a^2x^2)$ about an axis *Oz*; *e* – a surface of rotation of a curve $z = -a\cosh(x/b)$ about an axis *Oz*;
f – "fairing of cycloidal type"

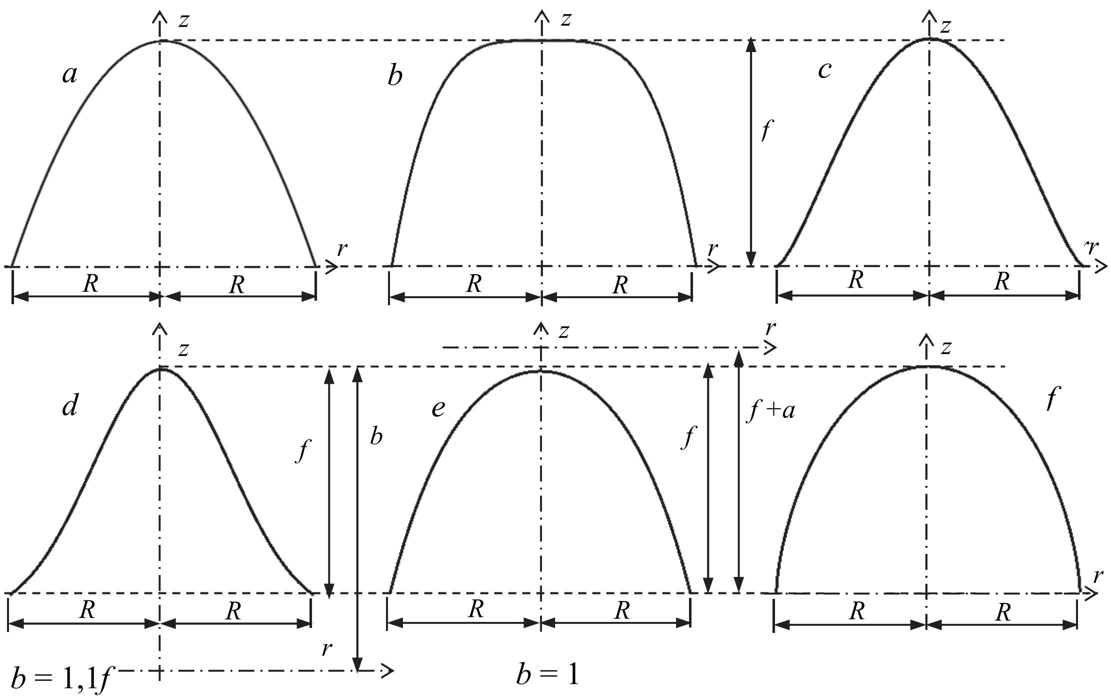


Figure 2. Meridians of six surfaces of revolution with the same overall dimensions *R* = 2 m and *f* = 3 m:
a – paraboloid of revolution; *b* – the fourth order paraboloid of revolution; *c* – a surface of revolution "Soucoupoid";
d – a surface of rotation of a curve $z = b\exp(-a^2x^2)$ about an axis *Oz*; *e* – a surface of rotation of a curve $z = -a\cosh(x/b)$ about an axis *Oz*;
f – "fairing of cycloidal type"

The determination of stress-strain state of chosen shells of revolution is the next stage of simplified selection of an optimal shell of revolution.

2. Stress-strain state of shells of revolution with similar geometrical parameters

Find the parameters of stress-strain state of the selected shells (figure 2) subjected to a constant uniform over the surface load q = 1000 N/m^2 directed along the Oz axis. Let f = 3 m, R = 2 m, $0 \le \beta \le 2\pi$, $0 \le r \le R$ (figure 3). The shells have a thickness equal to 0.05 m and the radius of the top opening equal to 0.25 m. Taking into account that the shells have the top opening, we can determine that the heights f of the shells will change slightly (figure 3).

If shells of revolution are subjected to axisymmetric loading, then the surface uniform load Y = 0, normal (membrane) forces N_r, N_β, shearing forces Q_r, bending moments M_r, M_β, strains ε_r, ε_β, κ_r, κ_β, and displacements $W = u_z$, u_r are axisymmetric, i.e. they do not depend on an angle of longitude β and

$$S = Q_\beta = M_{r\beta} = 0, \quad u_\beta = \varepsilon_{r\beta} = \kappa_{r\beta} = 0.$$

In figure 3 the results of calculation of the given shells of revolution by a finite difference energy method are presented. Let the shells have the hinged immovable supports along the round lower edges $r = R$ but the edges of the upper openings are free. Assume the modulus of elasticity of the shell material $E = 3.5 \cdot 10^4$ MPa, Poisson's ratio ν = 0.17. The special computer program was written by V.N. Ivanov for a finite difference energy method of calculation.

A finite difference energy method and a finite element method (FEM) are based on the Lagrange's principle. A principle of minimum of the total strain energy was assumed by Lagrange as a basis. In FEM, a shell is divided into the finite elements and displacements are approximated by the shape functions. In finite difference energy method, a difference lattice is marked on a shell structure and the derivatives in the total strain energy functional are substituted for difference relations. The functional of total strain energy becomes a function of node displacements. Having minimized this functional, one can derive a system of algebraic equations and after that the displacements in the nodes of the difference lattice. For the determination of strains and internal forces in the lattice nodes, difference derivatives are used again. In a program complex of a finite difference energy method worked out in the RUDN University, a library of curves and surfaces is used for the determination of geometrical characteristics of thin-walled shell. It gives the possibility to take into account a real geometry of thin-walled shells in a process of analysis of their stress-strain state.

A finite difference energy method demands the obligatory accurate satisfaction of kinematical boundary conditions only.

The total strain energy expression can be given by

$$F = U - T,$$

where U is internal work, T is external work,

$$U = \frac{1}{2}\int_\Omega\int\left(N_1\varepsilon_1 + N_2\varepsilon_2 + 2S\varepsilon_{12}\right)d\Omega + \frac{1}{2}\int_\Omega\int\left(M_1\chi_1 + M_2\chi_2 + 2H\chi_{12}\right)d\Omega,$$

$$T = \int_V\int\left(Xu + Yv + Z\,w\right)dV + \int_\Omega\int\left(q_1u + q_2u + q_z w\right)d\Omega,$$

where Ω is an area of the middle surface of a shell; $d\Omega = A_1A_2drd\beta$; A_1, A_2 is the coefficients of the first fundamental form of the surface; X, Y, Z are volume external forces; q_1, q_2, q_z are external surface loads; u, v, w are displacements in the direction of curvilinear coordinate r and β, and in the direction of the normal z; V is a volume of the body.

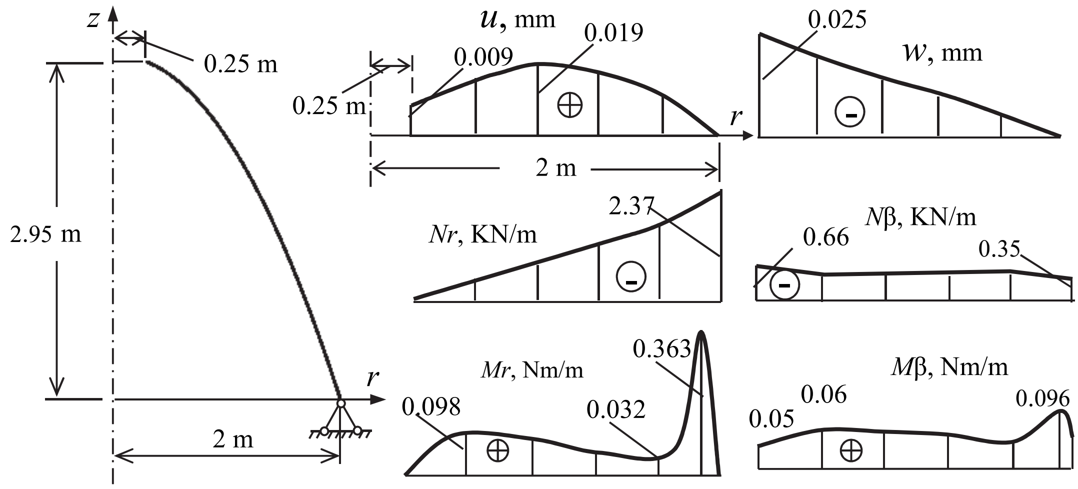
In our case, $X = Y = Z = 0$.

The geometrical and physical equations of a shell theory in the principal curvatures are well known and can be taken in [1].

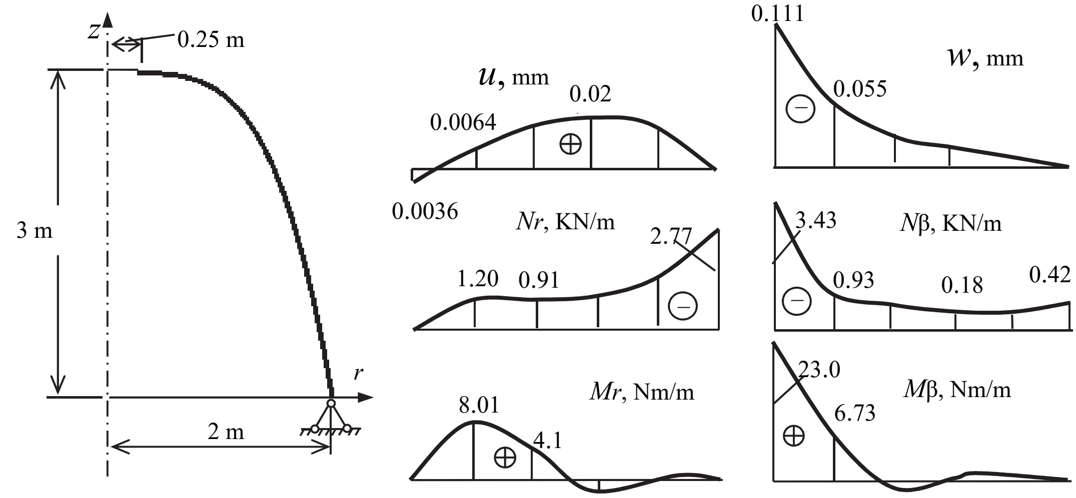
Compile a table (see table) of maximum values of displacements $u_r = u$, $u_z = w$, bending moments M_r, M_β and normal stresses σ_{Mr}, $\sigma_{M\beta}$ coming into existence from these moments, and normal (membrane) forces N_r, N_β and normal stresses σ_r, σ_β coming into existence from these forces.

It is obvious that almost all calculated parameters of stress-strain state of shells in the form of paraboloid of revolution and in the form of surface of rotation of a curve $z = -a\cosh(x/b)$ about an axis Oz are nearly equal and they assume minimal values in comparison with other shapes of the shells with the same geometrical parameters. This can be by the final stage of selection of an optimal shell of revolution.

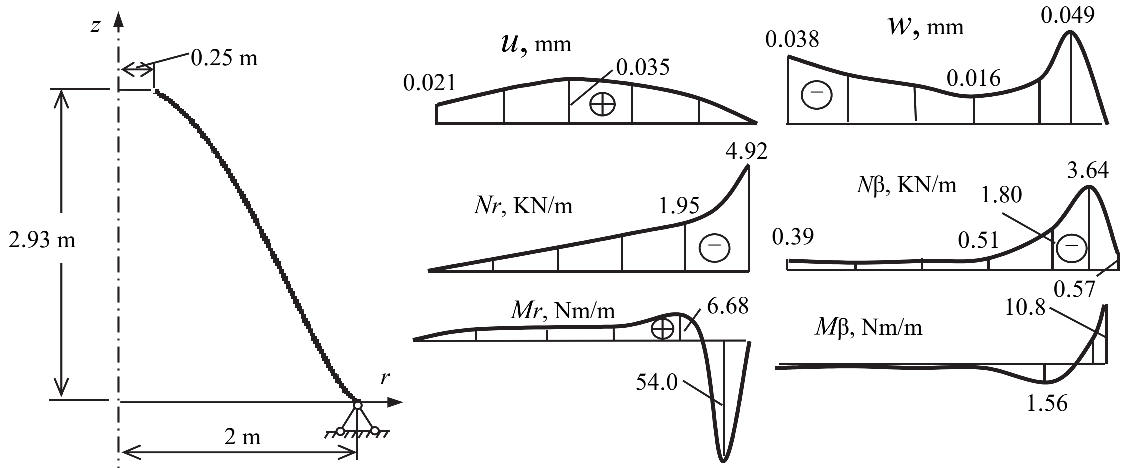
After the determination of the stress-strain state of chosen shells of revolution and selection of the shell with minimal normal stresses in comparison with other shapes of the shells with the same geometrical parameters, it would be useful to get to know about application and researches of the selected domes described in other published works. This can have an influence on final decision.



a) The shell in the shape of paraboloid of revolution

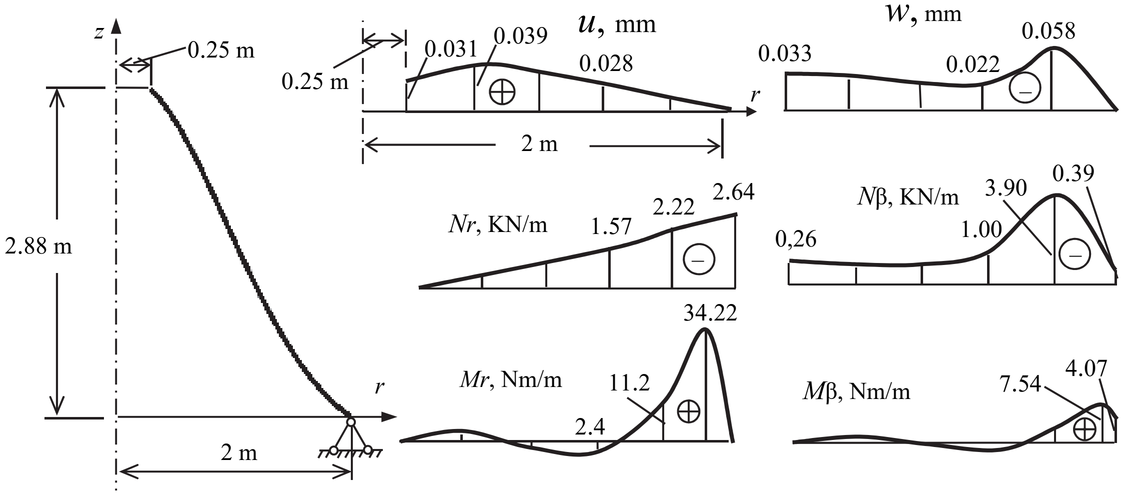


b) The shell in the shape of the fourth order paraboloid of revolution

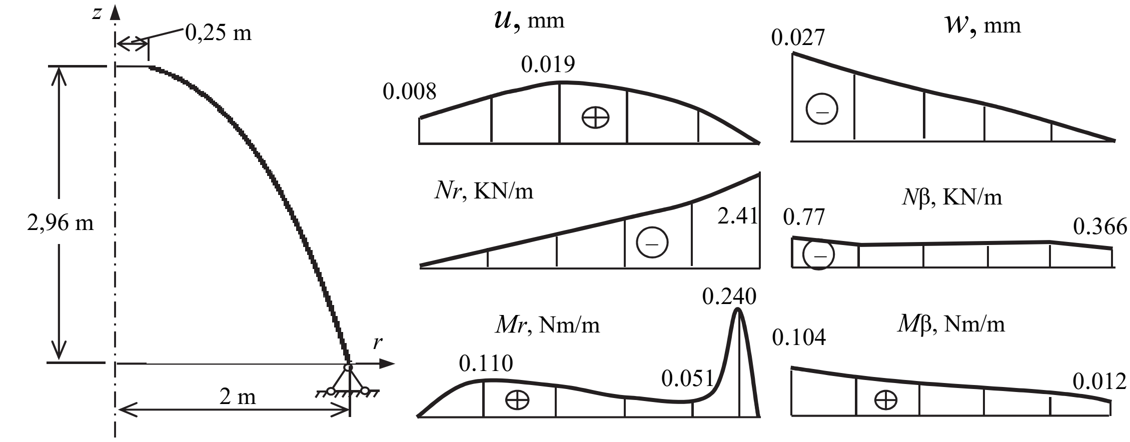


c) The shell in the shape of a surface of revolution “Soucoupoid”

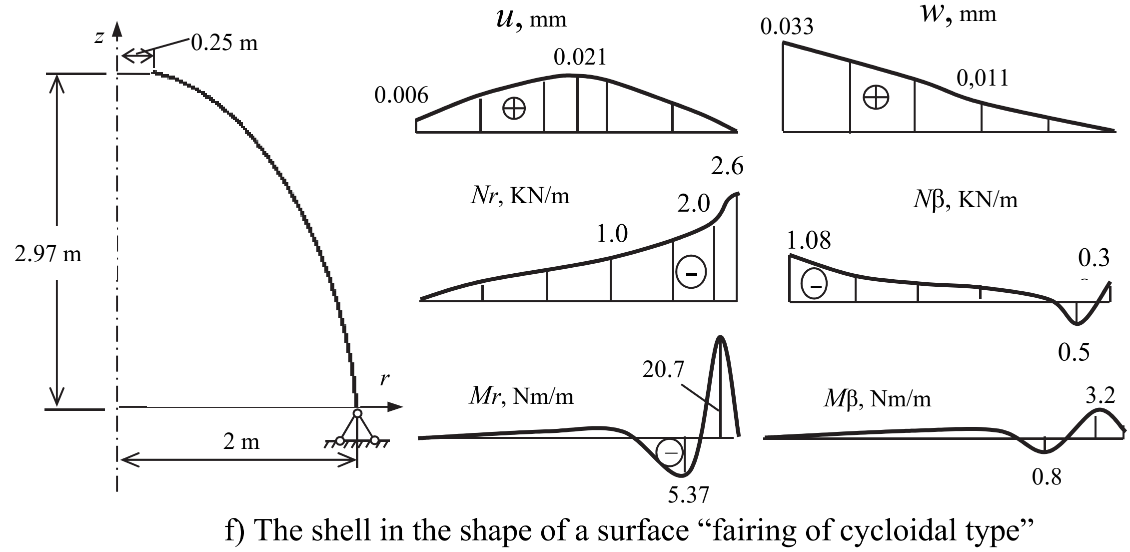
Figure 3. Orthographic representation of displacements and internal force factors of the shells of revolution



d) The shell in the shape of a surface of rotation of a curve $z = b\exp(-a^2x^2)$ about an axis Oz



e) The shell in the shape of a surface of rotation of a curve $z = -a\cosh(x/b)$ about an axis Oz



f) The shell in the shape of a surface "fairing of cycloidal type"

Figure 3. Orthographic representation of displacements and internal force factors of the shells of revolution (*continuation*)

Table

The comparison of maximum results obtained by the calculation with the help of a variational-difference energy method

Figure 3	$u_r = u$, mm	$u_z = w$, mm	N_r, KN/m	σ_r, KPa	N_β, KN/m	σ_β, KPa	M_r, KN·m/m	σ_{Mr}, KPa	M_β, KN·m/m	$\sigma_{M\beta}$, KPa
a	0.019	0.025	–2.37	–47.4	–0.66	–13.2	0.363	0.87	0.096	0.23
b	0.024	**0.111**	–2.77	–55.4	–3.43	–68.6	8.01	1.92	**23.0**	**55.2**
c	0.035	0.049	**–4.92**	**–98.4**	–3.64	–72.8	**–54**	**–129.6**	10.8	25.92
d	**0.039**	0.058	–2.64	–52.8	**–3.90**	**–78**	34.22	82.13	7.54	18.1
e	0.019	0.027	–2.41	–48.2	–0.77	–15.4	0.24	0.58	0.104	0.25
f	0.021	0.033	–2.6	–52	–1.08	–21.6	20.76	49.8	3.2	7.68

3. Recommendations for the application of six types of the selected domes of revolution in construction and machine-building

In this paper, six types of the shells of revolution are submitted for consideration and only paraboloids of revolution obtained a widespread recognition [14]. A surface "fairing of cycloidal type" which found application in machine-building, in particular, in aircraft industry [13] is known much less. Z.V. Belyaeva [15] offers to use catenary line $z = -a\cosh(r/a)$ as a generatrix curve of the surface of revolution (1) with an Oz axis. She noted that it is not possible to connect a rise and a diameter of the dome in explicit form and that is why it is necessary to introduce the additional parameter b into a formula (2). Firstly, Antonio Gaudi used catenary line in his projects, in particular, for the design of the form of the church dome [16].

The rest of the surfaces of revolution (figure 1, *b, c, d*) are known to mathematicians only [17]. For example, geometrical modeling being one of direction of mathematical modeling is used for solution of complex problems of design of different objects and processes by the descriptive geometry methods. In a work [18], the 4th order paraboloid of revolution is employed for these aims (figure 1, *b*).

4. Results and discussions

In this paper, parametrical equations of the middle surfaces of shells of revolution containing two constant geometrical parameters f and R are presented. So, designers can realize six shapes for one object and can choose the most attractive shape for their object (figure 1).

The domes on round plan, shown in figure 1, *c, d*, the upper part of the middle surface of which is the surface of positive Gaussian curvature ($K > 0$) but the lower part is the surface of negative Gaussian curvature ($K < 0$), are studied in this paper for the first time.

The behavior of the shell in the shape of the fourth order paraboloid of revolution (figure 3, *b*) under loading can be explained by a low stiffness of its middle surface [19]. The upper part of this shell is very similar to a plate rested upon an elastic support of large rigidity and that is why we have significant bending moment M_r quite far from the upper edge of the shell.

The emergence of circular tensile normal forces N_β near the lower support in the shell "fairing of cycloidal type" compels to use designed reinforcement in reinforced concrete shells.

If to change geometrical parameters *f, R, h* in proportion to each other, then the character of the diagrams shown in figure 3 will not change. Hence, conclusions and recommendations remain valid.

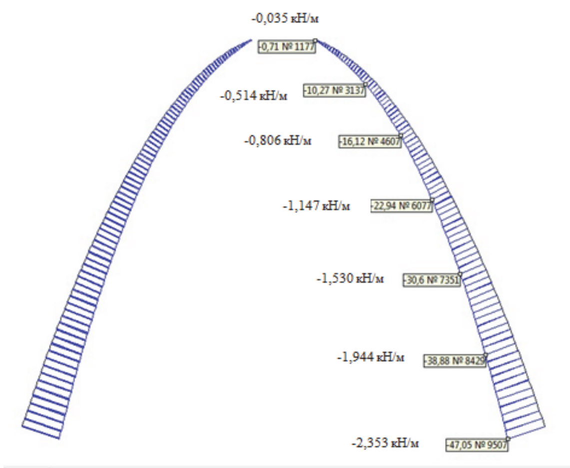


Figure 4. The paraboloid of revolution (the normal stresses σ_r and the normal forces N_r)

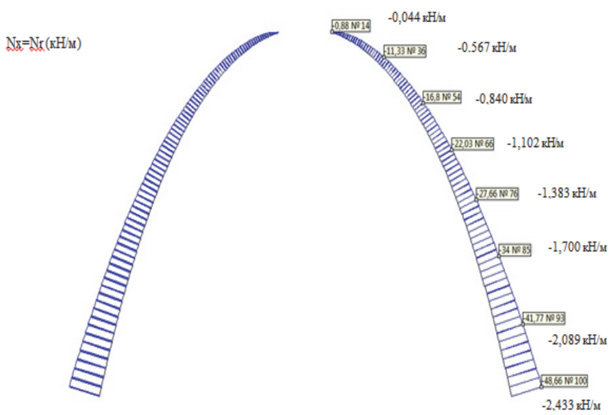


Figure 5. The shell in the shape of a surface of rotation of a curve $z = -a\mathrm{ch}(x/b)$ about an axis Oz (the normal stresses σ_r and the normal forces N_r)

Designers can use the results of static strength analysis of six types of the shells of revolution in question, given in this paper, and can choose the suitable shape for their objects with point of view of a shell stress-state state. The normal stresses σ_r and normal forces N_r for two types of the shells is shown in figures 4 and 5.

It should be noted that comparison of results of strength analyses of six domes showed that a paraboloid of revolution (figure 1, *a*) and a dome with a middle surface in the form of the surface of revolution of the $z = -a\cosh(x/b)$ curve about the *Oz* axis (figure 1, e) have the better indices of the stress-strain state. These domes work almost in the momentless state and it is very well for thin-walled shell structures.

For control of the obtained results of calculation, test analyses of the shells, presented in figure 1, *a, e*, were fulfilled with the help of the standard computer program SCAD that uses a FEM. Practically identical results were derived. For instance, the orthographic representation of normal stresses σ_r and normal forces N_r for two types of the shells is shown in figures 4 and 5.

The additional information on other twelve shapes of domes of revolution is given in the papers [7; 8]. Influence of the geometrical researches of surfaces of revolution on design of unique structures was studied in a paper [20].

Conclusion

A great quantity of published works was devoted to geometrical modeling of surfaces of revolution, to determination of stress-strain state of shells of revolution subjected to static and dynamic loads, to investigation of buckling problems but in spite of it, in recent years, these shells attract attention of geometricians [15], architects [21], civil engineers [22], and machine builder [23].

Designers continue to search for optimal forms of meridians for given load using both the membrane theory [21] and the moment shell theory of the analysis [24]. They search for a meridian shape that secures the constant meridional and circular normal forces in the shell [22]. Domes on round, elliptical, oval, and rectangular plans are studied. In general, numerical methods are used for the determination of stress-strain state of the shells. For instance, in a paper [25], a finite element analysis is applied for the examination of a shell in the form of paraboloid of revolution.

The famous architect and engineer E. Torroja has told that the best erection is such one the reliability of which is ensured, mainly, by its shape but not at the expense of strength of its material [12]. Agreeing with this conclusion, the authors go on with the search of the most optimal shape of a shell of revolution the middle surface of which should be given by analytical formulae, because form finding and optimization presents contemporary design methods for shell and gridshell structures [26]. The simplified method of selection of optimal dome can be called “minimum normal stresses in shells of revolution with the same overall dimensions, boundary conditions, and external load”.

It should be repeated that under results of this new research the shape of a paraboloid of revolution (figure 1, a) and a dome with a middle surface in the form of the surface of revolution of the $z = -a\cosh(x/b)$ curve about the *Oz* axis (figure 1, *e*) have the most optimal shape from six selected domes with the given geometrical parameters which subjected to axisymmetric uniformly distributed constant surface load acting in the direction of an axis of rotation of the meridians. These two types of meridians have the rather like form. Probably, these meridians are the most rational curves with rational distribution of curvatures.

The obtained results of calculations show the conflicting properties of shells of alternating total curvature (figure 1, *c, d*). Being geometrically invariable structures, they show features peculiar to geometrically variable shells [27], i.e. they have low rigidity (a small flexible stiffness) and heightened part of factors of bending (figure 3, *c, d*; the curves of M_r). A momentless shell theory is unacceptable for such shells.

Another approach was used in a work [28] where a dome is analyzed for different external loads and for different supports. As a result, optimal type of loading of a considered dome is chosen.

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НАУЧНАЯ СТАТЬЯ

Упрощенный выбор оптимальной оболочки вращения

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Аннотация

Актуальность. Архитекторы и инженеры, работающие с оболочками вращения, используют в своих проектах в основном хорошо зарекомендовавшие себя сферические оболочки, параболоиды, гиперболоиды и эллипсоиды вращения, хотя известны около сотни поверхностей вращения, которые могут быть с успехом применены в строительстве и машиностроении.

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Конфликт интересов
Авторы подтверждают, что данная статья не содержит конфликта интересов.

Методы. Рассматривается оптимизационная задача в проектировании осесимметричной оболочки, подверженной действию внешней нагрузки. Обычно решение этой проблемы заключается в нахождении формы меридиана и в распределении толщины оболочки вдоль меридиана. В статье исследуется более узкая задача, которая заключается в выборе формы оболочки вращения из нескольких известных подклассов, срединные поверхности которых могут быть заданы параметрическими уравнениями. Приводятся результаты статических расчетов куполов различной гауссовой кривизны с одинаковыми габаритными размерами на осесимметричную поверхностную распределенную нагрузку типа собственного веса. Используется вариационно-разностный метод. ***Результаты.*** Сравнительный анализ результатов расчета шести куполов показал, что с точки зрения напряженно-деформированного состояния лучшие результаты у параболоида вращения и у оболочки вращения кривой $z = -a\text{ch}(x/b)$ вокруг оси Oz. Эти оболочки работают почти в безмоментном состоянии, к чему стремятся проектировщики тонкостенных оболочечных структур. Предложенный критерий оптимальности предлагается назвать «минимальные нормальные напряжения в оболочках вращения с одинаковыми базовыми размерами, граничными условиями и внешними нагрузками».

Ключевые слова: купол; оболочка вращения; параболоид вращения; параболоид вращения четвертого порядка; цепная линия; вариационно-разностный метод расчета; линейная теория оболочек; геометрическое моделирование; оптимальное проектирование

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