Analytical and finite element modeling in the calculation and design of reinforcements of stretched elements by fiber-reinforced polymers based on high-strength fiber using adhesive joints

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(received: September 10, 2018; accepted: October 22, 2018)

Abstract. Subject. Analysis of applicability and effectiveness of various complexity level models in design of reinforcement of stretch elements by gluing on their surface high-strength fiber reinforced polymers (HSFRP).

Research objectives. Determine the necessary level of complexity of the calculation model based on the comparison of calculation results obtained on models of various complexity within the elastic behavior of the reinforced element and analysis of features of its elastoplastic behavior in case of its overload.

Materials and methods. Few relatively simple variants of HSFRP-reinforcement structures with application of four Finite Element Method (FEM) simulation models of varying complexity and an analytic approach. Plane and spatial Finite Element (FE) models with PC LIRA (SCAD) and FEMAP (NASTRAN) apply in considered series of numerical experiments. Comparative analysis of results of elastic FEM calculation based on various FE models with the results obtained using analytical expressions. A number of diagrams and tables represent the results of calculations. Nonlinear FEM analysis reveals some features of the reinforced elements response under extreme loads.

Results. The effect of various factors on the bonded joint behavior observed, the equations and formulae for the analysis and design are applied, the analytical approach based numerical results well correspond with those obtained using FEM. A number of nonlinear FEM calculations discover some features of elastic-plastic response of joints.

Conclusions. All the considered here FE models within the limits of elastic design are quite compatible mutually and with an approximate analytical approach as well. The least time- and effort-expensive for the stage of preliminary assessment of the various parameters effect on the glued joint behavior in the elastic design of the stretched elements reinforcement is an analytical approach allowing instantaneously obtain the resulting main components of stresses and forces in the components of joint to scroll through parameter values. FEM simulation for elastic calculation is expedient for verification of results. The simplified plain FEM simulation seems to be quite reliable here. In inelastic state of the reinforced element material yet, the features of its stress-strain distribution not observable in the elastic stage of its loading and requiring special attention and refined FEM simulation may dominate.

Keywords: bonded connection, adhesive layer, shear strength, shear modulus, analytical solution, FEM

\textbf{General}

HSFRP-reinforcement of steel elements (by high-strength fiber reinforced polymeric materials) using the adhesive and combined joints can radically simplify and accelerate the technology of reinforcement operation for steel structures in many cases without interrupting the process of their normal service.

The implementation of HSFRP-reinforcement with adhesive joints involves the reinforced element surface preparation for gluing. Allowing for the real dimensions of building structures, an important parameter is the area of gluing contact surfaces value
that may the dimensions of glue layer dictate. In case of rod elements reinforcement, the rational length of the bonding area may be dominant parameter, since the shear stress distribution in the adhesive layer may be extremely uneven along its length. This is especially true for adhesive joints when reinforcing stretched elements. Surface preparation, as the foreign experience of strengthening steel elements of bridge structures shows, is the most labor-intensive taking a lot of time operation of the reinforcement process.

Carbon high-strength fibers, having a tensile strength many times exceeding the ultimate tensile strength, have a modulus of elasticity close to the elastic modulus of the metal of the reinforced elements. However, the total cross-section of high-strength fibers in the reinforcement elements is very insignificant in comparison with the cross-sectional dimensions of the reinforced elements of building structures, and therefore such reinforcement elements can be significantly included in the work of the reinforced element only under pre-tension, which in case of glue joints is highly problematic. Yet, with significant plastic deformations, the polymer elements can be actively involved in the work of the main element, redistributing a significant part of the acting force, while remaining in the elastic state, thus preventing the transition of local damage of elements in the global destruction of the structure.

Allowing for the noted problems, one of its solution factors is the development of a technique for analyzing the polymer reinforcement structure response at the stages of preliminary assignment of parameter values and evaluation of the so obtained layout solution (search through the permissible parameter values for the joint components) and at the stage of taking the final constructive solution. At the first stage, it is reasonable to use the least labor-intensive and time saving FEM and analytical elastic models. The second stage realization may need more accurate calculation methods based on nonlinear models.

**Publications review**

Along with traditional methods of reinforcing elements of steel structures using welded and bolted joints, the use of high-strength fiber-reinforced polymer materials (for example, carbon fiber reinforced polymers (CFRP)) can be quite effective, providing, with an insignificant increase in the weight of the structure, that there is no weakening of cross sections, additional stress concentrators or additional potential areas of corrosion, as well as a less labor intensive reinforcement process than traditional ones.

In [1] the authors consider the application of carbon polymers in building structures. The [2] and [3] present the recently obtained results of experimental studies of the behavior of adhesive joints in the strengthening of damaged steel elements working in tension. The work [4] considered principal aspects of ensuring the structure reliability level control by incorporating reserve (intercepting) elements that do not take on the loads in normal service conditions but become active in emergencies and start working as bearing in the overall structure at load level when some elements lose their bearing capacity. Polymer materials, due to their unique properties, are ideal for such applications in stretch zones.

Steel reinforcement elements increase the weight of structures, are prone to corrosion, and lead to the forming of additional zones of stress concentration in the base metal resulting from welding or drilling [5]. Welding may involve additional problems of quality control of welded joints, welding in hard-to-reach places, welding residual stresses, cracking in welded zones subject to heating, and a significant decrease in fatigue strength [6].

Welding in an explosive atmosphere is possible only with long breaks in normal operation of the facility and strict safety measures, which significantly increases the cost of repairs [7].

Polymers have high strength and wear resistance, as well as resistance to aggressive environment. The weight of polymer reinforcement system is several times lower than that of steel of similar reinforcement [8]. At the same time, the resistance of a high-strength fiber to stretching can be many times higher than that of steel, and the modulus of elasticity may be close to or even much higher than that of steel [9]. In [10–18], various aspects of use of polymers in the repair and strengthening of building structures for improving fatigue strength, stability with the use of adhesive and mechanical connections are considered.

**Materials and methods**

The use of adhesive joints in bearing structures, for all its attractiveness due to a number of features, causes plausible doubts in designers, as a little-studied field.

For the confident use of glued joints, the condition is the reflection in the normative documents of the requirements for the physical and mechanical properties of the materials used and rules and recommendations for design of glue joints. The performance of the bearing glue joints that the application of existing and/or development of new adhesive
compositions must provide should be regulated as well.

The analysis of various parameters and properties of the adhesive layer effect on the adhesive joint response and the structure reinforcement effect on structure resistance forecast is possible on the basis of a comparison of the results of numerical and physical experiments with approximate calculations on the basis of analytic relations allowing at relatively little time and effort to consider any required number of different combinations of joint parameters, especially when planning an experiment.

For numerical realization and comparison of the results, let consider a steel strip 10 mm width and 6 mm thick. For a strip of greater width, say 100 mm, nothing fundamentally changes, except for some uneven stress distribution over its width. The thickness of the element increase leads to increase in the length of the gluing according to increase of the element limit load. The assumed reinforcement length of 100 mm is convenient for FE modeling and may be quite realistic for local compensation for “point” damage. The reinforcement length increase, as will be discussed below, increases the shear stress in the adhesive layer and the longitudinal force in the CFRP. Nevertheless, one can adequately project the presented numerical results onto the symmetric reinforcement of any cross section shape element under axial tension.

The main attention concentrates, first, to a comparative analysis of the calculation results based on FE models of different complexity level and, respectively, time and resource expensive, as well as using analytical expressions. The effect of the length of the reinforcement section and the thickness of the adhesive layer is also considered. The second issue is the reinforcement assembly response to the tension force, much higher than the plastic cross-section tension resistance in the reinforcement region, with the steel transition to the elastoplastic stage of the material response.

Five (four FE and analytic) models of the CFRP reinforcement joint:

1. the steel strip and reinforcement tape are both modeled by rod elements with a section of 10×3 and 10×1 mm, respectively (figure 1, model 1);
2. the steel strip and CFRP reinforcing tape are modeled by flat shell elements 10 mm thick oriented in the plane of the screen (figure 1, model 2);
3. the steel strip and CFRP reinforcing tapes are modeled by flat shell elements 10 mm thick, oriented along the normal to the screen plane (figure 1, model 3);
4. steel and CFRP tape are modeled by 3-dimensional solid type FE (1×1×1 mm);
5. analytical expressions.

The equilibrium conditions of the adhesion layer element of glued joint lead to the differential equation in the form (1)

\[ \tau'' - \beta^2 \tau = 0, \]

(1)

\[ \beta^2 \equiv \frac{G_a b_o}{t_a}, \left( \frac{1}{A_p A_f} + \frac{1}{A_x A_s} \right), \]

(2)

where \( \tau \) – shear stress in the adhesive layer; \( G_a \) – adhesive shear modulus, \( R_o \) – shear strength; \( t_a \) – adhesive layer thickness; \( b_o \) – width of the adhesive layer and CFRP tape; \( d \) – length of the adhesive layer on one side of the joint; \( E_s \) – elasticity modulus of the tension element material; \( A_s \) – cross-section area of tension element; \( E_p \) – elasticity modulus of the CFRP material; \( A_p \) – cross-section area of CFRP material.

The solution of this equation follows

\[ \tau(x) = P \left( B_{11} \sinh(\beta x) + B_{21} \cosh(\beta x) \right), \]

(3)

Due to the double symmetry of the joint, its upper left quarter with corresponding boundary conditions is considered. At the free end of the steel strip, an axial force of 10 kN is applied. Allowing for the adhesive layer analysis features, it is simulated in 1 and 2 models by flat elements (in the screen plane), and in 3 and 4 models by 3-dimensional solid type FE (1×1×1 mm).

Fragments with glue layer 1 mm depth. Top line on the model 1 – CFRP, bottom – steel with applied 10 kN tension force on the free end (10 mm long), the bold dark nodes have no in plane horizontal constraints. The opposite ends (50 mm from the end of the glue layer) of steel and polymer are both fixed.

Here the choice of FE type for modeling is obvious in figures 3 and 4. Also the figure 4 represents the bonded joint structure (1 mm node step).

Model 5 – analytic.
The tension force in the CFRP reinforcement tape at the gluing length may be defined from

\[ N_p(x) = P \frac{b_p}{\beta} \left( (B_{11} + B_{21}) \right) \]

The x coordinate is measured from the left end of the gluing length on the left side of connection.

From the condition of limiting shear stresses in the adhesive, it is easy to obtain a constraint on the applied force \( P \).

\[ \tau(x) = P \cdot [B_{11} + B_{21}] \leq R_a. \]

Results

Joint fragment model parameters:
- strip 60 mm long, steel (in nonlinear design later will be C255 from GOST RF), cross section 10×3 mm. The yield stress tension resistance for steel is 10×3×250 = 7500 N \( \approx \) 0.730 ton;
- the top (and bottom) CFRP tape 50 mm long is of 10×1 mm cross-section \( (E_p = 200 \text{ GPa}, R_{pu} = 2000 \text{ MPa}) \). Then 2000 MPa \times 10 \text{ mm}^2 = 20000 \text{ N} \approx 2 \text{ t} \) is the ultimate tension force for one tape;
- the adhesive layer length on one side of the joint is 20 mm, the total cross-section is 10×1 mm\(^2\) (the shear module \( G_s \) is taken 200 MPa (Poisson factor 0.20), the shear strength \( R_s \) supposed about 15 MPa or so;
- the tensile force applied to the steel strip is 7500 N.

The adhesive layer depth is 1 mm allowing for the conventionality of this quantity. With an increase in thickness of the glue layer, the guarantee of its strength is supposed to reduce due to an enhanced probability of internal defects occurrence. At the same time, its total compliance to shear strain, allowing reducing shear stresses in glue layer with lengthening it.

Figures 2–8 present some FE modeling results using PC LIRA, showing adequate mutual affinity.

![Figure 2. Tension and shear results in FE model 1 (figure 1)](image)

The effect of reinforcement may be estimated as 13% \((10 – 8.7) / 10\). The diagrams are shortened by cutting off their long right hand regions.

![Figure 3. FE results for model 2 (figures 1–2)](image)

The effect of reinforcement may be estimated as 13% \((10 – 8.7) / 10\). The diagrams are shortened by cutting off their long right hand parts with constant force values.

![Figure 4. FE results using model 3 (figures 1–3)](image)
Both diagrams have the right hand region cut off.

![Image of diagrams]

Figure 5. Glue layer shear stress contour plot for solid FE, MPa:
FE group along the glue layer in the middle of its width, values in the left second and third FEs are about 8.6

Model 4 (figures 1–3) type application in LIRA for the same sample and 3D FE dimensions gives very close results to those above and they are not presented here to save the paper space.

All presented above data result from using the PC LIRA-SAPR 13. Very close results supplies the using FEMAP with NASTRAN PC for the same 3D model.

![Image of diagrams]

Figure 6. FEMAP (NASTRAN) – for the same 3D model as in LIRA (on 3 tear stress – bottom curve, shear – top)

The results (MPa) in figures correspond to solid FE middle points. The FEMAP results are very close to previous.

Analytic model 5 results using eqv. (1) – (6):

For above input data the formulae give the following values: in steel 8.696 kN (290 MPa tension stress); in CFRP 1.304 kN (130 MPa tension stress); in glue 9.079 MPa to 4.531 MPa shear stress.

So, very close results may be obtained through absolutely incomparable expenses (time and effort).

These analytical expressions, though being constrained in application, if programmed using any available means (Excel, MathCad or any alike) allow to obtain immediately the above results for any number of design cases just by varying input parameters (materials, geometry, properties, applied tension load). It facilitates analyzing the effect of various parameters values and their combinations on the reinforcement structure behavior and to select the most effective combination.

As a sample one may analyze the effect of reinforcement length ($L$, mm) on the shear stress ($T_x$, MPa) in glue and tension reduction in steel (%). All the rest input parameters are as in previous samples.

Table 1

<table>
<thead>
<tr>
<th>$L_0$, mm</th>
<th>L</th>
<th>50</th>
<th>100</th>
<th>200</th>
<th>300</th>
<th>500</th>
<th>1000</th>
<th>1500</th>
<th>2000</th>
<th>3000</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>%</td>
<td>13.0</td>
<td>17.6</td>
<td>20.7</td>
<td>21.8</td>
<td>24.3</td>
<td>23.6</td>
<td>23.9</td>
<td>24.0</td>
<td>24.2</td>
</tr>
<tr>
<td></td>
<td>$T_{50}$</td>
<td>0.000</td>
<td>1.876</td>
<td>2.483</td>
<td>2.655</td>
<td>2.784</td>
<td>2.875</td>
<td>2.905</td>
<td>2.920</td>
<td>2.934</td>
</tr>
<tr>
<td></td>
<td>%</td>
<td>16.7</td>
<td>21.6</td>
<td>23.2</td>
<td>23.6</td>
<td>24.0</td>
<td>24.2</td>
<td>23.3</td>
<td>24.3</td>
<td>24.4</td>
</tr>
<tr>
<td></td>
<td>$T_{100}$</td>
<td>0.000</td>
<td>0.367</td>
<td>0.412</td>
<td>0.438</td>
<td>0.454</td>
<td>0.459</td>
<td>0.462</td>
<td>0.464</td>
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<tr>
<td></td>
<td>%</td>
<td>23.2</td>
<td>24.2</td>
<td>24.3</td>
<td>24.4</td>
<td>24.4</td>
<td>24.4</td>
<td>24.4</td>
<td>24.4</td>
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</table>
These values correspond to applied tension 10 kN. Elastic calculation mode.

These data are the result of model 5 (analytic) approach numerical realization tested partially on FE models approaches and allow concluding in particular that the rational glue layer length value is around 100–150 mm if possible.

The applied load is much greater than the reinforced part of steel element tension resistance.

The CFRP reinforcement task here is to provide the structure resistance even if it cannot be provided for the steel element.

Let the strip is twice as wide outside reinforcement length as inside. The applied load is twice higher than needed to reach the yield stress in the reinforced steel part. The CFRP must compensate and provide structure resistance.

<table>
<thead>
<tr>
<th>$L_{gl}$, mm</th>
<th>$L$</th>
<th>50</th>
<th>100</th>
<th>200</th>
<th>300</th>
<th>500</th>
<th>1000</th>
<th>1500</th>
<th>2000</th>
<th>3000</th>
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<tbody>
<tr>
<td>50</td>
<td>$T_0$</td>
<td>6.33</td>
<td>6.77</td>
<td>6.92</td>
<td>6.96</td>
<td>6.99</td>
<td>7.011</td>
<td>7.02</td>
<td>7.02</td>
<td>7.03</td>
</tr>
<tr>
<td></td>
<td>$T_{50}$</td>
<td>0.00</td>
<td>1.41</td>
<td>1.86</td>
<td>1.99</td>
<td>2.09</td>
<td>2.16</td>
<td>2.18</td>
<td>2.19</td>
<td>2.20</td>
</tr>
<tr>
<td></td>
<td>$%$</td>
<td>16.7</td>
<td>21.6</td>
<td>23.6</td>
<td>24.0</td>
<td>24.0</td>
<td>24.0</td>
<td>24.3</td>
<td>24.3</td>
<td>24.4</td>
</tr>
<tr>
<td>100</td>
<td>$T_0$</td>
<td>–</td>
<td>6.66</td>
<td>6.68</td>
<td>6.69</td>
<td>6.68</td>
<td>6.63</td>
<td>6.68</td>
<td>6.68</td>
<td>6.68</td>
</tr>
<tr>
<td></td>
<td>$T_{100}$</td>
<td>–</td>
<td>0.00</td>
<td>0.28</td>
<td>0.31</td>
<td>0.33</td>
<td>0.34</td>
<td>0.34</td>
<td>0.35</td>
<td>0.35</td>
</tr>
<tr>
<td></td>
<td>$%$</td>
<td>–</td>
<td>23.2</td>
<td>24.2</td>
<td>24.3</td>
<td>24.4</td>
<td>24.4</td>
<td>24.4</td>
<td>24.4</td>
<td>24.4</td>
</tr>
<tr>
<td>150</td>
<td>$T_0$</td>
<td>–</td>
<td>–</td>
<td>6.67</td>
<td>6.67</td>
<td>6.67</td>
<td>6.67</td>
<td>6.67</td>
<td>6.67</td>
<td>6.67</td>
</tr>
<tr>
<td></td>
<td>$T_{150}$</td>
<td>–</td>
<td>–</td>
<td>0.04</td>
<td>0.05</td>
<td>0.05</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
</tr>
<tr>
<td></td>
<td>$%$</td>
<td>–</td>
<td>–</td>
<td>24.4</td>
<td>24.4</td>
<td>24.4</td>
<td>24.4</td>
<td>24.4</td>
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<td>24.4</td>
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</table>

These results show that here the most effective glue layer length value is 150 mm where possible.

In nonlinear analysis below after a number of tests, the tension forces $1.6 \times 7.5 = 12.24$ kN applies. Divided by steel section it gives $12240 / (10 \times 3) = 408$ MPa. It is much higher than yield and just a little lower than ultimate stress. It may be a kind of overload or on the contrary damaged steel element (cross-section reduction). Anyway some extraordinary situation is meant and the CFRP reinforcement must save the situation.

The key factor here is obviously the strength of glue layer translating the exhaust tension portion from steel to polymer reinforcement. Assuming carbon fiber total section $10 \times 1$ mm and ultimate stress 2000 MPa it must bear $20000 N = 20$ kN = 2 ton. Therefore, the plasticity or even breakdown of steel in this case doesn’t mean the total reinforcement structure collapse.

Sample in PC FEMAP (Nastran): reinforcement (half-length) 200, glue $60 \times 10 \times 1$, steel S250 (diagram from SP 16.13330.2017, Application, $f_y = 255$ MPa), steel strip quarter cross section $10 \times 3$, corresponding load 7500 H ($7500 / 30 = 250$ MPa).

| Longitudinal coordinate $x = 0$ is assumed at the left end of CFRP (and glue layer). Left end of steel element $-x = -10$ mm and the right $x = 200$ mm | 419 |

2. Glue tear (top) & shear (bottom) stress. Shear: –6.282 (left) and –1.232 (right), tear: 3.213

Figure 9. Model 4 (in NASTRAN) results. Load 7500 kN, results (stress) – MPa

Compare with table 2 (50–200).

<table>
<thead>
<tr>
<th>Sample ( L_{gl} ) = 200 mm</th>
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</table>
| 1 – steel (\( \sigma_s = 284.5 \)) | 2 – CFRP (\( \sigma_p = 370.5 \)) | 3 – glue (tear \( \sigma_{gl,z} \) = 4.638, shear \( \tau_{gl,xy} \) = 8.767 ÷ 0.4)
| \( \sigma_s \) – tension stress in steel (reduced); \( \sigma_p \) – tension stress in CFRP (same section); \( \sigma_{gl,z} \) – glue vertical tear stress (top) |

In effect this is just 1/4 of total symmetric reinforcement structure: 420 mm steel, 400 mm CFRP, 20 mm width in the middle part and 34 mm at the ends (10 + 50 = 60 mm).

Let consider the results for load level \( 255 \times 30 \times 1.6 = 7650 \times 1.6 = 12240 \) kN for a number of \( L_{gl} \) values.

All parameters are the same but glue layer is 200 mm long. Therefore, it has no gap on the total length.

Sample \( L_{gl} \) = 150 mm

1. Steel tension stress 190 MPa – reduced

2. CFRP tension stress 179 MPa – max.

Figure 10. (Cont.) Model 4 (in NASTRAN) results

Reinforcement effect: \( 187 \times 30 = 5610 \) N, \( 178.9 \times 10 = 1789 \) N, total nearly 7500 N, \( 1789/7500 \Rightarrow 23.85\% \). Compare with table 2 (50–200).

So the model 5 (analytic), model 2 (LIRA) and model 4 (NASTRAN) provide close enough results.

In the next series of samples, the glue layer length \( L_{gl} \) is 200, 150, 100, 50, 40, 30, 20 mm and the steel strip width varies: for \( x = –10 \) to \( x_1 = 0 \) is taken \( b = 17 \) mm, \( x_2 = 50 \) to 200 mm – \( b = 10 \) mm.

Sample \( L_{gl} \) = 150 mm

1. Steel tension stress 190 MPa – reduced

2. CFRP tension stress 179 MPa – max.

Figure 11. Stress distribution in the joint components (\( L_{gl} = 200, x_2 = 50 \)):

3. Figure 12. Stress distribution in the joint components (\( L_{gl} = 200, x_2 = 50 \)):

1 – steel (\( \sigma_s = 283.6 \)); 2 – CFRP (\( \sigma_p = 372.5 \)); 3 – glue (tear \( \sigma_{gl,z} \) = 4.638, shear \( \tau_{gl,xy} \) = 8.767 ÷ 0.0)

Sample $L_{gl} = 100$ mm

Figure 13. Stress distribution in the joint components ($L_{gl} = 100, x_2 = 50$):

1. steel ($\sigma_s = 284.2$);
2. CFRP ($\sigma_p = 371.3$);
3. glue (tear $\sigma_{gl}, z = 4.64$, shear $\tau_{gl,xy} = 8.77 \pm 0.46$)

Sample $L_{gl} = 75$ mm

Figure 14. Stress distribution in the joint components ($L_{gl} = 75, x_2 = 50$):

1. steel ($\sigma_s = 285.4$);
2. CFRP ($\sigma_p = 367.7$);
3. glue (tear $\sigma_{gl}, z = 4.66$, shear $\tau_{gl,xy} = 8.803 \pm 0.71$)

Sample $L_{gl} = 50$ mm

Figure 15. Stress distribution in the joint components ($L_{gl} = 50, x_2 = 50$):

1. steel ($\sigma_s = 290$);
2. CFRP ($\sigma_p = 356.6$);
3. glue (tear $\sigma_{gl}, z = 4.943$, shear $\tau_{gl,xy} = 9.308 \pm 5.757$)

Sample $L_{gl} = 40$ mm

Figure 16. Stress distribution in the joint components ($L_{gl} = 40, x_2 = 50$):

1. steel ($\sigma_s = 292$);
2. CFRP ($\sigma_p = 348.5$);
3. glue (tear $\sigma_{gl}, z = 5.445$, shear $\tau_{gl,xy} = 10.19 \div 7.987$)

Sample $L_{gl} = 30$ mm

Figure 17. Stress distribution in the joint components ($L_{gl} = 30, x_2 = 50$):

1. steel ($\sigma_s = 296$);
2. CFRP ($\sigma_p = 338$);
3. glue (tear $\sigma_{gl}, z = 6.558$, shear $\tau_{gl,xy} = 12.12 \div 10.74$)
Sample $L_{gl} = 20$

![Figure 18. Stress distribution in the joint components ($L_{gl} = 20, x_2 = 50$):](image)

1. steel ($\sigma = 300$); 2. CFRP ($\sigma_p = 324$); 3. glue (tear $\sigma_{gl,z} = 8.964$, shear $\tau_{gl,x,y} = 16.27 \div 15.62$)

In figures 13–16 (1) there are splashes ($\sigma \approx 305$ MPa) at the point $x_2 = 50$ where the steel width $b_s$ becomes equal 10 mm. It results from the discontinuity of second order derivative of edge of steel strip curve just at the points $x_1 = 0$ and $x_2 = 50$ mm. However, the splash is much less than ultimate stress of steel.

Below are a few sample results for the case. The splash is again at the point of the jump point of second derivative of the steel plate width $x_2 = 100$ mm. For cases where $L_{gl} \leq x_2$ there are no splashes on tension diagrams.

![Figure 19. Stress distribution in the joint components ($L_{gl} = 200, x_2 = 100$):](image)

1. steel ($\sigma = 285$); 2. CFRP ($\sigma_p = 369.2$); 3. glue (tear $\sigma_{gl,z} = 3.453$, shear $\tau_{gl,x,y} = 6.662 \div 0.421$)

The splash moved to $x = x_2 = 100$ mm, where now the starts $b_s = 10$ mm.

![Figure 20. Stress distribution in the joint components ($L_{gl} = 150, x_2 = 100$):](image)

1. steel ($\sigma = 285$); 2. CFRP ($\sigma_p = 370.4$); 3. glue (tear $\sigma_{gl,z} = 3.453$, shear $\tau_{gl,x,y} = 6.662 \div 0.235$)

Next case is $L_{gl} = x_2 = 100$ mm.

![Figure 21. Stress distribution in the joint components ($L_{gl} = 100, x_2 = 100$):](image)

1. steel ($\sigma = 289$); 2. CFRP ($\sigma_p = 357$); 3. glue (tear $\sigma_{gl,z} = 3.487$, shear $\tau_{gl,x,y} = 6.723 \div 1.47$)

The splash now disappeared and it won’t appear again while $L_{gl} \leq x_2$ takes place.

At the load levels lower than plastic resistance of $10 \times 3$ cross section this effect does not work. For parameters of figure 34 (glue $150, x_2 = 100$) and load 7500 the result is in figure 36 with no splashes.
Thus, the results showed that elastic-plastic response of joint may be quite unlike the elastic one and it needs a special attention in design.

Conclusions

1. The high strength polymer glued reinforcement joint response and its efficiency determine such parameters as the glue shear modulus, strength and thickness of the adhesive layer and its length, reinforcement region length etc.

2. The FEM modeling in specialized PC with relatively acceptable accuracy is a time- and effort-consuming procedure. Therefore, at least at the preparatory stage, one should keep in mind the expediency of simplified models application to ride through the variety of the joint parameter combinations to select their initial values.

3. The analytic approach facilitates the process of varying the reinforcement joint parameters yet providing acceptable accuracy of results for further design.

4. The most rational approach seems to select the effective values of geometry and the material parameters of the reinforcement joint using analytic expressions, followed by a possible numerical FEM and, if necessary, an experimental verification.

5. The most effective application of the reinforcement structures under consideration can be in case of the danger of a material of the reinforced elements transition into an inelastic state.

6. The design of reinforcement taking into account the physical nonlinearity of the material of the reinforced elements can lead to specific results not typical for linear design.

7. Specific qualities of high-strength polymeric materials and glue joints determine the prospects for the development of effective reinforcement design variants for multi-purpose application in building structures.

References


Аналитическое и конечно-элементное моделирование при расчете и проектировании усиления растянутых элементов фиброармированными полимерами на основе высокопрочного волокна с применением клеевых соединений

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(поступила в редакцию: 10 сентября 2018 г.; принята к публикации: 12 октября 2018 г.)

Предмет исследования. Анализ применимости и эффективности применения моделей различного уровня сложности для расчета и проектирования усиления растягиваемых элементов путем наклеивания их поверхности полимеров на основе высокопрочных волокон.

Цель исследования. Определение необходимого уровня сложности расчетной модели путем сравнения полученных на моделях различной сложности численных результатов в рамках упругого поведения материалов и анализ особенностей упругопластической работы в случае повышенной нагрузки.

Материалы и методы. Рассматривается несколько относительно простых вариантов конструкции усиления высокопрочными волокнами с применением четырех конечно-элементных моделей (КЭ-моделей) различной сложности и аналитического подхода. В представленной серии численных экспериментов с применением ПК «ЛИРА» (СКАД) и FEMAP (NASTRAN) использовались двумерные и трехмерные КЭ-модели. Сравнение результатов упру-
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gого расчета различных КЭ-моделей с результатами, полученными с помощью аналитических выражений. Результаты расчета представлены в графической и табличной форме. Нелинейный анализ обнаруживает некоторые особенности поведения усиленных элементов при напряженных нагрузках.

Результаты. Рассмотрено влияние различных факторов на работу клеевого соединения, применение уравнений и формул для расчета и проектирования. Результаты на основе аналитического подхода хорошо согласуются с результатами расчета методом конечных элементов (МКЭ). Расчеты МКЭ в физически нелинейной постановке обнаруживают некоторые особенности упругопластической работы соединений.

Выводы. Все рассмотренные в статье КЭ-модели и приближенный аналитический подход в пределах упругого расчета дают близкие результаты. Наиболее экономичным по затратам усилий и времени на стадии предварительной оценки влияния различных параметров на работу узла в упругой стадии является аналитический подход. Применение МКЭ в упругой стадии целесообразно для уточнения результатов. Упрощенные плоские модели здесь достаточно надежны. Однако за пределами упругости материала усиливающего элемента проявляется некоторые особенности НДС, не наблюдаемые в упругой стадии его нагружения и требующие особого внимания и уточненного расчета МКЭ.

Ключевые слова: клеевое соединение, адгезионный слой, сдвиговая прочность, модуль сдвига, аналитическое решение, МКЭ

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Для цитирования